Behavior of Beams under Bending Moment only

خواص الكمرات تحت تأثير عزوم الانحناء فقط

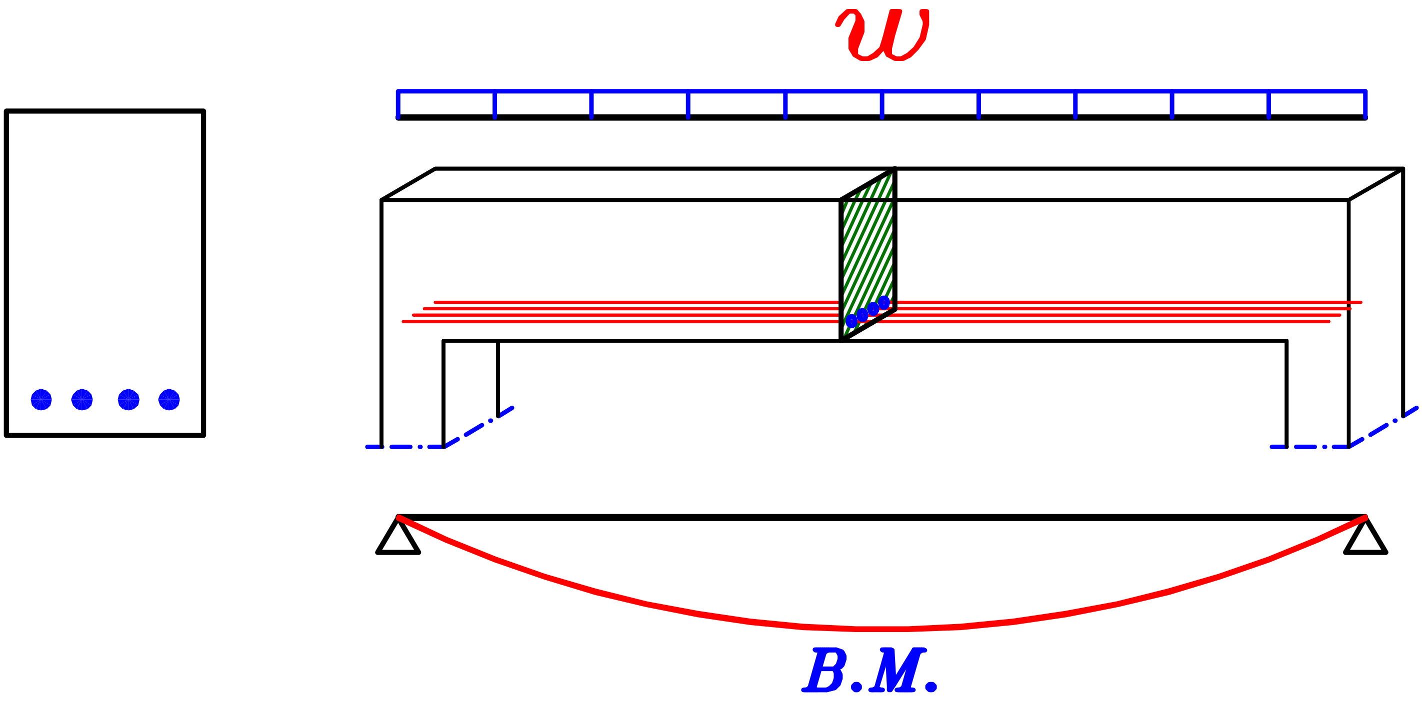
نسألكم الدعاء

IF you download the Free ***APP.*** **RC Structures** YASSER EL LEATHY *on your smart phone or tablet, you will be able to play illustrative movies For any paragraph that has a QR code icon*

اذا حملت تطبيق **RC Structures** YASSER EL LEATHY على تليفونك المحمول او اللوح السطحى ستستطيع أن تشغل أفلام شرح للمقاطع التى تحتوى على رمز

Behavior of Beams Table of Contents.

Introduction.



كثيرا ما نحتاج لحساب قوه تحمُل قطاعات الكمره للعزوم المؤثره عليها .

أى نحتاج لحساب أكبر عزوم يستطيع القطاع تحملها فى الحالات المختلفه مثل :

1- ($M_{cr.}$) Cracking Moment

($M_{cr.}$) هو العزم الذى تبدأ عنده الخرسانه من جهه الشد فى التشرخ .

2- (M_w) Working Moment

(M_w) هو أكبر عزم مسموح به للكمرات الشغاله و الذى يجعلها Just safe

و اذا عرض القطاع لعزم أكبر من (M_w) يكون unsafe فى طريقه W.S.D.M.

Working Stress Design Method

3- (M_{ult}) Ultimate Moment.

(M_{ult}) هو أكبر عزم يتحمله القطاع و اذا تعرض القطاع لعزم أكبر ينهار .

4- ($M_{U.L.}$) Ultimate Limits Moment.

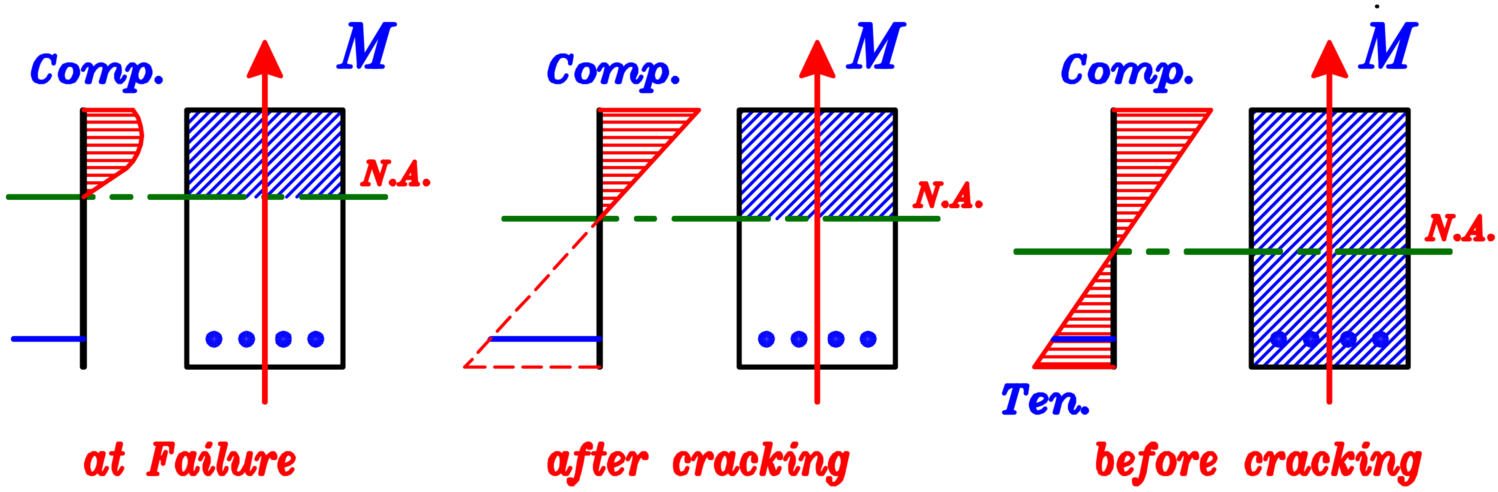
($M_{U.L.}$) هو أكبر عزم مسموح به للكمرات الشغاله و الذى يجعلها Just safe

و اذا عرض القطاع لعزم أكبر من ($M_{U.L.}$) يكون unsafe فى طريقه U.L.D.M.

Ultimate Limits Design Method

و لكى نستطيع أن نحسب العزوم التى يتحملها القطاع .
يجب أولا دراسه بعض خواص الخرسانه و الحديد المستخدمين فى القطاع .
و أيضا دراسه بعض الخواص الهندسيه للقطاع و معرفه بعض المبادئ الاساسيه للعناصر الانشائيه .

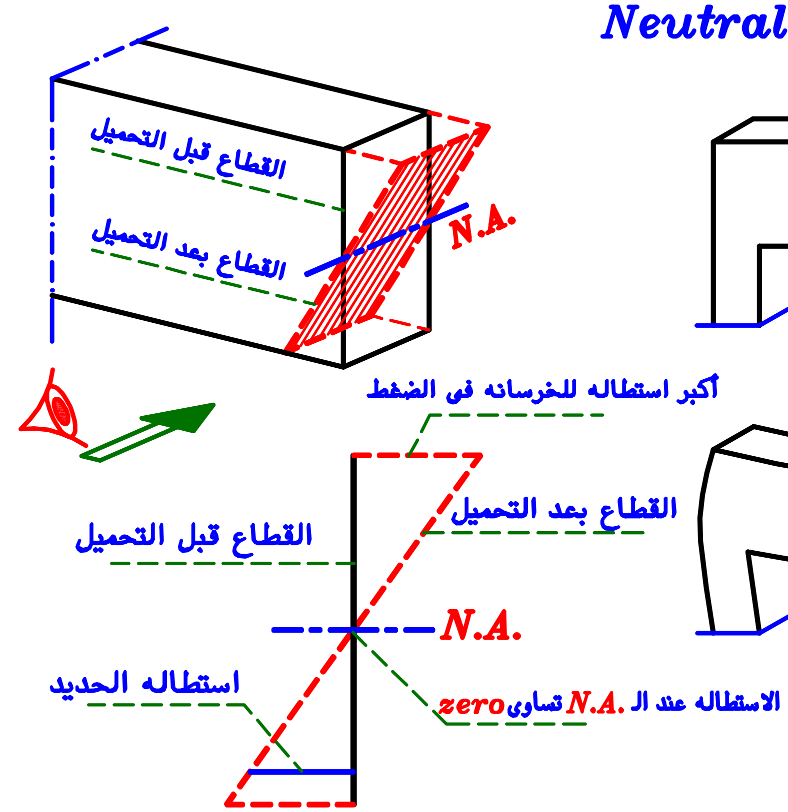
Stress Diagram For section under Bending Moment only.



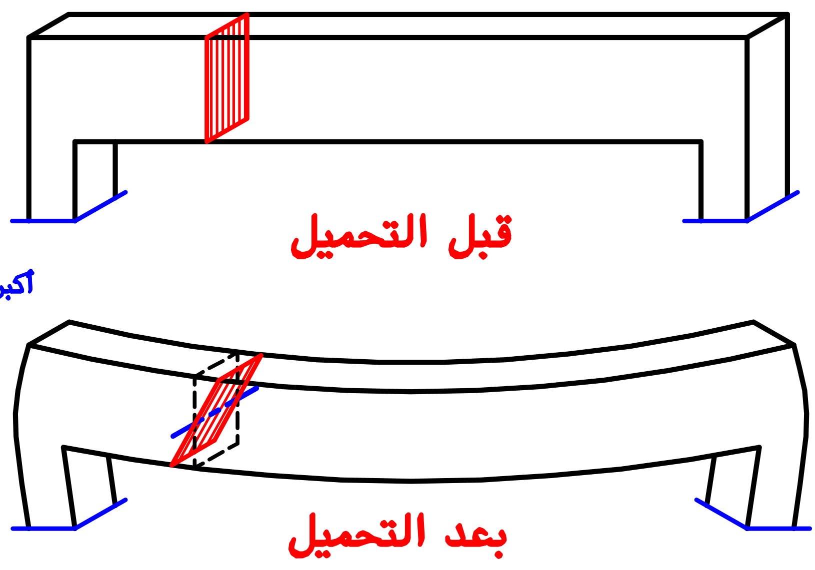
Strain Diagram For sections.

Elastic Theory.

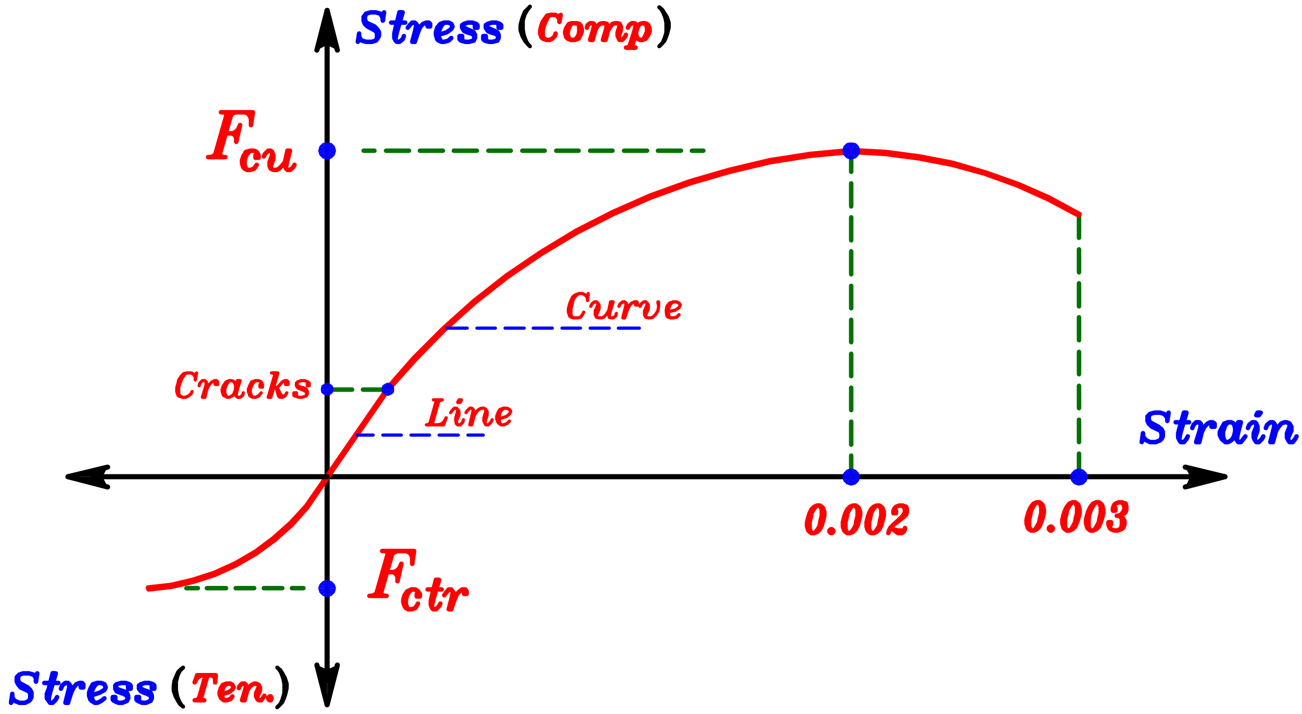
هى نظريه تعتمد على أن شكل القطاع المستوى قبل تحميل الكمره يظل مستوى بعد التحميل .
و يلف القطاع دائما حول الـ (*N.A.*) *Neutral Axis*



Strain Diagram.



Stress – Strain Curve For Concrete.



F_{cu} هى أكبر اجهاد تتحمله الخرسانه فى الضغط

و تتوقف قيمتها على تصميم الخلطه الخرسانيه .

رتبه الخرسانه							
F_{cu} ($N\backslash mm^2$)	18	20	25	30	35	40	45

F_{ctr} هى أكبر اجهاد تتحمله الخرسانه فى الشد .

واذا زاد اجهاد الشد فى الخرسانه عن هذه القيمه تحدث شروخ فى الخرسانه.

$$\boxed{F_{ctr} = 0.6\sqrt{F_{cu}}} \quad N\backslash mm^2$$

F_{ctr} (Concrete Tension Rupture)

Modulus of elasticity. (E) معاير المرونه

$$\boxed{E = \frac{stress}{strain}}$$

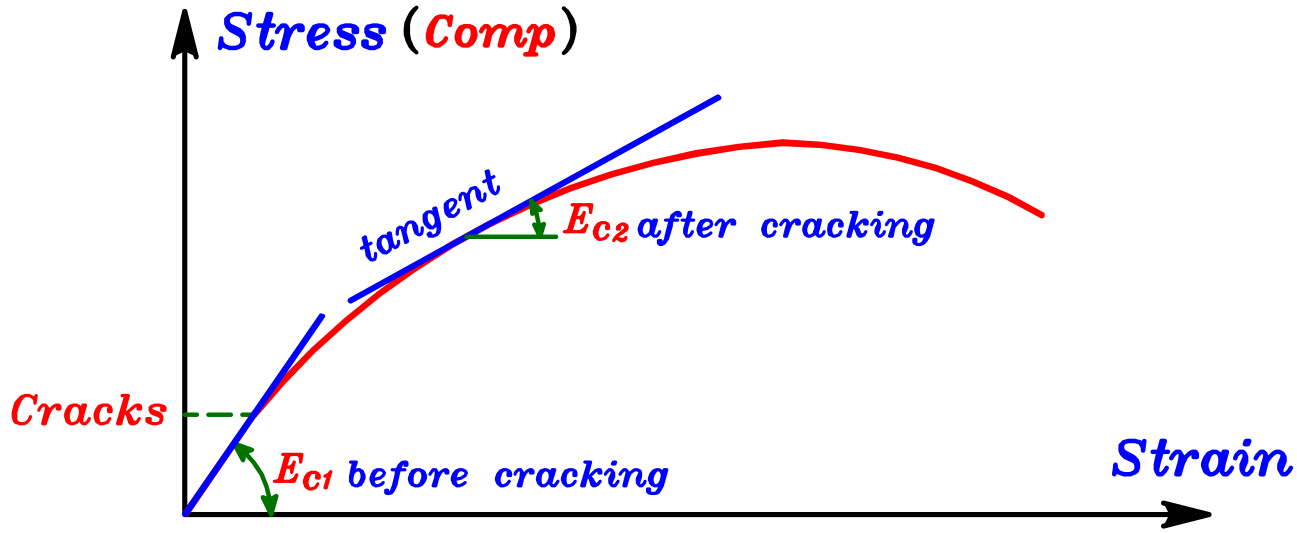
(E) أى ماده لها معاير مرونه و يسمى

و كلما زادت قيمه (E) كلما كانت الماده أصلب أى أن مرونتها أقل

و لحساب قيمه (E) سيتم استنتاج قيمتها من شكل

من شكل Stress–Strain Curve لهذه الماده .

Modulus of elasticity of Concrete. (E_c)



$$\boxed{E_{c_1} = 4400\sqrt{F_{cu}}} \quad N\backslash mm^2$$

E_{c_1} = modulus of elasticity of concrete before craking.

و هو عباره عن ميل خط الـ stress–strain curve قبل التشرخ .

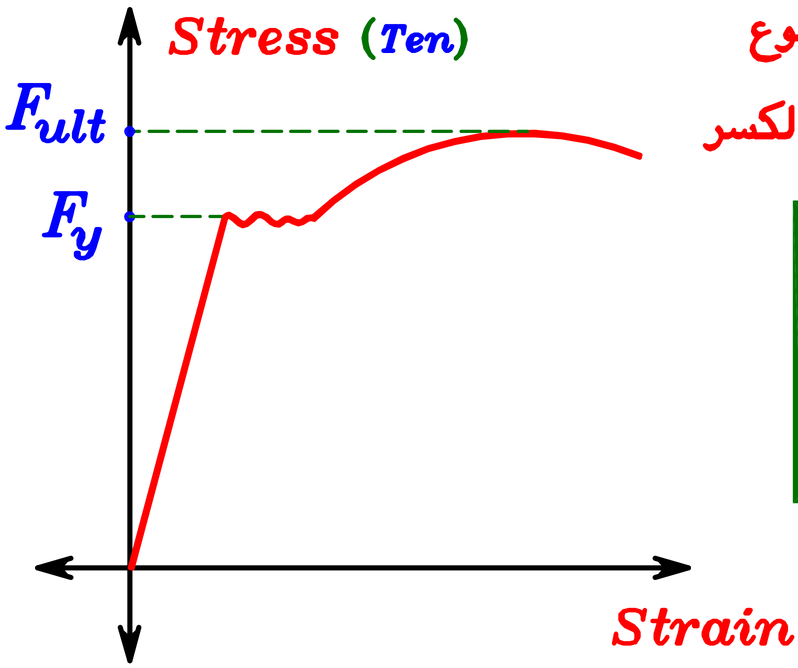
E_{c_2} = modulus of elasticity of concrete after craking.

و هو عباره عن ميل المماس لـ curve عند أى نقطه بعد التشرخ .

و لا يوجد لها معادله هى فقط ميل مماس الـ curve عند النقطه المحسوب عندها E

$$\boxed{E_{c_2} < E_{c_1}}$$

Stress–Strain Curve For Steel in Tension.



اجهاد الخضوع yield stress (F_y)

اجهاد الكسر ultimate Stress (F_{ult})

انواع الحديد المستخدم فى مصر	
Mild Steel	240\350
High strength Steel.	360\520
High strength Steel.	400\600

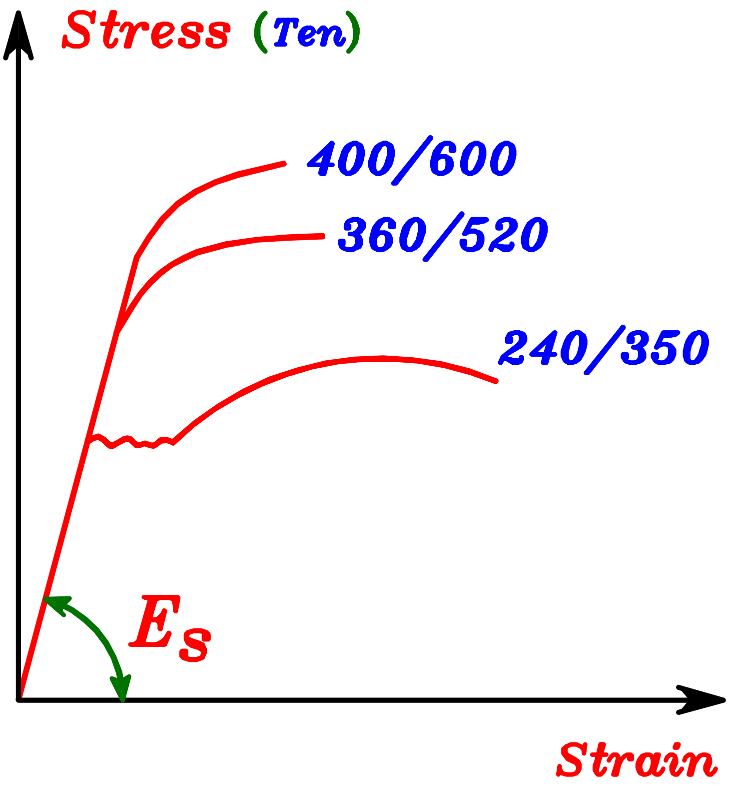
Modulus of elasticity of Steel. (E_s)

معاير مرونه الحديد

For all types of steel

$$\boxed{E_s \simeq 2 * 10^5} \quad N\backslash mm^2$$

E_s (Young's Modulus)



Modular Ratio (n)

$$\boxed{n = \frac{E_s}{E_c}}$$

$E_s = constant = 2 * 10^5\ N\backslash mm^2$

$E_{c_1} = 4400\sqrt{F_{cu}}\ N\backslash mm^2$ --- before cracking

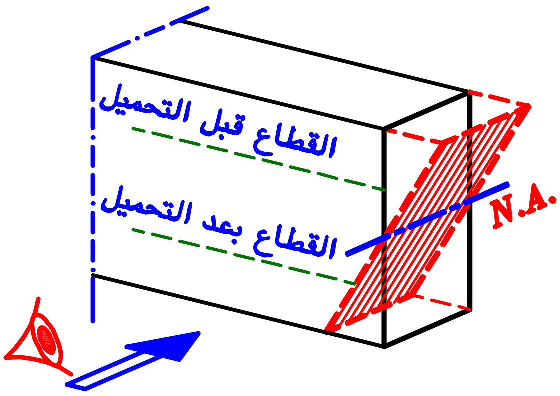
$E_{c_2} < E_{c_1}$ --------------- after cracking

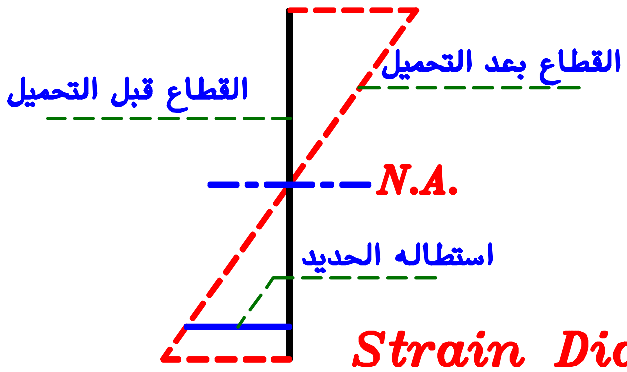
Before cracking $\quad n = \frac{E_s}{E_{c1}} = \frac{2 * 10^5}{4400\sqrt{F_{cu}}} \simeq 10$

After cracking $\quad n = \frac{E_s}{E_{c2}} \simeq 15$

و معناه إنه إذا حدث للحديد نفس الإستطاله الحادثه للخرسانه سوف يكون على الحديد إجهادات (n) مره الإجهادات الواقعه على الخرسانه.

$$n = \frac{E_s}{E_c} = \frac{(stress\backslash \cancel{strain})\ steel.}{(stress\backslash \cancel{strain})\ conc.} = 10$$



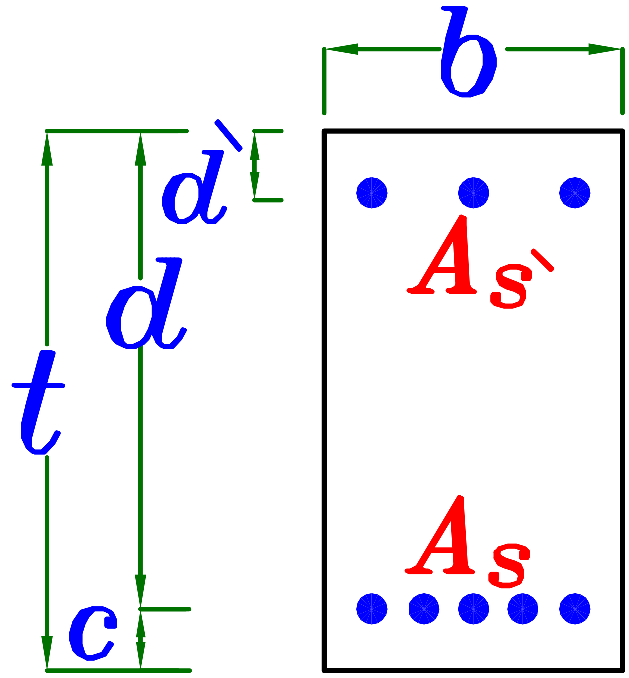


Strain Diagram.

و لأنة من المفترض أن القطاع المستوى قبل التحميل يظل مستوى بعد التحميل فهذا معناه أن الإستطاله Strain الحادثه فى الحديد هى نفس الإستطاله الحادثه فى الخرسانه الملاصقه للحديد.

و هذا معناه أن الاجهادات الواقعه على الحديد تساوى (n) مره الاجهادات الواقعه على الخرسانه الملاصقه له .

Important Symbols. رموز هامه

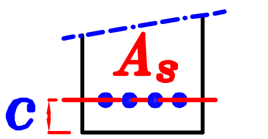


Width عرض القطاع $= b$

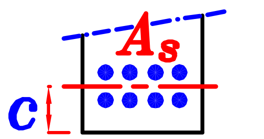
Depth عمق القطاع $= t$

Area of tension steel مساحه حديد الشد $= A_s$

Area of compression steel مساحه حديد الضغط $= A_s`$



$c \simeq 50\,mm$



$c \simeq 75\,mm$

Tension cover غطاء حديد الشد $= c$

و يحسب من *C.G.* أسياخ الحديد

$$\boxed{d = t - c}$$

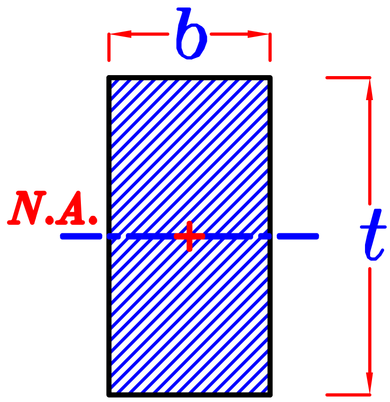
Effective depth العمق الفعلى $= d$

$d` \simeq 50\,mm$ *Compression cover* غطاء حديد الضغط $= d`$

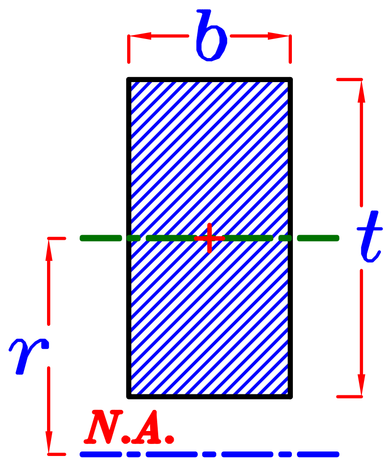
Moment of Inertia.

ملحوظه دائما نحسب الـ Inertia (I) للقطاع حول الـ (N.A.) Neutral Axis

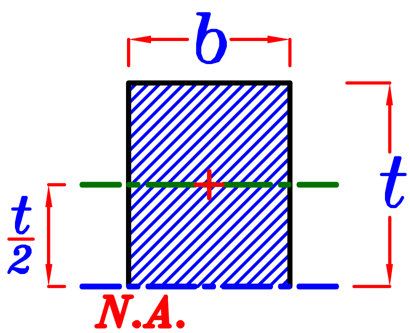
$$\boxed{I = \frac{b\,t^3}{12}}$$



$$\boxed{I = \frac{b\,t^3}{12} + (b\,t)\,(r)^2}$$



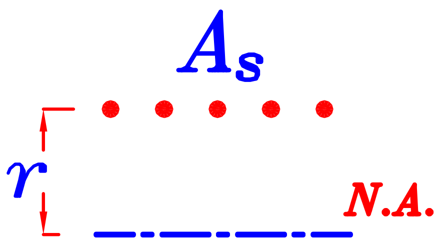
$$\boxed{I = \frac{b\,t^3}{3}}$$



For Steel Bars.

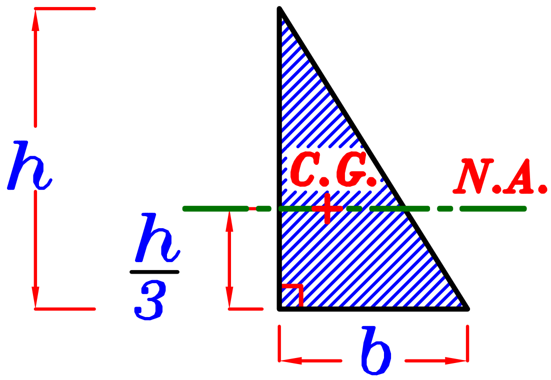
نعمل الـ I حول الـ C.G. أسياخ الحديد و نأخذ فقط نقل المحاور

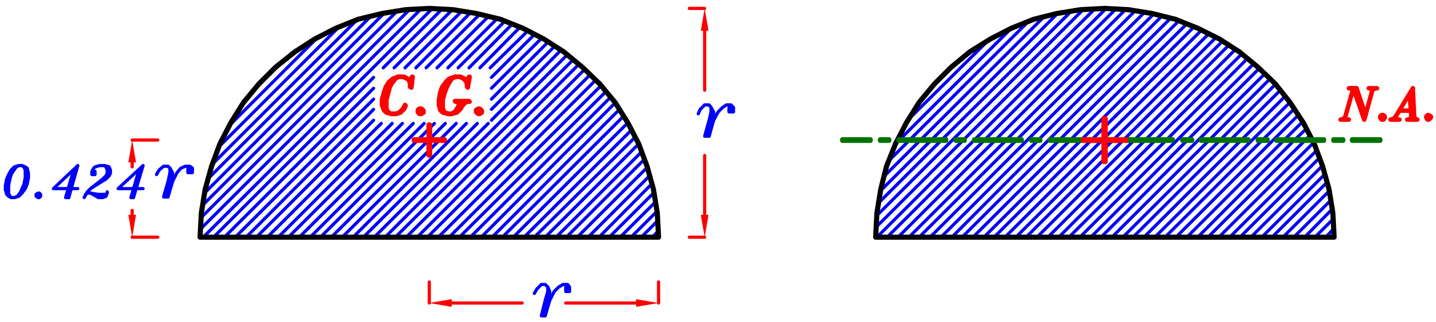
$$\boxed{I = A_s\,(r)^2}$$



Special Cases.

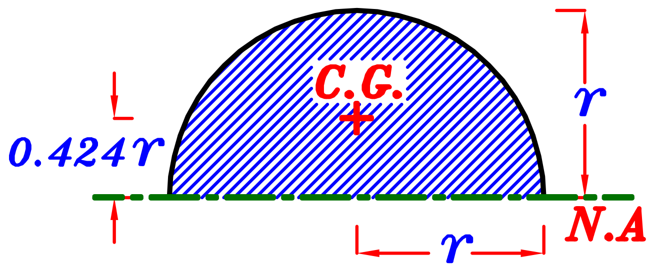
$$I_x = \frac{b\,h^3}{36}$$

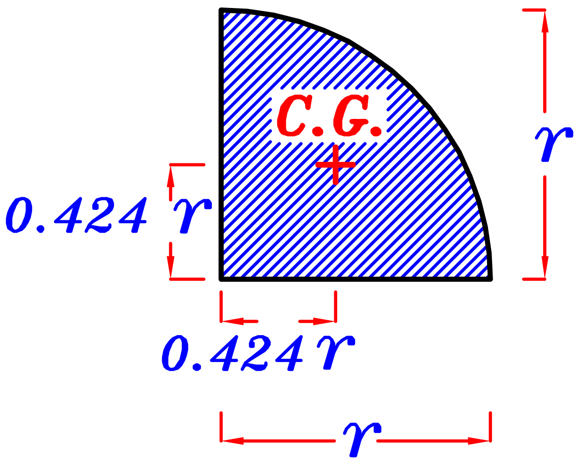


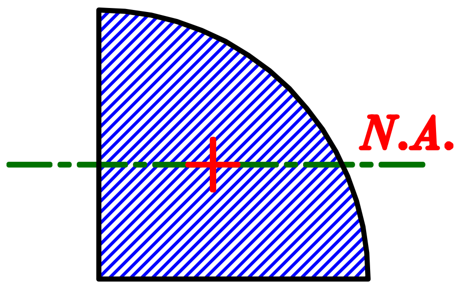


$$I = 0.11\,r^4$$

$$I = 0.11\,r^4 + \left(\frac{\pi r^2}{2}\right)\left(0.424\,r\right)^2$$







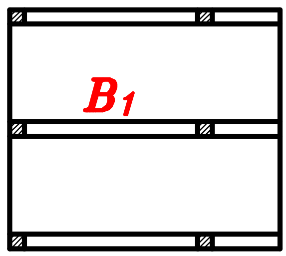
$$I_x = 0.055\,r^4$$

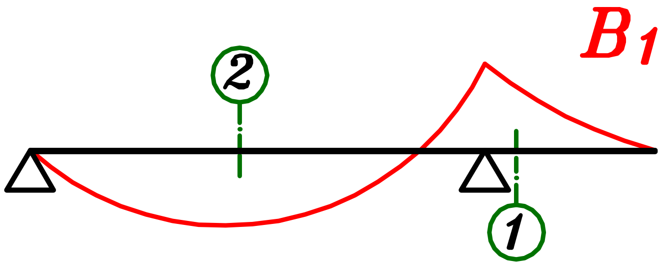
Types of Sections.

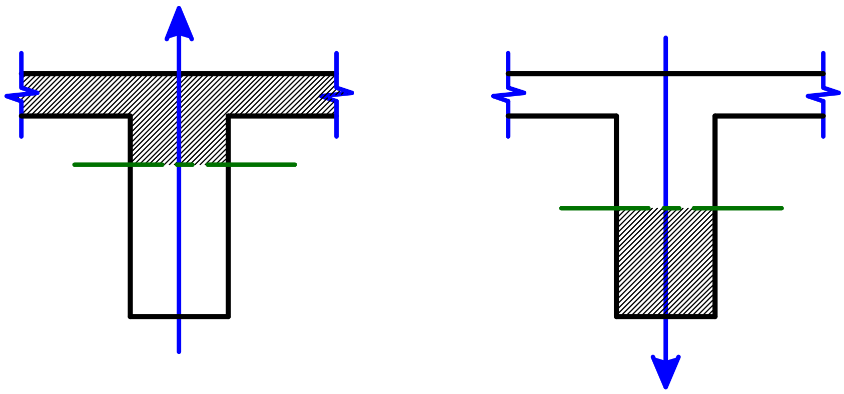
R–Sec. , T–Sec. , L–Sec.

ⓐ Intermediate Beam. كمره وسطيه (أى أن البلاطه من الإتجاهين)

plan





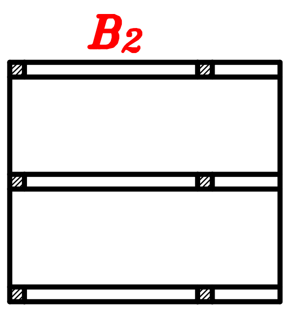


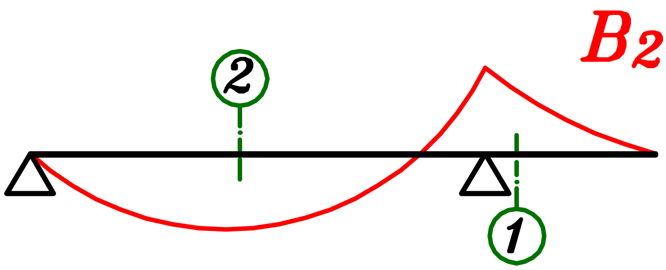
Sec. (2–2) — T – section

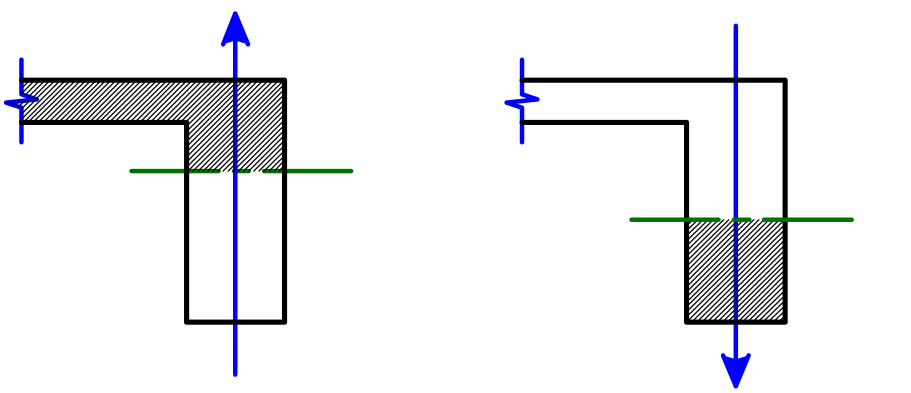
Sec. (1–1) — R – section

ⓑ Edge Beam. كمره طرفيه (أى أن البلاطه من جهه واحده)

plan







Sec. (2–2) — L – section

Sec. (1–1) — R – section

Virtual Section. القطاع التخيلى

لحساب ال *Inertia* (I) لقطاع بالقوانين السابقه

يجب أن يكون القطاع متجانس (*homogeneous section*) أى يتكون من ماده واحده فقط

أما اذا كان القطاع غير متجانس (*heterogeneous section*) أى يتكون من أكثر من ماده

فيجب عمل حل تخيلى و هو بأفتراض أن القطاع يتكون من ماده واحده فقط و هى الخرسانه

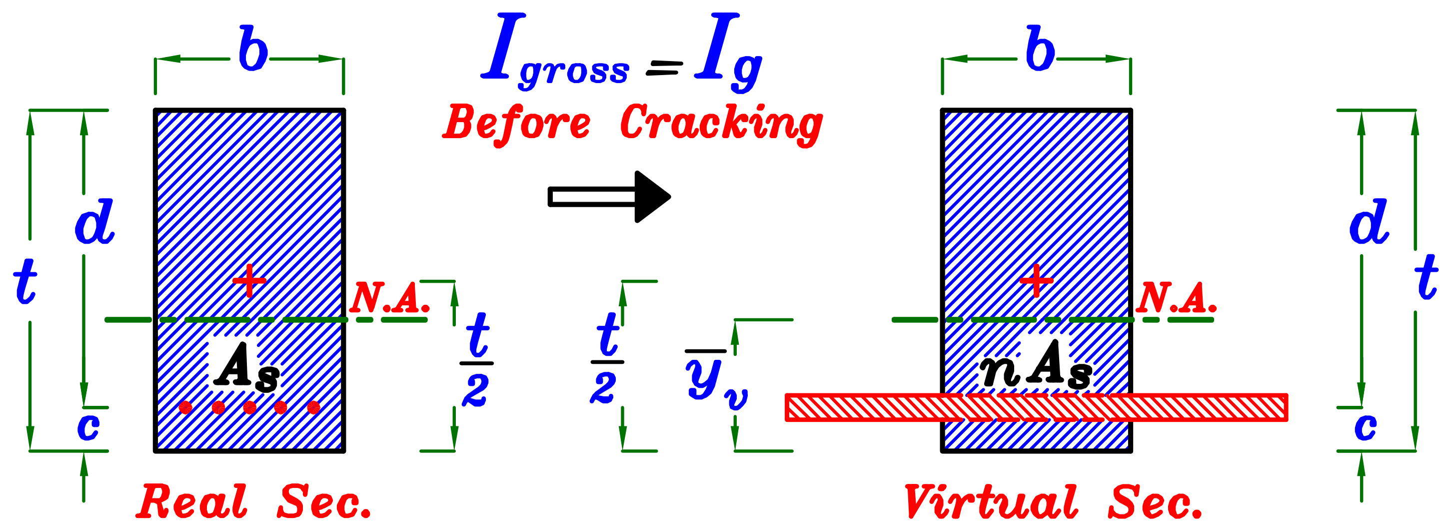
و لان الاجهادات الواقعه على الحديد تساوى (n) مره الاجهادات الواقعه على الخرسانه الملاصقه له

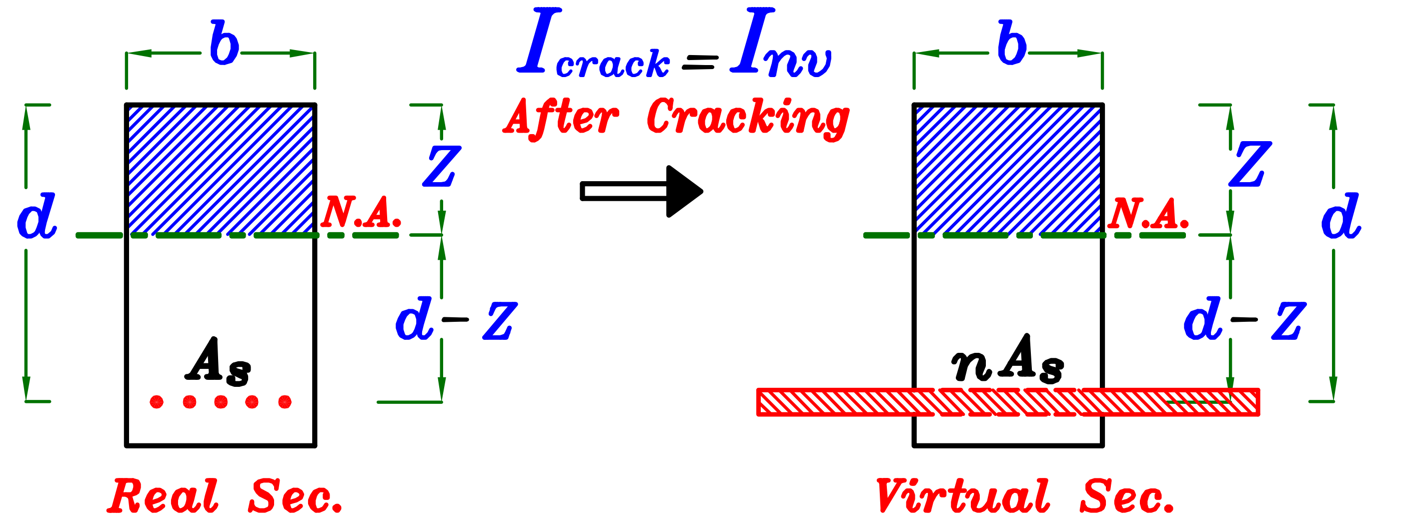
فمن الممكن ان نتخيل انه بدل الحديد الموجود فى القطاع يوجد مكانه خرسانه مساحتها (n) مره مساحه

الحديد و موضوعه فى نفس المكان لكى تتحمل نفس القوى الواقعه على الحديد تماما فنستطيع

حساب العزم الذى يتحمله القطاع التخيلى و يكون مساوى تماما للعزم الذى سيتحمله القطاع الاصلى .

بهذه الطريقه نستطيع حساب ال (I) للقطاع التخيلى فتكون هى نفس ال (I) للقطاع الحقيقى .

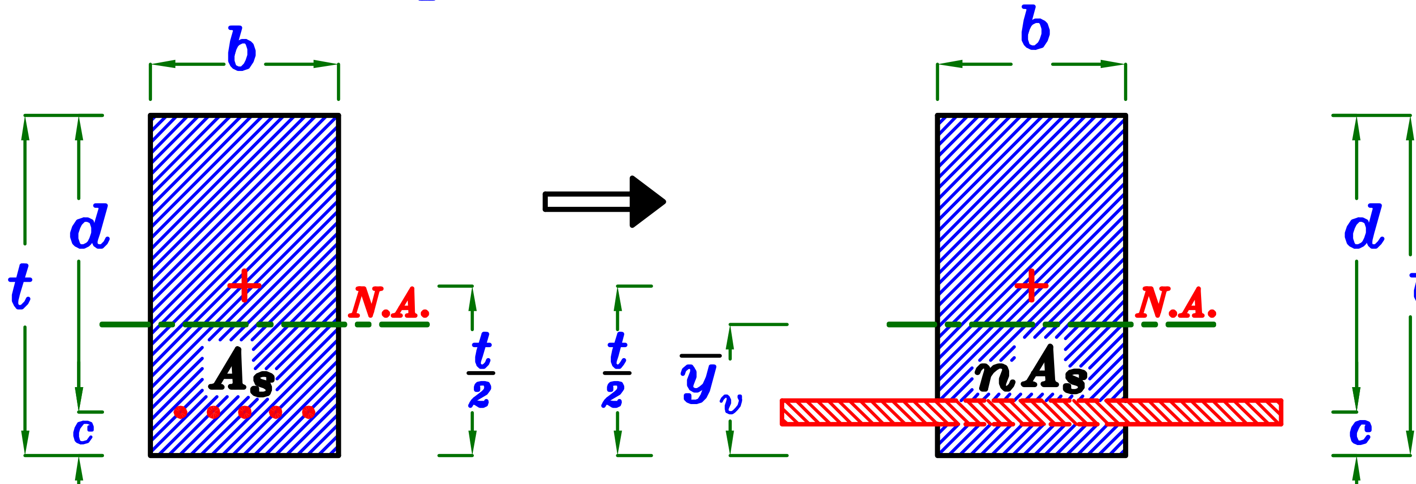




ملحوظه دائما نحسب ال *Inertia* (I) للقطاع حول ال (N.A.) *Neutral Axis*

Before cracking. I_g

without compression steel $A_{s}^{`}$



Real Sec. — Virtual Sec.

n (before cracking) $\simeq 10$

c = cover From tension steel $\simeq (40 \longrightarrow 50)$ mm.

d = distance From tension steel to max compression Fibers.

قبل أن تتشرخ الخرسانه يكون مكان ال (N.A.) عند ال (C.G.) للقطاع لذا نحدد $\overline{y}$ قبل حساب ال (I)

$$A_c = b*t - A_s$$

$$A_v = A_c + nA_s = b*t - A_s + nA_s = b*t + (n-1)A_s$$

$$\boxed{A_v = b*t + (n-1)A_s}$$

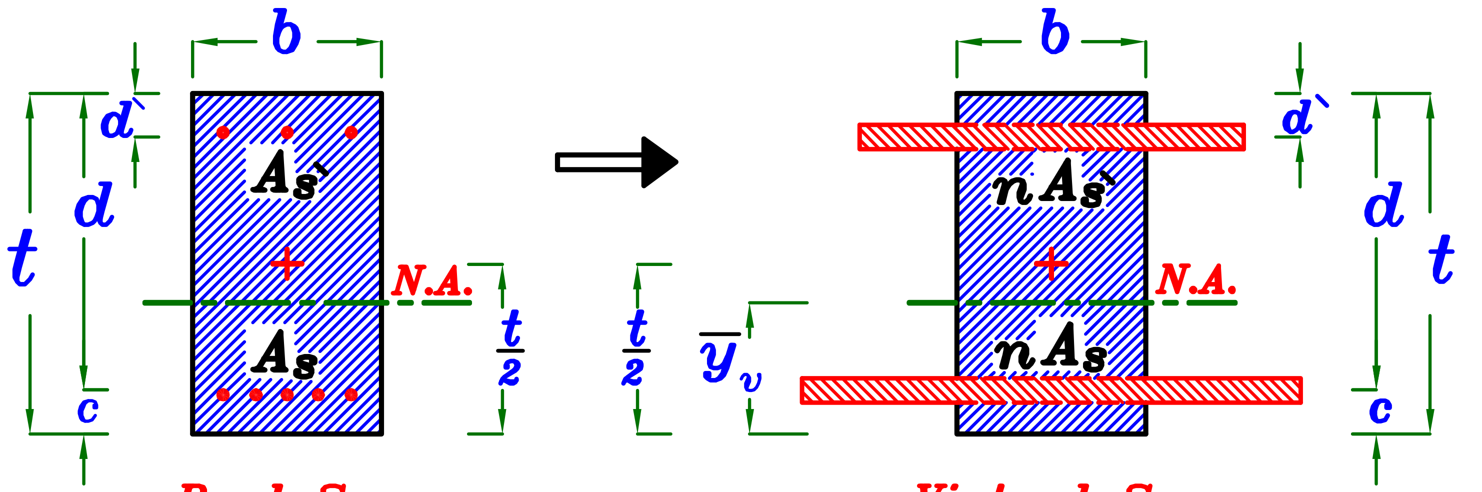
$\overline{y}_t$ = The distance From the C.G. of virtual Sec. to Tension side.

$$\boxed{\overline{y}_t = \frac{b*t*\frac{t}{2} + (n-1)A_s*c}{A_v}}$$

I_g = moment of inertia about N.A. For virtual Sec.

$$\boxed{I_g = \frac{b*t^3}{12} + b*t\left(\frac{t}{2} - \overline{y}_v\right)^2 + (n-1)A_s\left(\overline{y}_v - c\right)^2}$$

with compression steel $A_s^{\backprime}$



Real Sec.   Virtual Sec.

$d^{\backprime}$ = distance From Compassion steel to max compression Fibers.

IF $A_s^{\backprime} < 0.2\ A_s$
∴ We can neglect $A_s^{\backprime}$

$$A_c = b*t - A_s - A_s^{\backprime}$$

$$A_v = A_c + nA_s + nA_s^{\backprime}$$

$$= b*t - A_s - A_s^{\backprime} + nA_s + nA_s^{\backprime}$$

$$\boxed{A_v = b*t + (n-1)A_s + (n-1)A_s^{\backprime}}$$

$\bar{y}_t$ = The distance From the C.G. of virtual Sec. to Tension side.

$$\boxed{\bar{y}_t = \frac{b*t*\frac{t}{2} + (n-1)A_s*c + (n-1)A_s^{\backprime}*(t-d^{\backprime})}{A_v}}$$

I_g = moment of inertia about N.A. For virtual Sec.

$$\boxed{I_g = \frac{b*t^3}{12} + b*t\left(\frac{t}{2}-\bar{y}_v\right)^2 + (n-1)A_s\left(\bar{y}_v - c\right)^2 + (n-1)A_s^{\backprime}\left[(t-d^{\backprime})-\bar{y}_v\right]^2}$$

After cracking. I_{nv}

عند تشرخ الخرسانه من جهه الشد يتحرك ال (N.A.) جهه الضغط قليلا ليوازن القطاع من جديد
و بالتالى لن يكون ال (N.A.) عند ال (C.G.) القديمه للقطاع
و لكى نستطيع أن نحدد مكان ال (N.A.) الجديد نحدده عن طريق الاتزان
أى يجب أن يكون مجموع ضرب المساحات فى بعد مركزها عن ال (N.A.) أسفل ال (N.A.)
تساوى مجموع ضرب المساحات فى بعد مركزها عن ال (N.A.) أعلى ال (N.A.)

Area * distance $= S_{nv.}$ (First Moment of Area)

$$\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$$

nv. means about (N.A.) For Virtual section

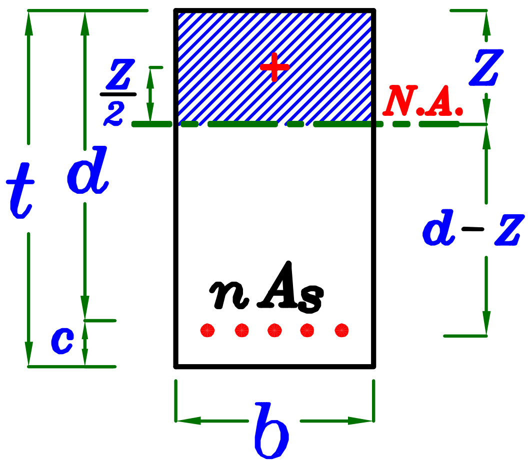
① For R−Sec.

without compression steel A_s'

n (after cracking) $\simeq 15$

Get Z (From Comp. side)

by taking $\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$



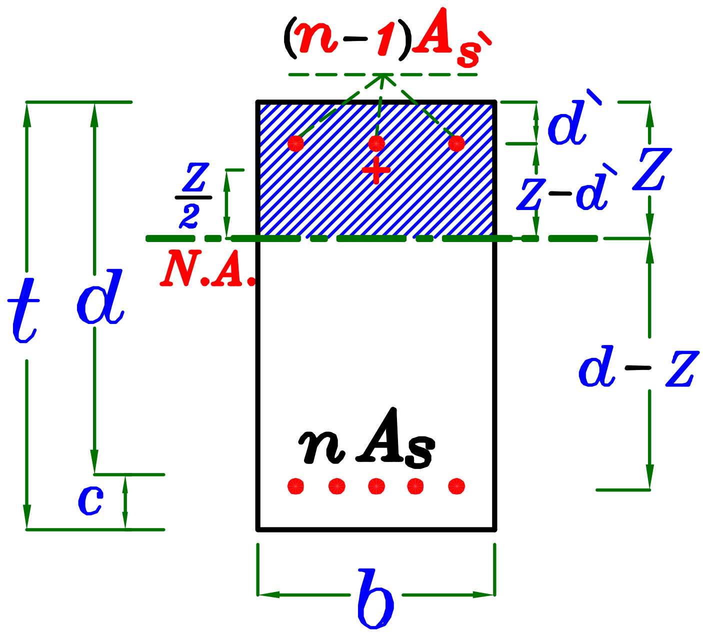
$$\boxed{b\,(z)\left(\frac{Z}{2}\right) = n\,A_s\,(d-Z)}$$

Get $I_{cr.} = I_{nv}$ (moment of inertia For cracked section)

$$\boxed{I_{nv} = I_{cr.} = \frac{bZ^3}{3} + n\,A_s\,(d-Z)^2}$$

with compression steel A_s' — IF $A_s' > 0.2\ A_s$

n (after cracking) $\simeq 15$



Get Z (From Comp. side)

$$\boxed{S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}}$$

$$\boxed{b\,(Z)\left(\frac{Z}{2}\right) + (n-1)\,A_s'\,(Z-d') = n\,A_s\,(d-Z)}$$

Get $I_{cr.} = I_{nv}$ (moment of inertia For cracked section)

$$\boxed{I_{nv} = I_{cr.} = \frac{bZ^3}{3} + (n-1)\,A_s'\,(Z-d')^2 + n\,A_s\,(d-Z)^2}$$

② For T−Sec. or L−Sec.

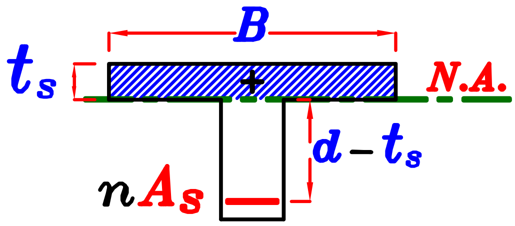
(Tension Steel only)

No Compression steel in T−sec. & L−sec.

To know IF the N.A. is above or under the Flange.

Assume that the N.A. is exactly at the Flange.

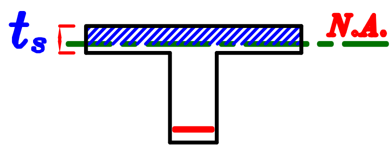
Calculate (First Moment of Area) $S_{nv.}$ above and under the Flange.



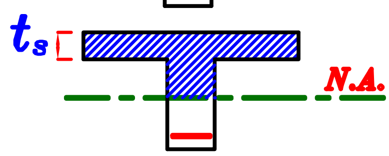
$$S_{nv.}(above) = (B * t_s) * \frac{t_s}{2}$$

$$S_{nv.}(under) = (n A_s) * (d - t_s)$$

IF $S_{nv.}(above) > S_{nv.}(under)$ $\therefore Z < t_s$



IF $S_{nv.}(under) > S_{nv.}(above)$ $\therefore Z > t_s$

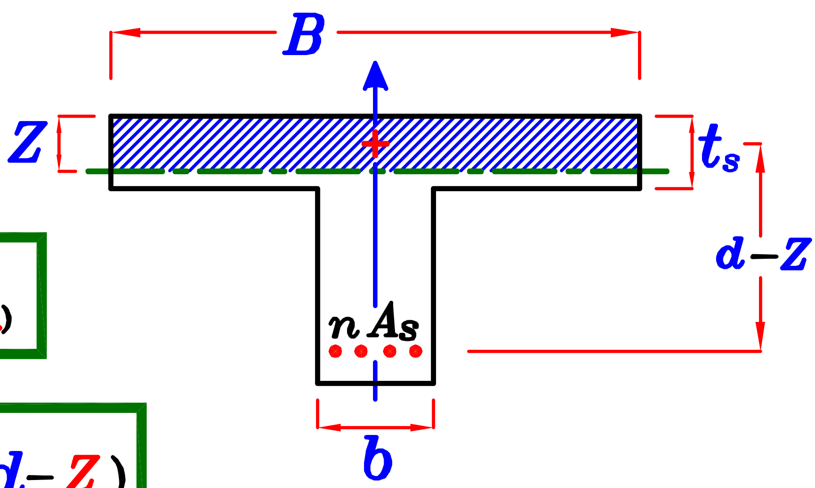


ⓐ IF $S_{nv.}(above) > S_{nv.}(under)$ $\therefore Z < t_s$

$\therefore$ The sec. will act the same as R−sec. but with width B

n (after cracking) $\simeq 15$

Get Z (From Comp. side)



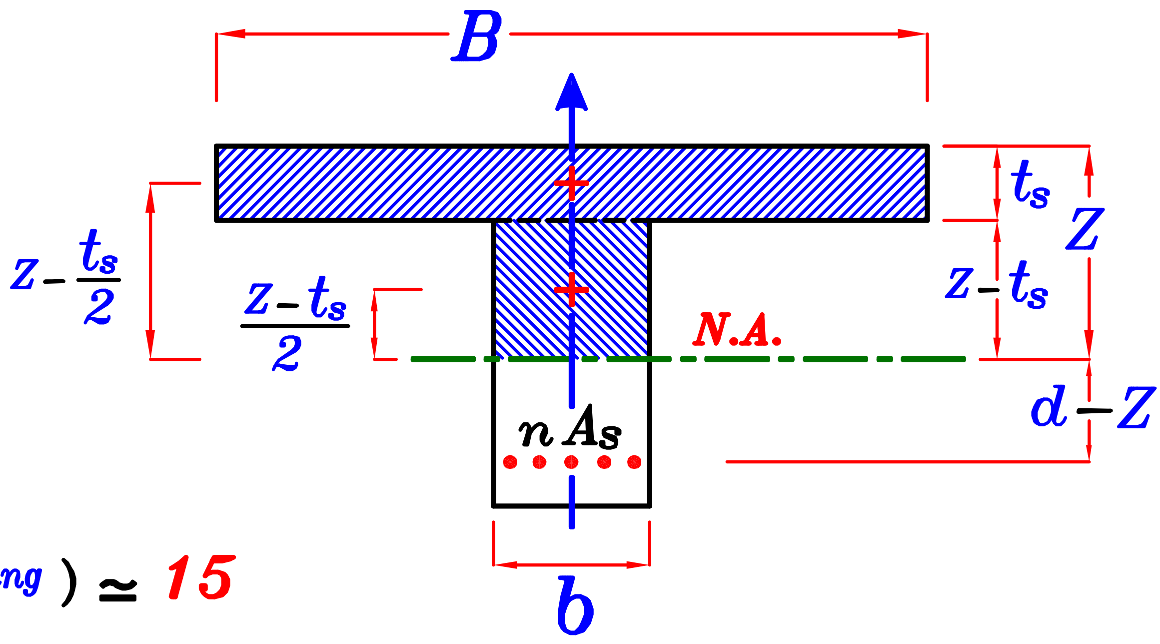
$$\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$$

$$\boxed{B(Z)\left(\frac{Z}{2}\right) = n A_s (d - Z)}$$

Get $I_{cr.} = I_{nv}$ (moment of inertia For cracked section)

$$\boxed{I_{nv} = I_{cr.} = \frac{B Z^3}{3} + n A_s (d - Z)^2}$$

ⓑ IF $S_{nv.}(above) < S_{nv.}(under)$ $\therefore Z > t_F$



n (after cracking) $\simeq 15$

Get Z (From Comp. side)

$$\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$$

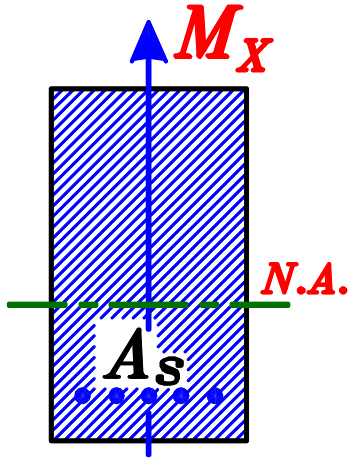
$$\boxed{B\,(t_s)\left(Z-\frac{t_s}{2}\right) + b\,(Z-t_s)\left(\frac{Z-t_s}{2}\right) = n\,A_s\,(d-Z)}$$

Get $I_{cr.} = I_{nv}$ (moment of inertia For cracked section)

$$\boxed{I_{nv} = I_{cr.} = \frac{B\,t_s^3}{12} + B\,(t_s)\left(Z-\frac{t_s}{2}\right)^2 + \frac{b\,(Z-t_s)^3}{3} + n\,A_s\,(d-Z)^2}$$

لحساب الـ Normal stress على الخرسانه فى أى قطاع نستخدم معادله :

$$F = -\frac{N}{A} \pm \frac{M_Y\, x}{I_Y} \pm \frac{M_X\, y}{I_X}$$



و لاننا نتحدث على كمرات فلا يوجد عليها قوى محوريه $N = Zero$

$$\therefore \quad F = \pm \frac{M_Y\, x}{I_Y} \pm \frac{M_X\, y}{I_X}$$

و لاننا نتحدث على أوزان فقط و لا نتحدث عن قوى افقيه فبالتالي يكون العزم رأسى فقط $M_Y = Zero$

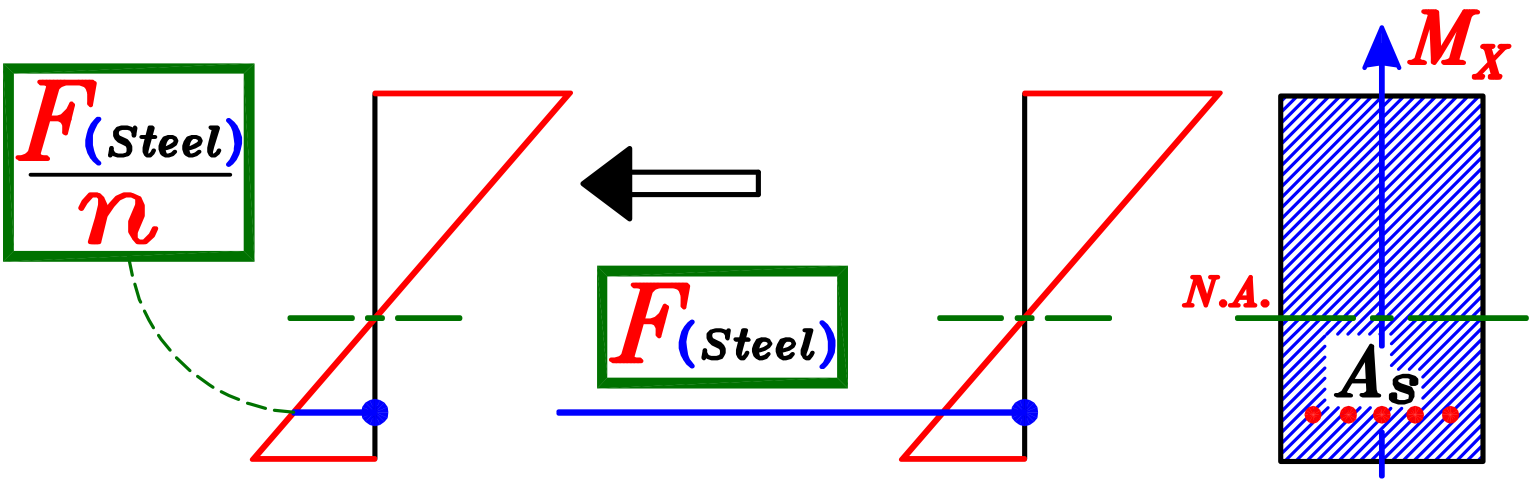
$$\therefore \quad \boxed{F = \pm \frac{M_X\, y}{I_X}}$$

Normal stress على الخرسانه

و لان الاجهادات الواقعه على الحديد تساوى (n) مره الاجهادات الواقعه على الخرسانه الملاصقه له.

$$\therefore \quad \boxed{F = \pm n * \frac{M_X\, y}{I_X}}$$

Normal stress على الحديد



$$\boxed{F = \frac{M\,y}{I}} \quad (Concrete)$$

$$\boxed{F = n\,\frac{M\,y}{I}} \quad (Steel)$$

Where :

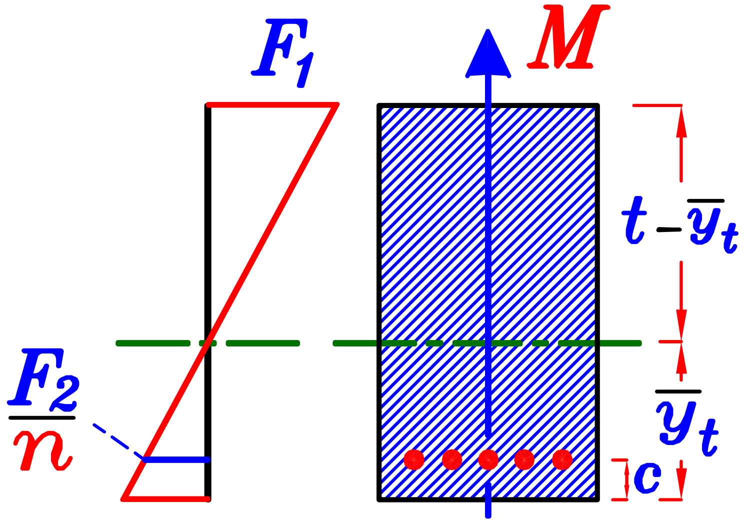
y هى المسافه من النقطه المحسوب عندها ال stress حتى ال N.A.

I هى ال moment of Inertia للقطاع الشغال حول ال N.A.

و تساوى $I = I_g$ للقطاع قبل التشرخ before cracking

و تساوى $I = I_{nv}$ للقطاع بعد التشرخ after cracking

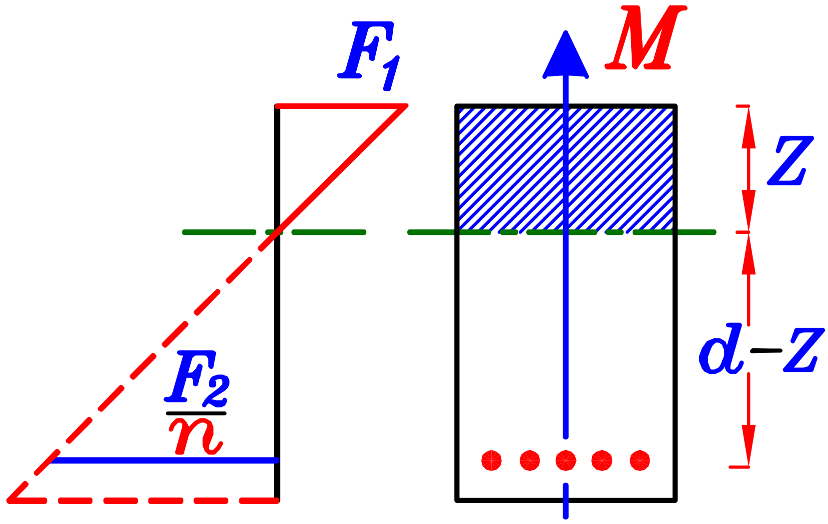
Before Cracking.



$$F_{1\,(Concrete)} = \frac{M * y}{I} = \frac{M * (t - \bar{y}_t)}{I_g}$$

$$F_{2\,(Steel)} = n\,\frac{M * y}{I} = 10 * \frac{M * (\bar{y}_t - c)}{I_g}$$

After Cracking.



$$F_{1\,(Concrete)} = \frac{M * y}{I} = \frac{M * Z}{I_{nv}}$$

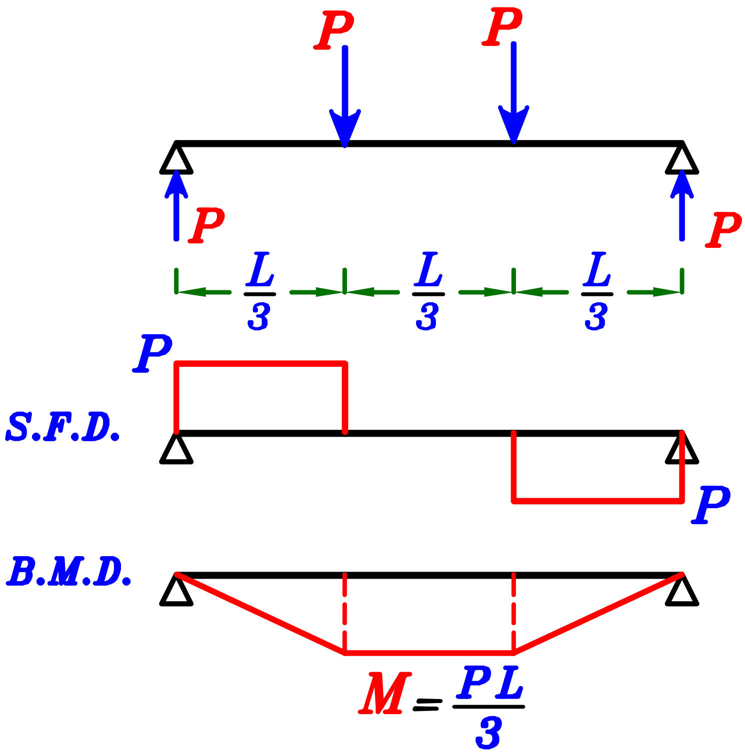
$$F_{2\,(Steel)} = n\,\frac{M * y}{I} = 15 * \frac{M * (d - Z)}{I_{nv}}$$

Stages of Beams under Variable Bending Moment.

لدراسه خواص الكمره تحت تأثير حالات التحميل المختلفه .

ندرس كمره (*Simply Supported*) كما هو مبين فى الشكل (مع اهمال وزنها *o.w.*) .

حيث يكون الثلث الأوسط من الكمره يوجد عليه *B.M.* فقط .
و لا يوجد علية *S.F.* و هذا هو الجزء هو الذى سندرسه .



بزياده مقدار القوى P يزداد مقدار العزم الواقع على على الكمره $M = \frac{PL}{3}$

و بدراسه الكمره مع زياده الحمل نجد أنها تمر بثلاث مراحل :

1 – 0.0 ⟶ Cracking.
2 – Cracking ⟶ Working.
3 – Working ⟶ Ultimate.

1– Cracking Stage.

$M = 0.0 \longrightarrow M_{cr.}$

$P_{cr.}$ هو الحمل الذى يحدث عنده أول شرخ فى الكمره من جهه الشد *Tension Side*

$M_{cr.} = (P_{cr.} * L) \backslash 3$

2– Working Stage.

$M_{cr.} \longrightarrow M_w$

P_w هو الحمل الذى يصل عنده الاجهاد على أى من الحديد أو الخرسانه الى $F_{allowable}$

$M_w = (P_w * L) \backslash 3$

3– Ultimate Stage.

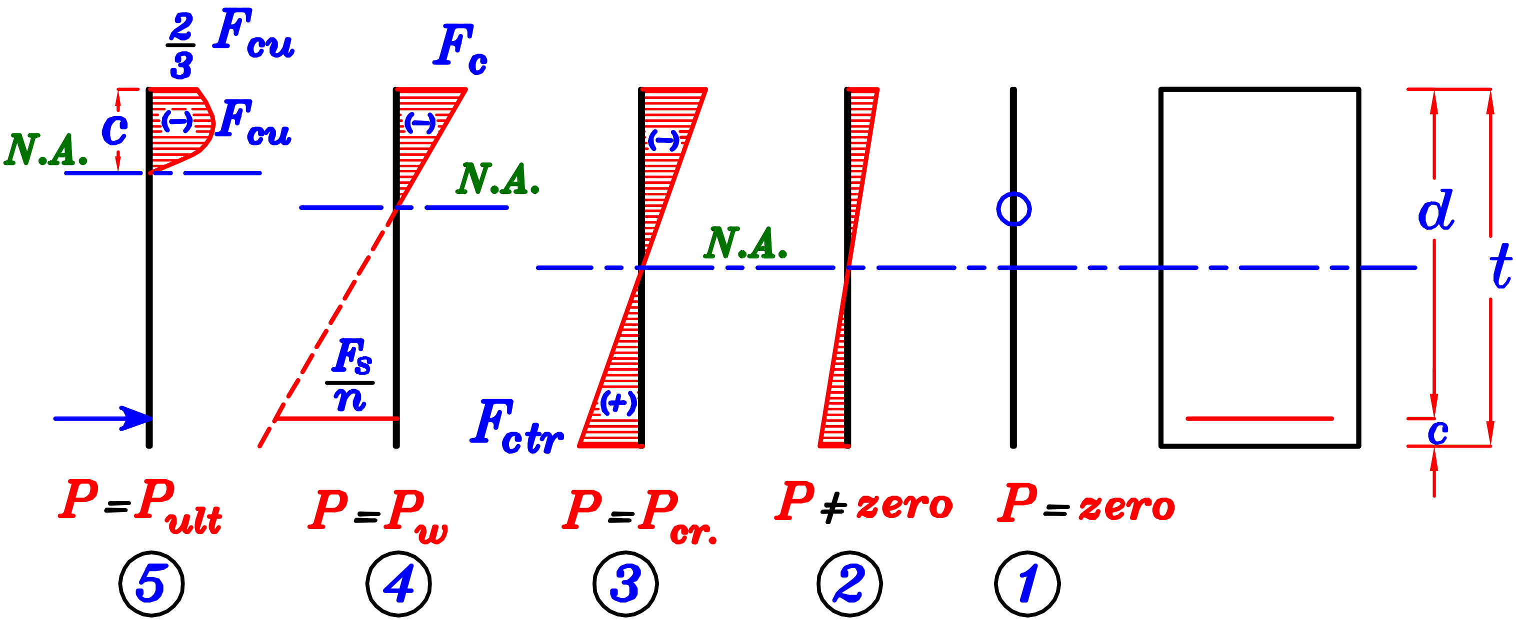
$M_w \longrightarrow M_{ult.}$

$P_{ult.}$ هو الحمل الذى يحدث عنده انهيار للكمره أى يصل الاجهاد على الخرسانه فى الضغط الى F_{cu}
أو يصل الاجهاد على الحديد فى الشد الى F_y

$M_{ult.} = (P_{ult.} * L) \backslash 3$

Normal Stresses Diagram

For beams subjected to Bending Moment only.



١ ـ قبل التحميل يكون الـ **normal stress = Zero**

٢ ـ فى بدايه التحميل يحدث شد فى السطح السفلى و ضغط فى السطح العلوى

٣ ـ مع زياده الحمل يزيد الـ **normal stress** حتى يصل فى منطقه الشد الى F_{ctr}

و عند هذه اللحظه يسمى الحمل $P_{cr.}$ و يسمى العزم $M_{cr.}$

٤ ـ مع زياده الحمل تظهر شروخ فى الخرسانه فى منطقه الشد

(الجزء المتشرخ من الخرسانه لا يؤخذ فى الحساب أى كأنه غير موجود)

و مع زياده الحمل يصل الاجهاد فى الخرسانه فى منطقه الضغط الى F_c **Allowable stresses**

أو يصل الاجهاد فى الحديد فى منطقه الشد الى F_s **Allowable stresses**

و عند هذه اللحظه يسمى الحمل P_w و يسمى العزم M_w

٥ ـ مع زياده الحمل يزداد الضغط على الخرسانه و يحدث تغير غير منتظم فى الاجهادات

non Linear stresses على الخرسانه

حتى يصل الاجهاد فى الخرسانه فى منطقه الضغط الى F_{cu}

أو يصل الاجهاد فى الحديد فى منطقه الشد الى F_y

و تبدأ الكمره فى الانهيار و عند هذه اللحظه يسمى الحمل P_{ult} و يسمى العزم M_{ult}

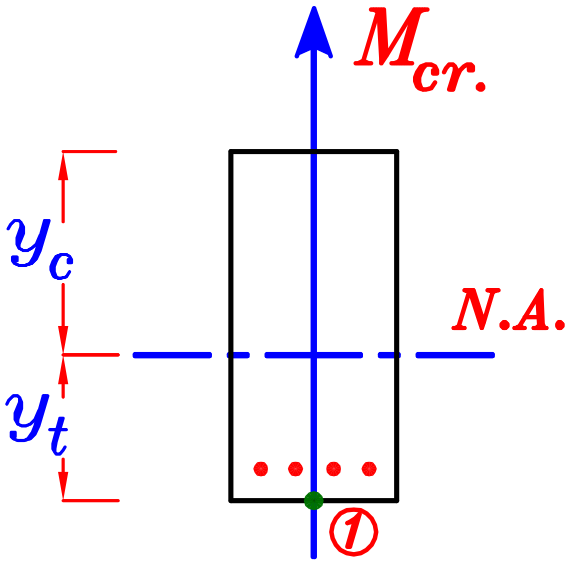
Cracking Moment ($M_{cr.}$)

هو قيمه العزم الذى يؤدى الى حدوث أول شرخ فى الخرسانه من جهه الشد و عنده يصل الإجهاد فى الخرسانه فى منطقه الشد الى F_{ctr}

$$\boxed{F_{ctr} = 0.6\sqrt{F_{cu}}} \quad N\backslash mm^2$$

Cracking Tensile stress. (**Concrete Tension Rupture**)

$$\because\ F = \frac{M * y}{I} \Longrightarrow M = \frac{F * I}{y}$$



at cracking

F at point ① $= F_{ctr}$

$\therefore$ Moment at this case $= M_{cr.}$

$$\boxed{M_{cr.} = \frac{F_{ctr} * I_g}{y_t}}$$

$M_{cr.}$ = Cracking moment

I_g = Moment of Inertia around N.A.
(For virtual sec.)

y_t = Distance between N.A. to extreme tension Fibers.
(For virtual sec.)

عندما يكون شكل المقطاع معطى و مطلوب $M_{cr.}$

أى يطلب قيمه العزم الذى سوف يسبب التشرخ للخرسانه فى منطقه الشد .

تكون خطوات الحل كالأتى :ـ

١ـ نحسب n

$$n = \frac{E_s}{E_{c1}} = \frac{2*10^5}{4400\sqrt{F_{cu}}} \simeq 10$$

٢ـ نحسب A_v المساحه التخيليه للقطاع بالكامل $A_v = A_c + (n-1)A_s + (n-1)A_{s'}$

٣ـ نحسب $\bar{y}_t = \bar{y}_v$ و تكون من جهه الشد Tension Side

٤ـ نحسب I_v و هو عزم القصور الذاتى للقطاع التخيلى بالكامل $I_v = I_g$

٥ـ نحسب F_{ctr}

$$\boxed{F_{ctr} = 0.6\sqrt{F_{cu}}}$$

٦ـ نحسب $M_{cr.}$

$$\boxed{M_{cr.} = \frac{F_{ctr} * I_g}{\bar{y}_t}}$$

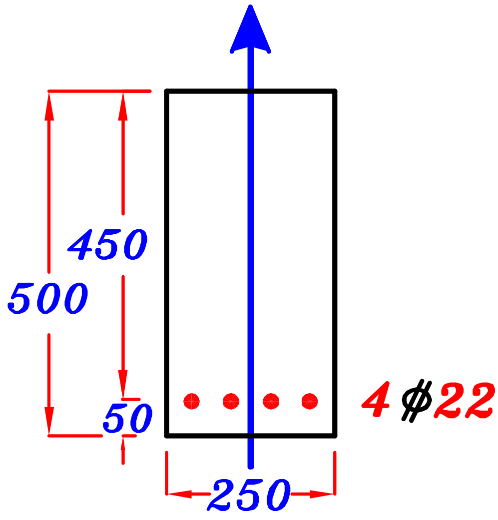
Example.

Data.

Mega Pascal

$F_{cu} = 25\ N\backslash mm^2 = 25\ Mpa$

st. 360/520



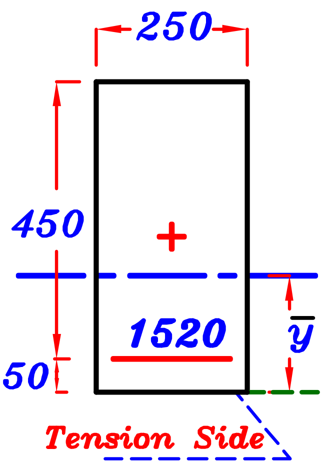
Req.

For the shown Cross-Section

Calculate $M_{cr.}$

Solution.

$$A_s = 4\phi22 = 4\left[\frac{\pi * 22^2}{4}\right] = 1520\ mm^2$$



① $$n = \frac{E_s}{E_c} = \frac{2 * 10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$$

② $$A_v = A_c + (n-1)A_s$$

$$A_v = 250 * 500 + (10-1)(1520) = 138680\ mm^2$$

③ $$\bar{y}_t = \frac{250 * 500 * 250 + (10-1)(1520)(50)}{138680} = 230.27\ mm$$

④ $$I_{gross} = \frac{250 * 500^3}{12} + 250 * 500(250-230.27)^2 + (10-1)(1520)(230.27-50)^2$$

$$= 3097388472\ mm^4$$

⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0\ N\backslash mm^2$$

⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.0 * 3097388472}{230.27} = 40353347.9\ N.mm$$

$$= \frac{40353347.9\ N.mm}{10^6} = 40.35\ kN.m$$

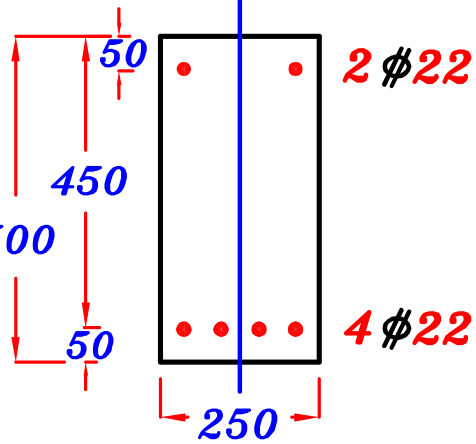
$$\boxed{M_{cr} = 40.35\ kN.m}$$

Example.

Data. $F_{cu} = 25\ N/mm^2 = 25\ Mpa$

st. 360/520

Req. Calculate M_{cr}.



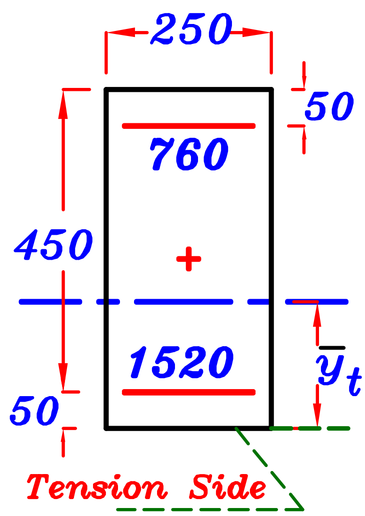
Solution.

$$A_s = 4\phi22 = 4\left[\frac{\pi * 22^2}{4}\right] = 1520\ mm^2$$

$$A_s` = 2\phi22 = 2\left[\frac{\pi * 22^2}{4}\right] = 760\ mm^2$$

IF $A_s` < 0.2\ A_s$ We can neglect $A_s`$

$\because \frac{A_s`}{A_s} = \frac{760}{1520} = 0.50 > 0.2$ $\therefore$ We can't neglect $A_s`$



① $$n = \frac{E_s}{E_c} = \frac{2*10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$$

② $$A_v = b*t + (n-1)A_s + (n-1)A_s`$$

$$A_v = 250*500 + (10-1)(1520) + (10-1)(760) = 145520\ mm^2$$

③ $$\overline{y}_t = \frac{250*500*250 + (10-1)(1520)(50) + (10-1)(760)(450)}{145520} = 240.6\ mm$$

④ $$I_{gross} = \frac{250*500^3}{12} + 250*500(250-240.6)^2 + (10-1)(1520)(240.6-50)^2$$
$$+ (10-1)(760)(450-240.6)^2 = 3412106414\ mm^4$$

⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0\ N\backslash mm^2$$

⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\overline{y}_t} = \frac{3.0 * 3412106414}{240.6} = 42544967.7\ N.mm$$

$$= \frac{42544967.7\ N.mm}{10^6} = 42.54\ kN.m$$

$$\boxed{M_{cr} = 42.54\ kN.m}$$

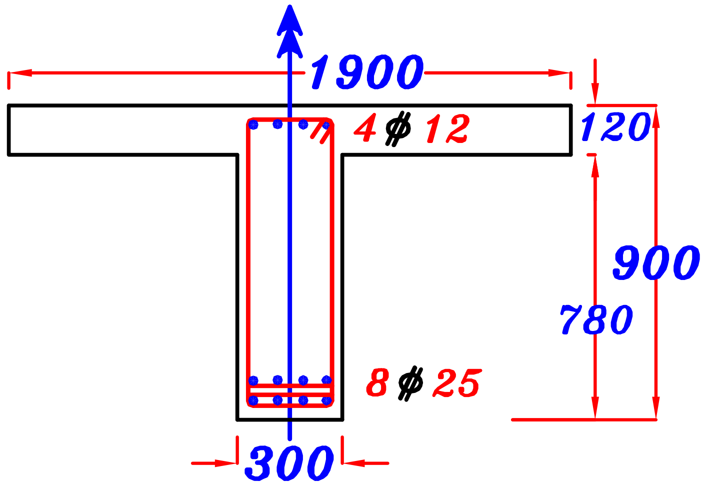
Example.

Data.

$F_{cu} = 25\ N/mm^2 = 25\,Mpa$

st. 360/520

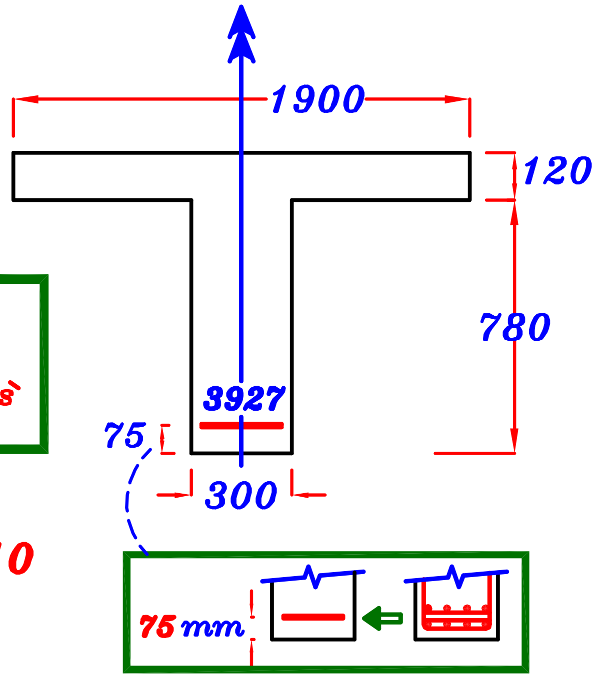
Req. Calculate M_{cr}.



Solution.

$$A_s = 8\phi 25 = 8\left[\frac{\pi * 25^2}{4}\right] = 3927\ mm^2$$

$$A_{s'} = 4\phi 12 = 4\left[\frac{\pi * 12^2}{4}\right] = 452\ mm^2$$



IF $A_{s'} < 0.2\ A_s$ We can neglect $A_{s'}$

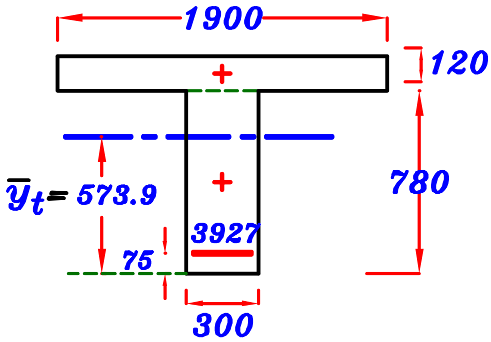
$\because \frac{A_{s'}}{A_s} = \frac{531}{3930} = 0.135 < 0.2 \quad \therefore$ We can neglect $A_{s'}$

① $$n = \frac{E_s}{E_c} = \frac{2 * 10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$$

② $$A_v = A_c + (n-1)A_s = 120 * 1900 + 780 * 300 + (10-1)(3927) = 497343\ mm^2$$

③ $$\bar{y}_t = \frac{120 * 1900 * (780+60) + 780 * 300 * \left(\frac{780}{2}\right) + (10-1)(3927)(75)}{497343} = 573.9\ mm$$

④ $$I_{gross} = \frac{1900 * 120^3}{12} + 1900 * 120\,(780+60-573.9)^2 + \frac{300 * 780^3}{12}$$
$$+ 300 * 780\left(573.9 - \frac{780}{2}\right)^2 + (10-1)(3927)(573.9-75)^2$$
$$= 44992510490\ mm^4$$

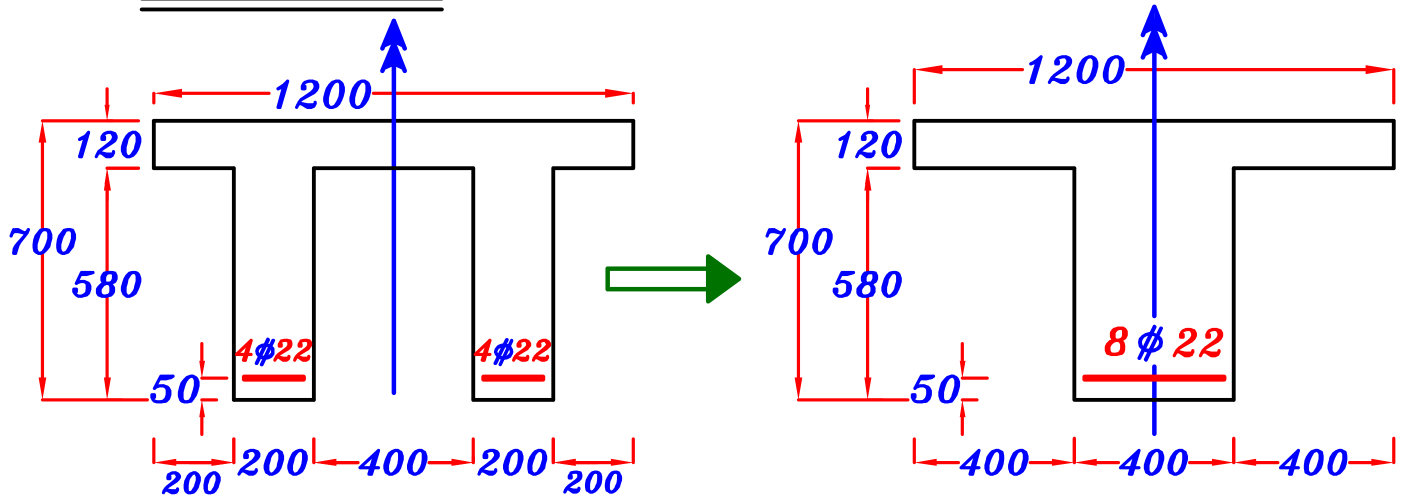


⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0\ N\backslash mm^2$$

⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.0 * 44992510490}{573.9} = 235193468.3\ N.mm$$
$$= 235.19\ kN.m$$

$$\boxed{M_{cr} = 235.19\ kN.m}$$

Example.



We can convert the Sec. to an easier Cross-Sec. and has the same properties. $(Area, \bar{y}, A_s, c, I$ & $M_{cr.})$

Data. $F_{cu} = 25\ N\backslash mm^2$ st. 360/520

Req. For the shown Cross-Section Calculate $M_{cr.}$

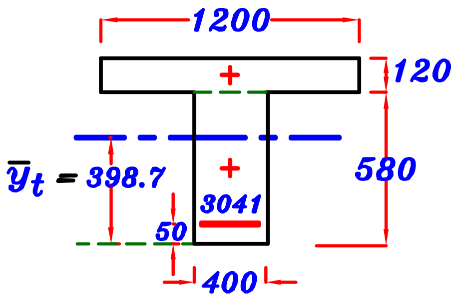
Solution. $A_s = 8\phi 22 = 8\left[\frac{\pi * 22^2}{4}\right] = 3041\ mm^2$

① $n = \frac{E_s}{E_c} = \frac{2*10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$

② $A_v = A_c + (n-1)A_s = 120*1200 + 580*400 + (10-1)(3041) = 403369\ mm^2$

③ $\bar{y}_t = \frac{1200*120*(580+60) + 580*400*\frac{580}{2} + (10-1)(3041)(50)}{403369}$

$= 398.7\ mm$



④ $I_{gross} = \frac{1200*120^3}{12} + 1200*120(580+60-398.7)^2 + \frac{400*580^3}{12} + 400*580\left(398.7 - \frac{580}{2}\right)^2$

$+ (10-1)(3041)(398.7-50)^2 = 21130115740\ mm^4$

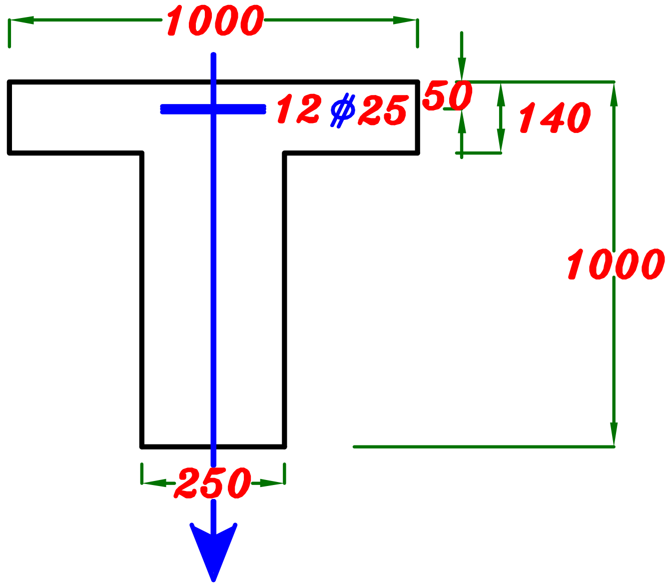
⑤ $F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0\ N\backslash mm^2$

⑥ $M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.0 * 21130115740}{398.7} = 158992594\ N.mm$

$= 159.0\ kN.m.$

$$\boxed{M_{cr} = 159.0\ kN.m}$$

Example.



Data. $F_{cu} = 25 \ N\backslash mm^2$

st. 360/520

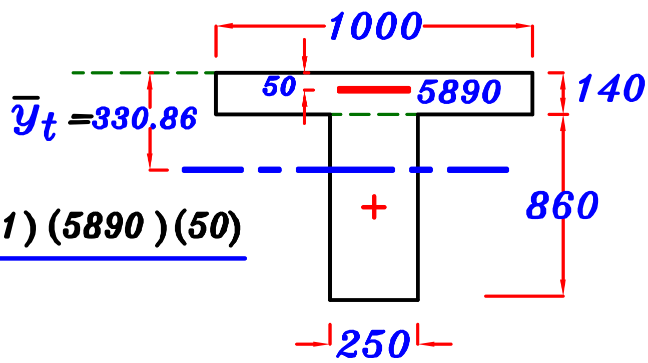
Req. Calculate $M_{cr.}$

Solution.

$$A_s = 12\,\phi 25 = 12\left[\frac{\pi * 25^2}{4}\right] = 5890\ mm^2$$

① $$n = \frac{E_s}{E_c} = \frac{2*10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$$

② $$A_v = A_c + (n-1)A_s = 140*1000 + 860*250 + (10-1)(5890) = 408010\ mm^2$$



③ $$\bar{y}_t = \frac{1000*140*70 + 250*860*\left(\frac{860}{2} + 140\right) + (10-1)(5890)(50)}{408010}$$

$$= 330.87\ mm$$

④ $$I_{gross} = \frac{1000*140^3}{12} + 1000*140\,(330.87-70)^2 + \frac{250*860^3}{12} + 250*860\left(\frac{860}{2} + 140 - 330.87\right)^2$$

$$+ (10-1)(5890)(330.87-50)^2 = 39483504630\ mm^4$$

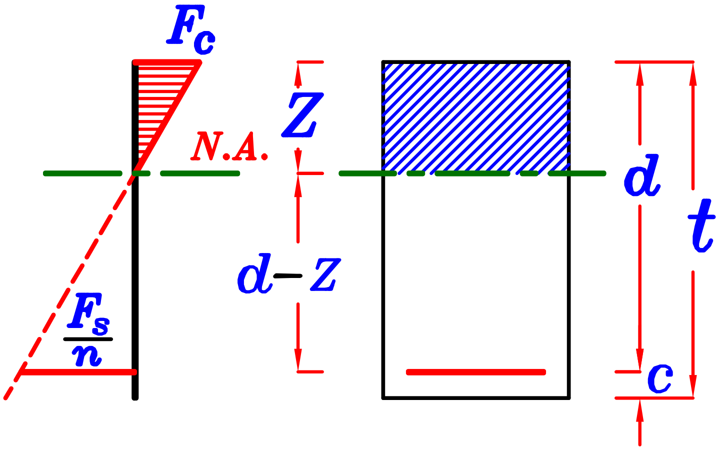
⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0\ N\backslash mm^2$$

⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.0 * 39483504630}{330.87} = 357997140.5\ N.mm$$

$$= 358.0\ kN.m.$$

$$\boxed{M_{cr} = 358.0\ kN.m}$$

Working Moment (M_w)
OR Allowable Moment



عندما تتشرخ الخرسانه فى منطقه الشد
يتحول القطاع الفعال كما بالشكل
الى خرسانه فى منطقه الضغط
و حديد فى منطقه الشد .

أكبر إجهاد تتحمله الخرسانه فى الضغط F_{cu}

أكبر إجهاد يتحمله الحديد فى الشد أو الضغط F_y

و إذا زادت الإجهادات المؤثره على أى من الخرسانه أو الحديد عن F_{cu} أو F_y يحدث إنهيار للكمره .

لذا فنعمل على أن تكون الإجهادات المؤثره أقل من F_{cu} ، F_y حتى لا يحدث إنهيار للكمره .

و هذه الإجهادات تسمى (الإجهادات المسموح بها) **Allowable Stresses**

أى أنها أكبر إجهادات نسمح بها لكى تؤثر على الحديد و الخرسانه مع ضمان عدم الإنهيار.

Allowable Stresses For Concrete $= F_c$

Allowable Stresses For Steel $= F_s$

F_{cu} $(N\backslash mm^2)$	18.0	20.0	25.0	30.0	35.0	40.0
F_c $(N\backslash mm^2)$	7.0	8.0	9.5	10.5	11.5	12.5

F_y $(N\backslash mm^2)$	240	360	400
F_s $(N\backslash mm^2)$	140	200	220

Egyptian Code
Page (5−2)

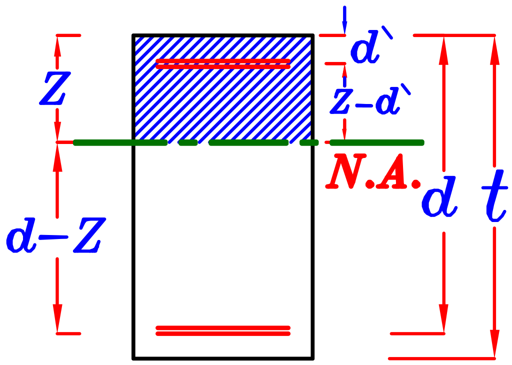
أنواع الإجهادات	المصطلحات	18	20	25	30
		إجهادات التشغيل وفقاً لرتب الخرسانة حسب مقاومتها المميزة للمكعب القياسي بعد ٢٨ يوماً (ن/مم٢)			
مقاومة الخرسانة المميزة (الرتبة)	f_{cu}	18	20	25	30
الضغط المحوري ($e=e_{min}$)	f_{co}	4.5	5	6	7
الانحناء أو الضغط كبير اللامركزية	f_c	7.0	8.0	9.5	10.5
القص					
مقاومة الخرسانة للقص					
بدون تسليح في البلاطات و القواعد	q_c	0.7	0.8	0.9	0.9
بدون تسليح في الأعضاء الأخرى	q_c	0.5	0.6	0.7	0.7
وجود تسليح جذعى في جميع الأعضاء (القص واللي معا)	q_2	1.5	1.7	1.9	2.1
القص الثاقب	q_{cp}	0.7	0.8	0.9	1.0
الصلب الفولاذ					
1-صلب طري 240/350	f_s	140	140	140	140
2- صلب 280/450		160	160	160	160
3-صلب 360/520		200	200	200	200
4-صلب 400/600		220	220	220	220
5-الشبك الملحوم 450/520 أملس		160	160	160	160
ذو النتوءات أو ذو العضات		220	220	220	220

تعريف ال (M_w) هو العزم الذى يجعل الاجهادات تصل على أى من الحديد أو الخرسانه الى **Allowable Stresses**

فى المسأله عندما يعطينا القطاع و يطلب تحديد M_w

تكون خطوات الحل كالأتى : ـ

١ ـ نأخذ $n \simeq 15$ **Modular ratio after cracking**



٢ ـ لتحديد مكان ال N.A.

نحسب قيمه Z و تكون من جهه الضغط

و ذلك بأن نأخذ $S_{nv} = Zero$

٣ ـ نحسب قيمه I_{nv} و هو عزم القصور الذاتى للقطاع الشغال حول ال N.A.

٤ ـ نحسب قيمه العزم الذى يجعل الإجهادات على الخرسانه فى الضغط F_c

$$\boxed{M_{wc} = \frac{F_c * I_{nv}}{Z}}$$

٥ ـ نحسب قيمه العزم الذى يجعل الإجهادات على الحديد فى الشد F_s

$$\boxed{M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z}}$$

٦ ـ نحسب قيمه العزم الذى يجعل الإجهادات على الحديد فى الضغط $F_{s`}$

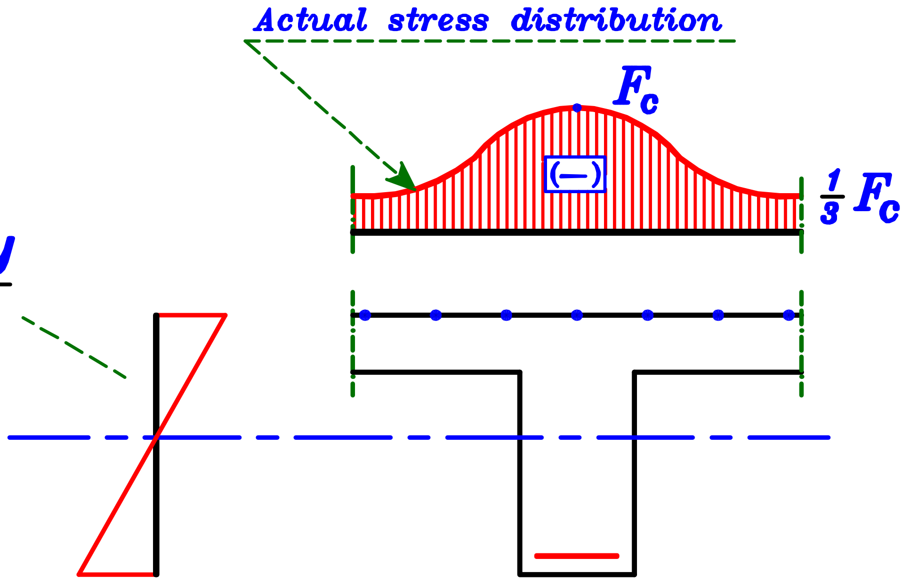
(ممكن إهمال هذه الخطوه)

$$\boxed{M_{ws`} = \frac{\left(\frac{F_{s`}}{n}\right) * I_{nv}}{Z-d`}}$$

٧ ـ نأخذ القيمه الاقل من M_{wc} , M_{ws} , $M_{ws`}$ فتكون هى M_w للقطاع

For T–Sec.

$$F = \pm \frac{M_x\, y}{I_x}$$

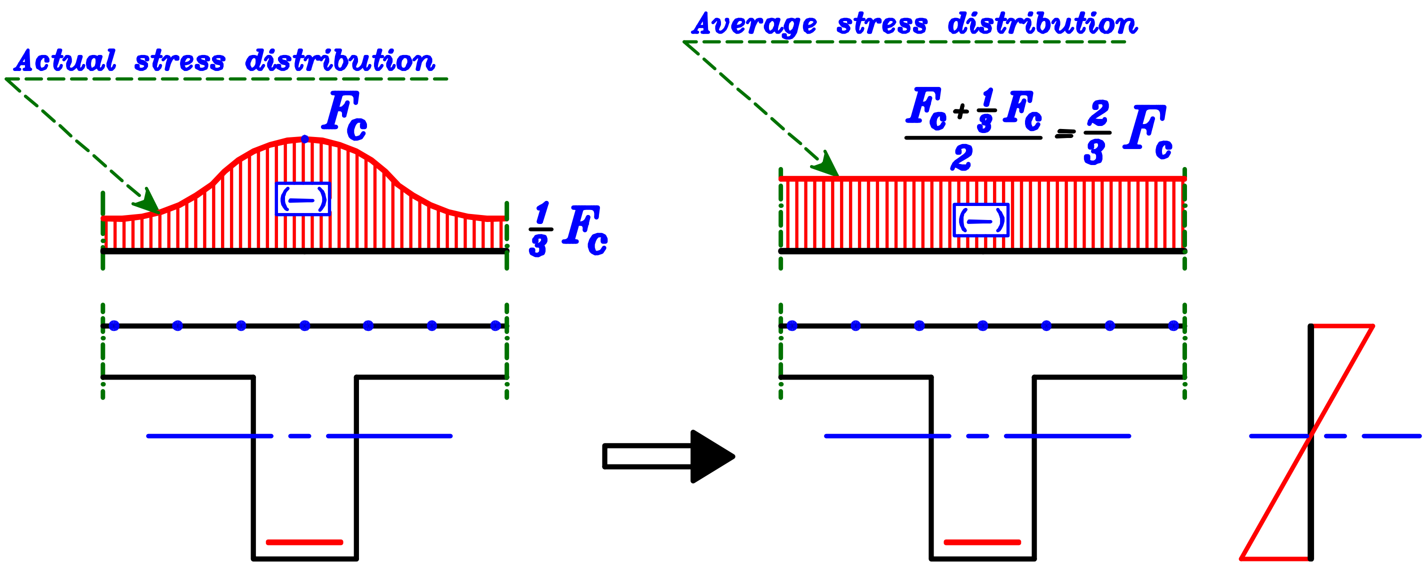


فى ال T–Sec. يحدث زياده كبيره فى الاجهادات على خرسانه البلاطه فوق الكمره مباشره فيكون شكل ال stress على اعلى خط أفقى فى القطاع غير منتظم (كما بالشكل) .

و لكى نستطيع ان نستخدم معادله $F = \dfrac{M\, y}{I}$

يجب ان يكون ال stress عند كل خط افقى فى القطاع منتظم (قيمته ثابته) .

لذا سنعتبر ان اعلى خط فى القطاع عليه stress منتظم بقيمه $\frac{2}{3} F_c$



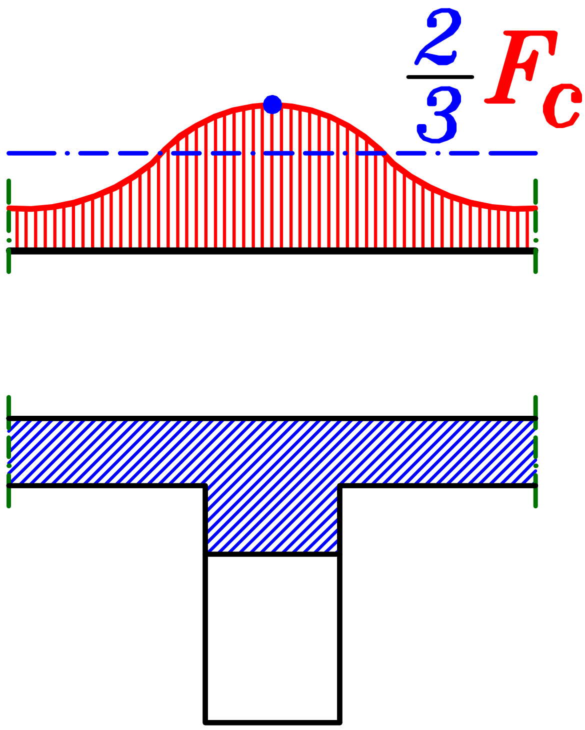
$\therefore$ For T–Sec.

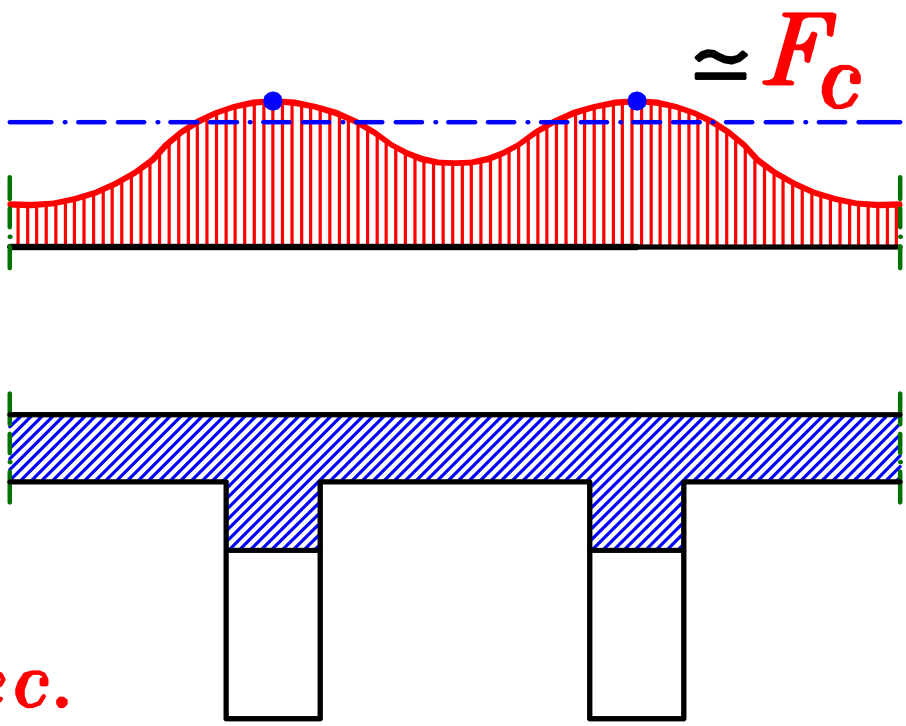
$$\boxed{M_{wc} = \frac{\left(\frac{2}{3} F_c\right) * I_{nv}}{Z}}$$

Special Case.

T–Sec.

$$M_{wc} = \frac{\left(\frac{2}{3}F_c\right) * I_{nv}}{Z}$$

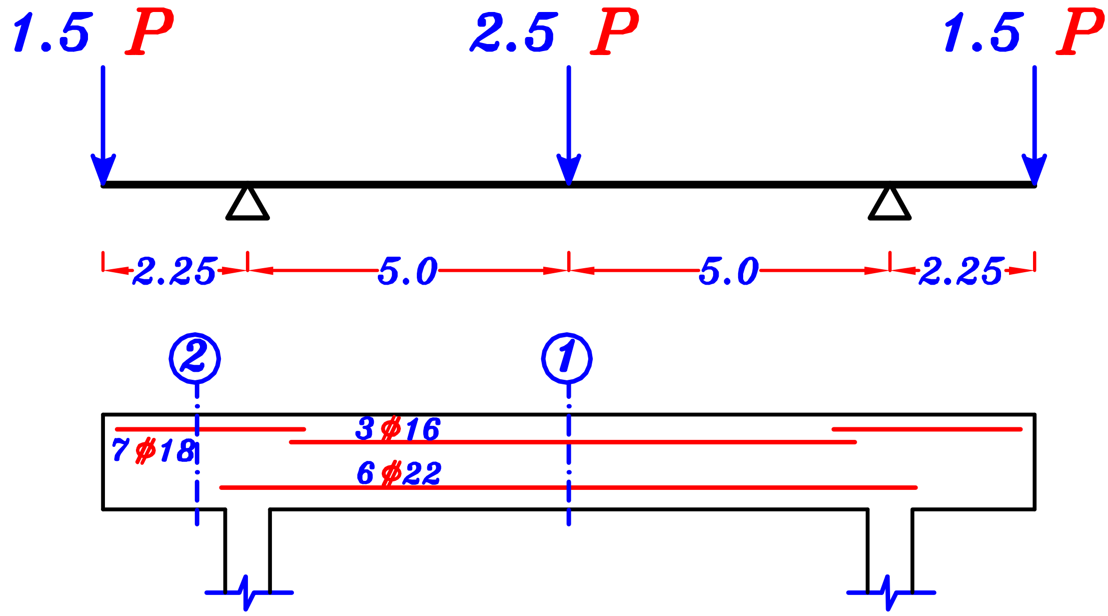


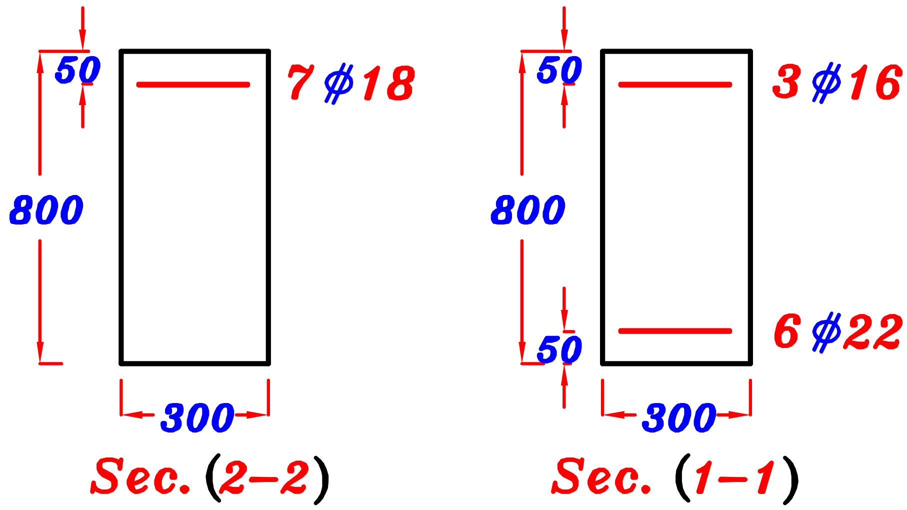


not as T–Sec.

$$M_{wc} = \frac{(F_c) * I_{nv}}{Z}$$

Example.





Data.

neglecting O.W.

$F_{cu} = 25 \; N\backslash mm^2 \qquad F_y = 360 \; N\backslash mm^2$

Req.

Find the allowable working loads (P_w) acting on the beam.

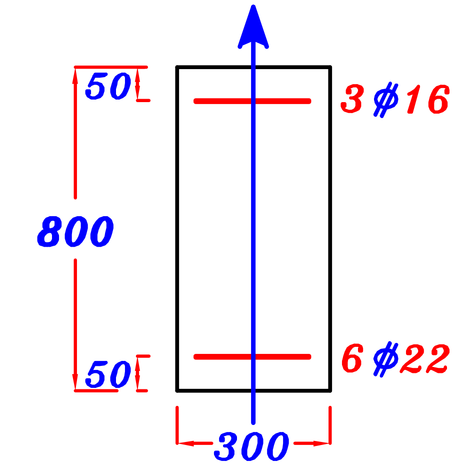
Allowable stresses

$F_{cu} = 25 \; N\backslash mm^2 \longrightarrow F_c = 9.5 \; N\backslash mm^2$

$F_y = 360 \; N\backslash mm^2 \longrightarrow F_s = 200 \; N\backslash mm^2$

Solution.

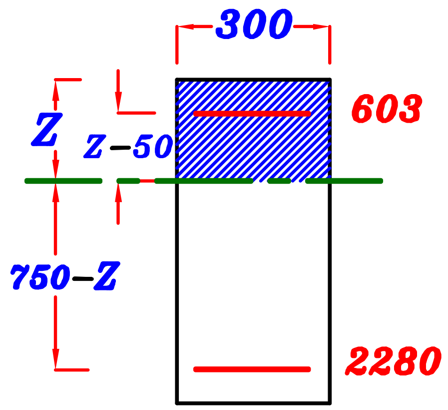
Sec. 1



$$A_s = 6\phi22 = 6\left[\frac{\pi * 22^2}{4}\right] = 2280\ mm^2$$

$$A_{s`} = 3\phi16 = 3\left[\frac{\pi * 16^2}{4}\right] = 603\ mm^2$$

$$\because \frac{A_{s`}}{A_s} = \frac{603}{2280} = 0.26 > 0.2 \quad \therefore \text{We can't neglect } A_{s`}$$



1. Take $n = 15$

2. Get Z by taking $\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$

$$b(Z)\left(\frac{Z}{2}\right) + (n-1)A_{s`}(Z-d`) = n A_s (d-Z)$$

$$300(Z)\left(\frac{Z}{2}\right) + (14)(603)(Z-50) = (15)(2280)(750-Z)$$

$$\boxed{Z = 298.3\ mm}$$

3. Get $I_{nv} = \frac{bZ^3}{3} + (n-1)A_{s`}(Z-d`)^2 + n A_s (d-Z)^2$

$$I_{nv} = \frac{300(298.3)^3}{3} + (14)(603)(298.3-50)^2 + (15)(2280)(750-298.3)^2$$

$$= 10152758140\ mm^4$$

4. $M_{wc} = \frac{F_c * I_{nv}}{Z} = \frac{9.5 * 10152758140}{298.3} = 323336246.6\ N.mm$

$$= 323.33\ kN.mm$$

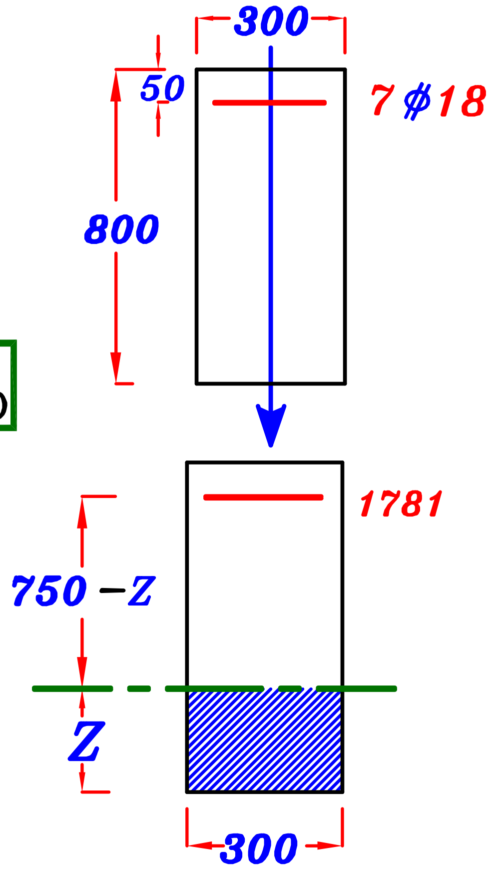
5. $M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z} = \frac{\left(\frac{200}{15}\right) * 10152758140}{750 - 298.3} = 299690300\ N.mm$

$$= 299.7\ kN.m$$

6. $$\boxed{M_{w1} = 299.7\ kN.m}$$

Sec. ②

$A_s = 7\phi18 = 7\left[\frac{\pi * 18^2}{4}\right] = 1781\ mm^2$



① Take $n = 15$

② Get Z by taking $S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}$

$$b(Z)\left(\frac{Z}{2}\right) = n A_s (d - Z)$$

$$300(Z)\left(\frac{Z}{2}\right) = (15)(1781)(750 - Z)$$

$$\boxed{Z = 287.1\ mm}$$

③ Get $I_{nv} = \frac{bZ^3}{3} + n A_s (d - Z)^2$

$$I_{nv} = \frac{300\ (287.1)^3}{3} + (15)(1781)(750 - 287.1)^2 = 8090856524\ mm^4$$

④ $M_{wc} = \frac{F_c * I_{nv}}{Z} = \frac{9.5 * 8090856524}{287.1} = 267722525\ N.mm$

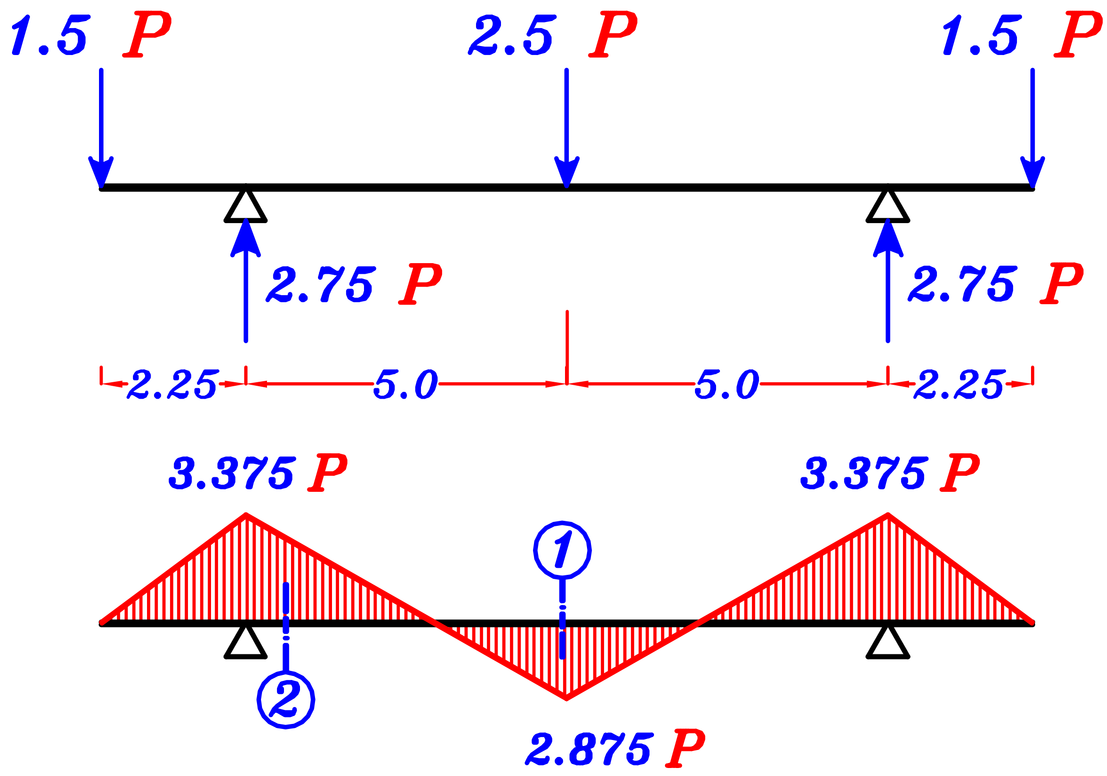
$= 267.72\ kN.m$

⑤ $M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \frac{\left(\frac{200}{15}\right) * 8090856524}{750 - 287.1} = 233048362\ N.mm$

$= 233.05\ kN.m$

⑥ $$\boxed{M_{w2} = 233.05\ kN.m}$$

Actual Moment.



Sec. 1 $M_{act.} = 2.875\ P$

To Get $P_w \longrightarrow M_{act.} = M_w$

$\therefore\ 2.875\ P_w = 299.7\ kN.m \longrightarrow P_{w1} = 104.24\ kN$

Sec. 2 $M_{act.} = 3.375\ P$

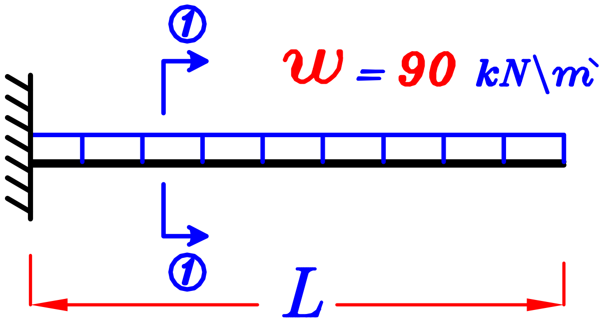
To Get $P_w \longrightarrow M_{act.} = M_w$

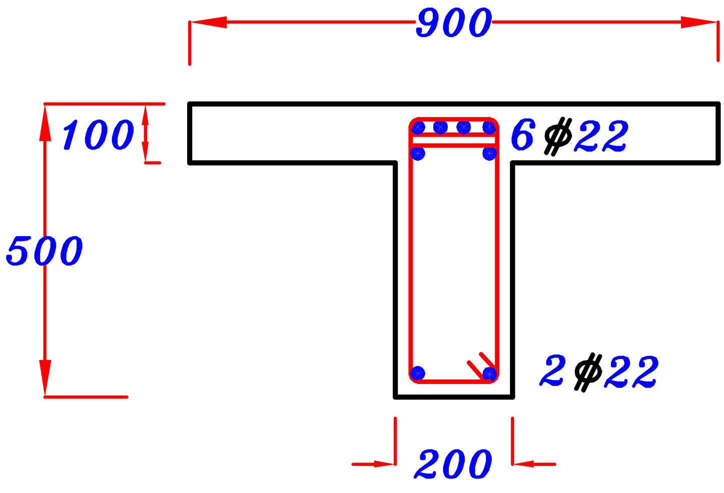
$\therefore\ 3.375\ P_w = 233.05\ kN.m \longrightarrow P_{w2} = 69.05\ kN$

P_w For all the beam is the least one of P_{w1}, P_{w2}

$$\boxed{P_w = 69.05\ kN}$$

Example.





Sec. (1–1)

Data.

$F_{cu} = 25 \; N\backslash mm^2$ $\qquad$ $F_y = 360 \; N\backslash mm^2$

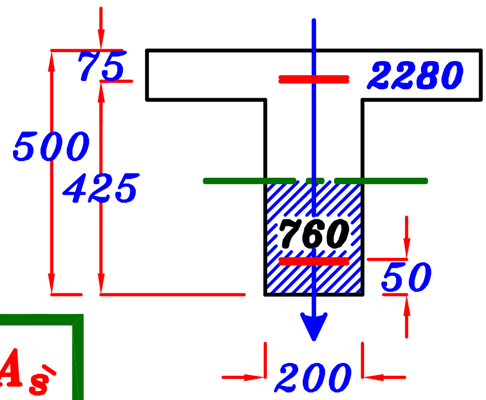
Req.

Find the maximum design length For the cantilever.

Solution.

$$A_s = 6\,\phi 22 = 6\left[\frac{\pi * 22^2}{4}\right] = 2280\ mm^2$$

$$A_s` = 2\,\phi 22 = 2\left[\frac{\pi * 22^2}{4}\right] = 760\ mm^2$$



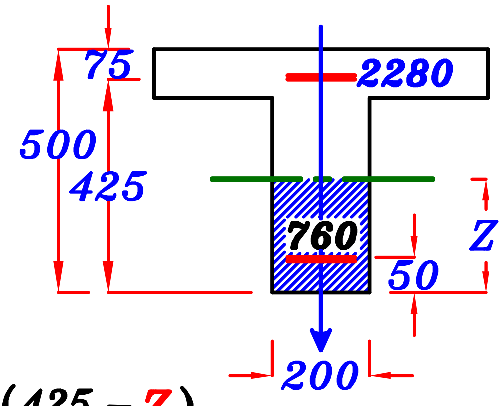
$$\boxed{\because \frac{A_s`}{A_s} = \frac{760}{2280} = 0.33 > 0.2 \quad \therefore \text{We can't neglect } A_s`}$$

Allowable stresses

$F_{cu} = 25 \; N\backslash mm^2 \longrightarrow F_c = 9.5 \; N\backslash mm^2$

$F_y = 360 \; N\backslash mm^2 \longrightarrow F_s = 200 \; N\backslash mm^2$

① Take $n = 15$



② Get Z by taking $S_{nv.\,\text{above (N.A.)}} = S_{nv.\,\text{under (N.A.)}}$

$$b(Z)\left(\frac{Z}{2}\right) + (n-1)A_{s`}(Z-d`) = n\,A_s(d-Z)$$

$$200(Z)\left(\frac{Z}{2}\right) + (14)(760)(Z-50) = (15)(2280)(425-Z)$$

$$\boxed{Z = 224.0 \text{ mm}}$$

③ Get $I_{nv} = \dfrac{bZ^3}{3} + (n-1)A_{s`}(Z-d`)^2 + n\,A_s(d-Z)^2$

$$I_{nv} = \frac{200(224.0)^3}{3} + (14)(760)(224.0-50)^2 + (15)(2280)(425-224.0)^2$$

$$= 2453145773 \text{ mm}^4$$

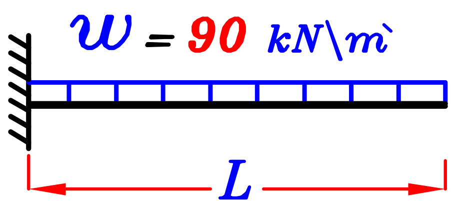
④ $M_{wc} = \dfrac{F_c * I_{nv}}{Z} = \dfrac{9.5 * 2453145773}{224.0} = 104039664.5 \text{ N.mm}$

$= 104.04 \text{ kN.m}$

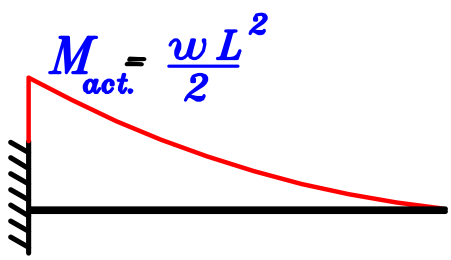
⑤ $M_{ws} = \dfrac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z} = \dfrac{\left(\frac{200}{15}\right) * 2453145773}{425-224.0} = 162729404.5 \text{ N.mm}$

$= 162.73 \text{ kN.m}$

⑥ $\boxed{M_w = 104.04 \text{ kN.m}}$



Actual Moment =



$$M_{act.} = \frac{wL^2}{2} = \frac{90\,L^2}{2} = 45\,L^2$$

To get the maximum design length $= L_w$

$$M_{act.} = M_w$$

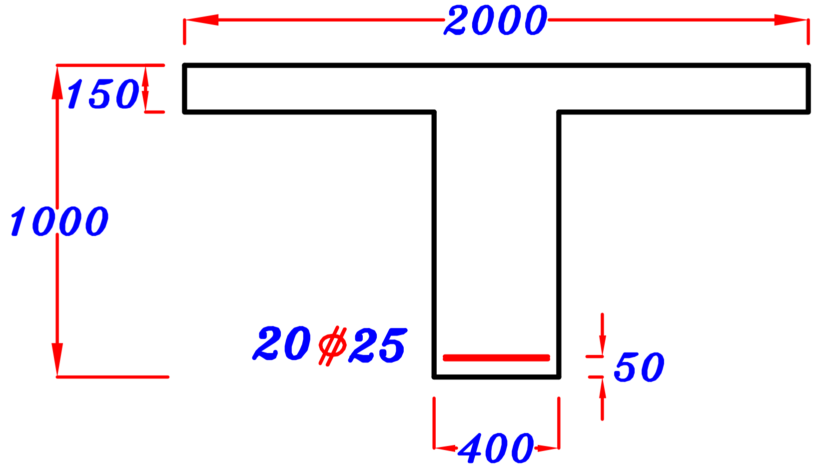
$$45\,L^2 = 104.04 \longrightarrow \boxed{L = 1.52 \text{ m}}$$

Example.

Data.

$F_{cu} = 25 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2$



Req. Calculate M_w

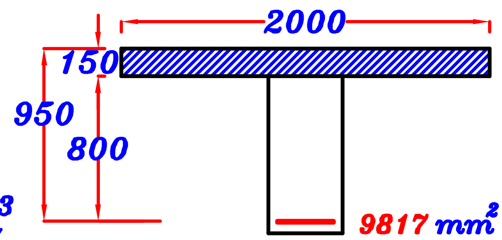
$$A_s = 20\,\phi 25 = 20\left[\frac{\pi * 25^2}{4}\right] = 9817\ mm^2$$

Allowable stresses

$F_{cu} = 25 \quad N\backslash mm^2 \longrightarrow F_c = 9.5 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2 \longrightarrow F_s = 200 \quad N\backslash mm^2$

To know IF Z is bigger or smaller than the Flange thickness $= 150\ mm$

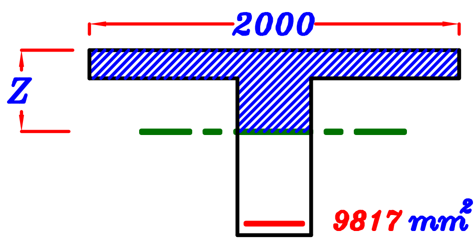


$S_{nv.}(\text{above}) = 150 * 2000 * (75) = 22500000\ mm^3$

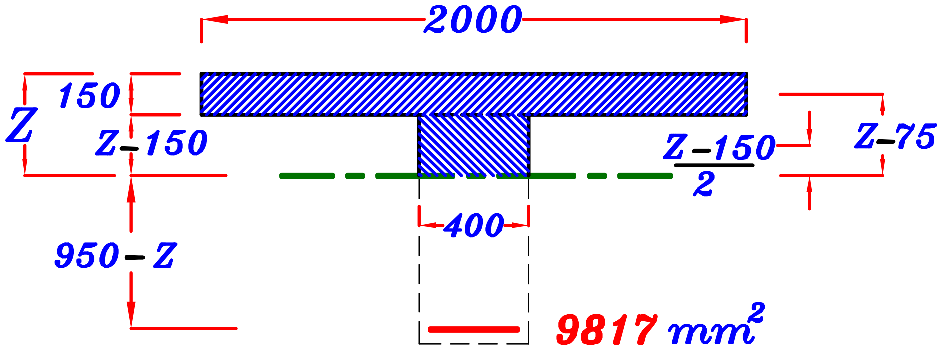
$S_{nv.}(\text{under}) = 15 * 9817 * (800) = 117804000\ mm^3$

$\because\ S_{nv.}(\text{under}) > S_{nv.}(\text{above})$

$\therefore\ Z > 150\ mm$



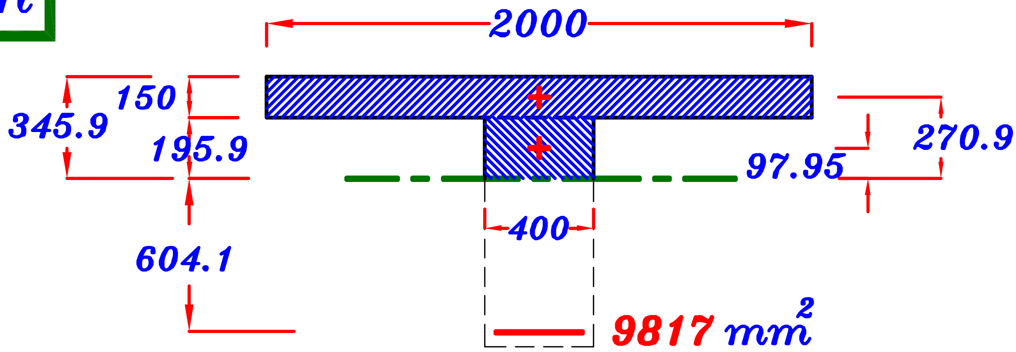
① Take $n = 15$



② Get Z by taking $S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}$

$$(2000)(150)(Z-75) + (400)(Z-150)\left(\frac{Z-150}{2}\right) = (15)(9817)(950-Z)$$

$$\boxed{Z = 345.9 \ mm}$$



③ $$I_{nv} = \frac{2000(150)^3}{12} + (2000)(150)(270.9)^2 + \frac{400(195.9)^3}{3} + (15)(9817)(604.1)^2 = 77319715230 \ mm^4$$

④ $$M_{wc} = \frac{\left(\frac{2}{3}\right) F_c * I_{nv}}{Z} \quad \text{------- } T\text{-}Sec.$$

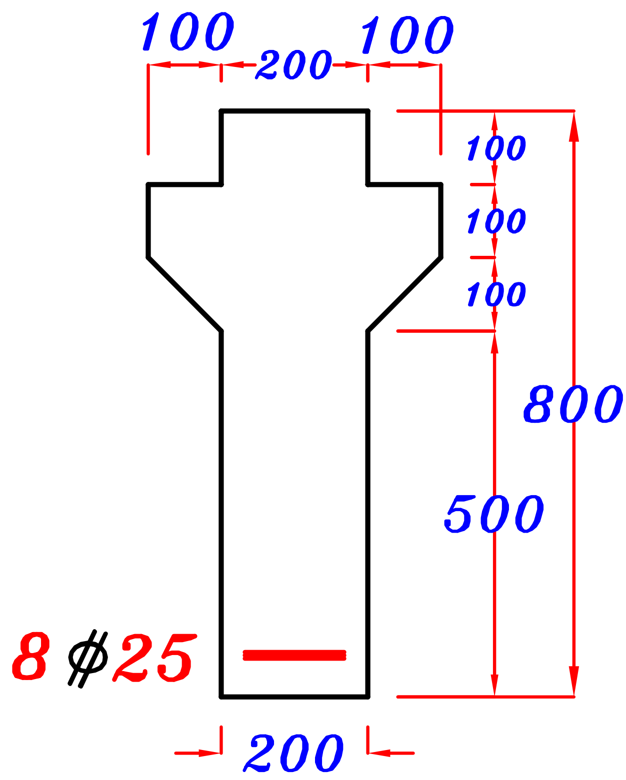
$$= \frac{\left(\frac{2}{3}\right) 9.5 * 77319715230}{345.9} = 1415702601 \ N.mm$$
$$= 1415.7 \ kN.m$$

⑤ $$M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z}$$

$$= \frac{\left(\frac{200}{15}\right) * 77319715230}{950 - 345.9} = 1706554439 \ N.mm$$
$$= 1706.55 \ kN.m$$

⑥ $$\boxed{M_w = 1415.7 \ kN.m}$$

Example.



Data.

$F_{cu} = 25 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2$

Req.

Calculate M_w

Solution.

$A_s = 8\phi 25 = 8\left[\frac{\pi * 25^2}{4}\right] = 3927\ mm^2$

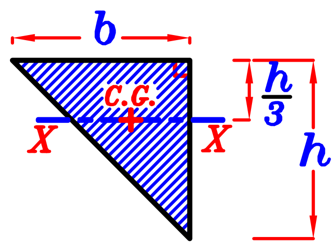
Allowable stresses

$F_{cu} = 25 \quad N\backslash mm^2 \longrightarrow F_c = 9.5 \quad N\backslash mm^2$

$F_y = 360\ N\backslash mm^2 \longrightarrow F_s = 200\ N\backslash mm^2$

Inertia For right angle Triangle

$$I_x = \frac{b\,h^3}{36}$$



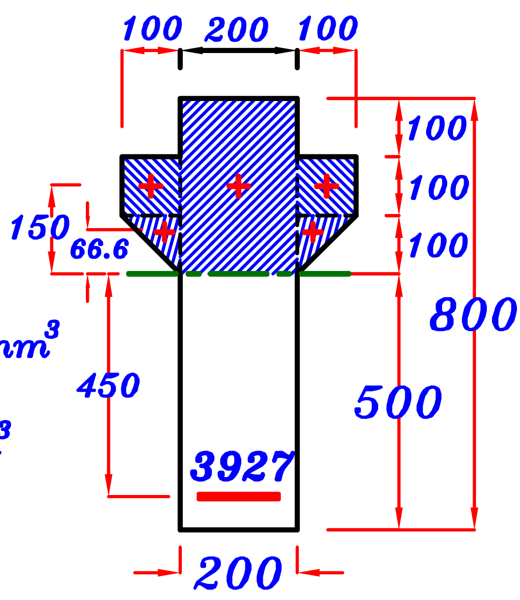
To know if Z is bigger or smaller than 300 mm

$S_{nv.}(above) = (200)(300)(150) + 2\,(100)(100)(150) + 2\left(\frac{1}{2}\right)(100)(100)(66.6) = 12666000\ mm^3$

$S_{nv.}(under) = 15 * 3927 * (450) = 26507250\ mm^3$

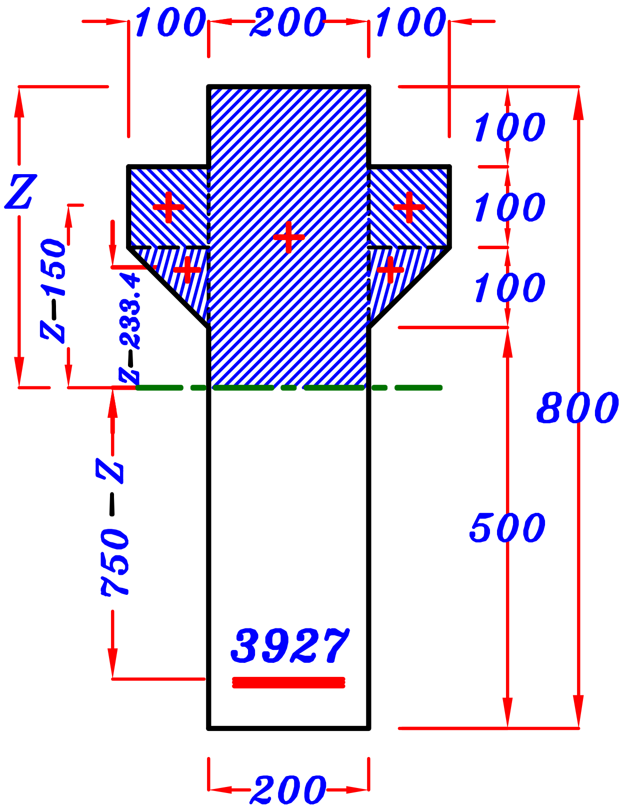
$\because\ S_{nv.}(under) > S_{nv.}(above)$

$\therefore\ Z > 300\ mm$



① Take $n = 15$

② Get Z by taking $\boxed{S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}}$



$$200(Z)\left(\frac{Z}{2}\right) + 2(100)(100)(Z-150)$$

$$+ 2\left(\frac{1}{2}\right)(100)(100)(Z-233.4)$$

$$= (15)(3927)(750-Z)$$

$$\boxed{Z = 387.77\ mm}$$

③ Get $I_{nv} = \dfrac{200(387.77)^3}{3} + 2\left(\dfrac{100*100^3}{12}\right) + 2(100)(100)(387.77-150)^2$

$$+ 2\left(\frac{100*100^3}{36}\right) + 2\left(\frac{1}{2}\right)(100)(100)(387.77-233.34)^2$$

$$+ (15)(3927)(750-387.77)^2 = 13007509270\ mm^4$$

④ $M_{wc} = \dfrac{F_c * I_{nv}}{Z} = \dfrac{9.5 * 13007509270}{387.77} = 318671733.5\ N.mm$

$= 318.67\ kN.m$

⑤ $M_{ws} = \dfrac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z} = \dfrac{\left(\frac{200}{15}\right) * 13007509270}{750 - 387.77} = 478793741.5\ N.mm$

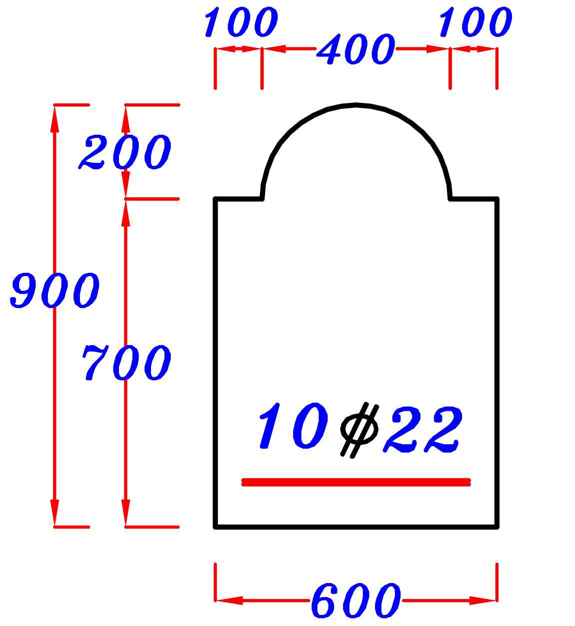
$= 478.7\ kN.m$

⑥ $\boxed{M_w = 318.67\ kN.m}$

Example.

Data. $F_{cu} = 25 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2$



Req.

Calculate M_w

Solution.

$$A_s = 10\phi 22 = 10\left[\frac{\pi * 22^2}{4}\right] = 3801\ mm^2$$

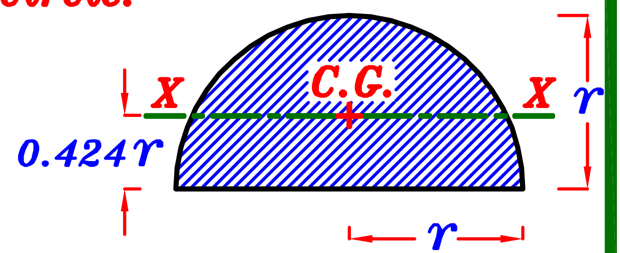
Allowable stresses

$F_{cu} = 25 \quad N\backslash mm^2 \longrightarrow F_c = 9.5 \quad N\backslash mm^2$

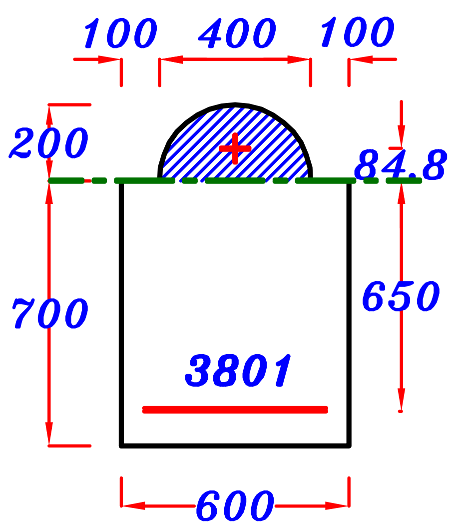
$F_y = 360 \quad N\backslash mm^2 \longrightarrow F_s = 200 \quad N\backslash mm^2$

Inertia For semi circle.

$$\boxed{I_x = 0.11\, r^4}$$



To know if Z is bigger or smaller than 200 mm

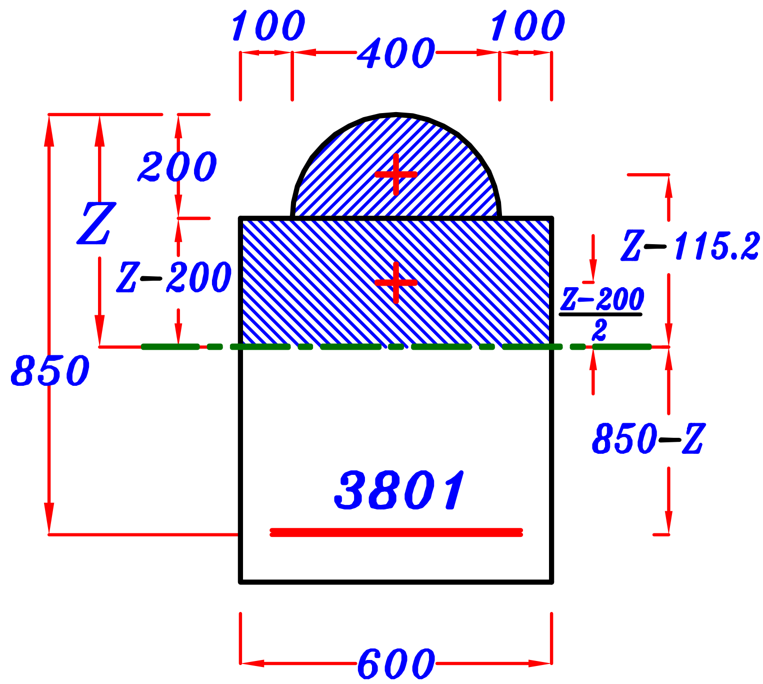


$$S_{nv.}(above) = \frac{\pi (200)^2}{2}(84.8) = 5328141.1\ mm^3$$

$$S_{nv.}(under) = 15 * 3801 * (650) = 37059750 \quad mm^3$$

$\because \; S_{nv.}(under) > S_{nv.}(above)$

$\therefore \; Z > 200\ mm$



① Take $n = 15$

② Get Z by taking $S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}$

$$\frac{\pi\,(200)^2}{2}\,(Z-115.2) + (600)(Z-200)\left(\frac{Z-200}{2}\right)$$

$$= (15)(3801)(850-Z)$$

$$\boxed{Z = 381.92\ mm}$$

③ Get $I_{nv} = 0.11\,(200)^4 + \frac{\pi\,(200)^2}{2}\,(381.92-115.2)^2$

$$+ \frac{600 * 181.92^3}{3} + (15)(3801)(850-381.92)^2$$

$$= 18341877640\ mm^4$$

④ $M_{wc} = \frac{F_c * I_{nv}}{Z} = \frac{9.5 * 18341877640}{381.92} = 456241719\ N.mm$

$= 456.24\ kN.m$

⑤ $M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z} = \frac{\left(\frac{200}{15}\right) * 18341877640}{850-381.92} = 522471305\ N.mm$

$= 522.47\ kN.m$

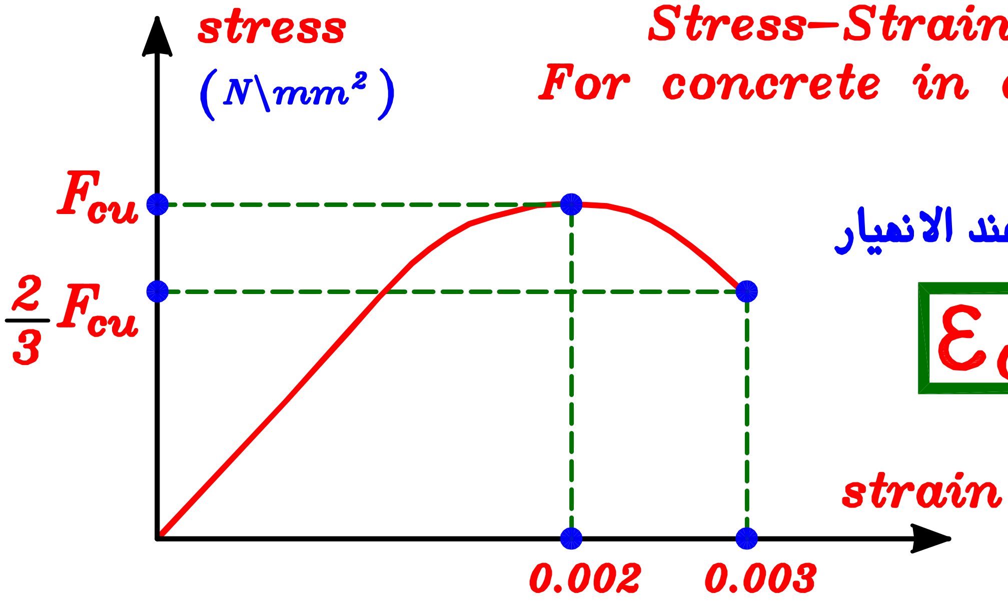
⑥ $\boxed{M_w = 456.24\ kN.m}$

(M_{ult})

Introduction of Ultimate Moment.

Types of Failure For Sections subjected to B.M. only.

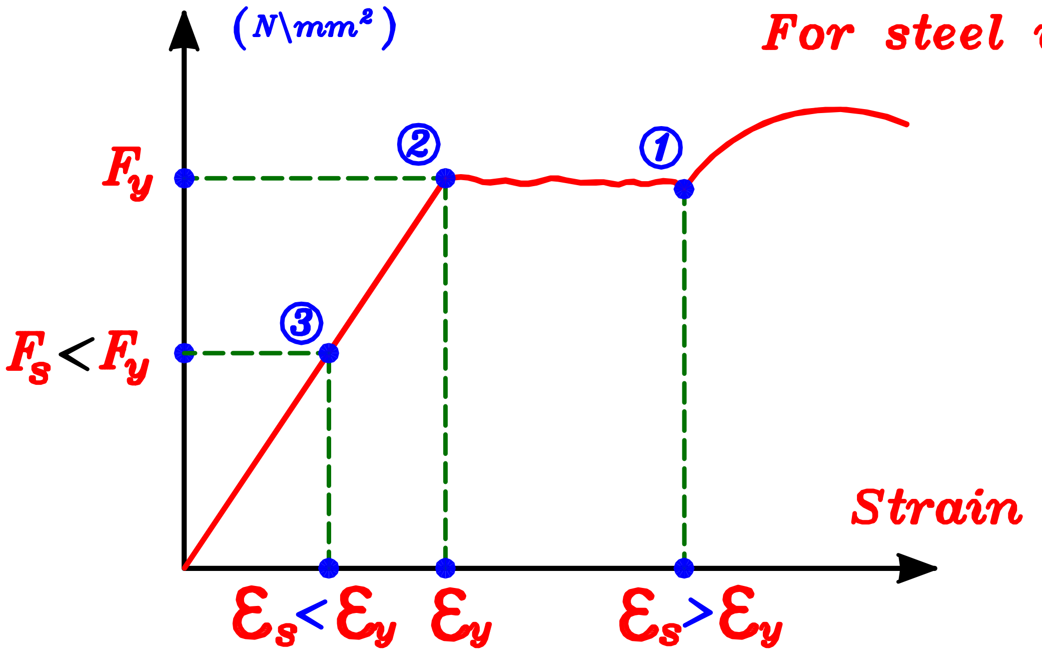
Stress–Strain curve For concrete in compression



استطاله الخرسانه عند الانهيار

$$\boxed{\varepsilon_c \simeq 0.003}$$

Stress–Strain curve For steel in tension



$$\varepsilon_s = \frac{F_s}{E_s} = \frac{F_s}{2*10^5} \quad , \quad \varepsilon_y = \frac{F_y}{E_s} = \frac{F_y}{2*10^5}$$

$$\text{when } \varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$$

Types of Sections at Failure.

① Under Reinforced Sections. كميه الحديد قليله

و فيه يصل الاجهاد على الحديد الى أقصى مقاومة له F_y

بينما لم يصل الاجهاد علىالخرسانه الى أقصى مقاومه لها F_{cu} .

Has a (Ductile Failure)

or called (Tension Failure)

إنهيار غير مفاجئ

② Balanced Sections. كميه الحديد متوسطه

و فيه يصل الاجهاد على الحديد الى أقصى مقاومة له F_y في نفس الوقت

الذى يصل فيه الاجهاد على الخرسانه الى أقصى مقاومه لها F_{cu} .

Has a (Brittle Failure)

or called (Balanced Failure)

انهيار مفاجئ

③ Over Reinforced Sections. كميه الحديد كبيره

و فيه يصل الاجهاد على الخرسانه الى أقصى مقاومه لها F_{cu}

قبل أن يصل الاجهاد على الحديد الى أقصى مقاومه له F_y .

Has a (Brittle Failure)

or called (Compression Failure)

انهيار مفاجئ

① Under Reinforced Sections.

و فيه يصل الاجهاد على الحديد الى أقصى مقاومة له F_y

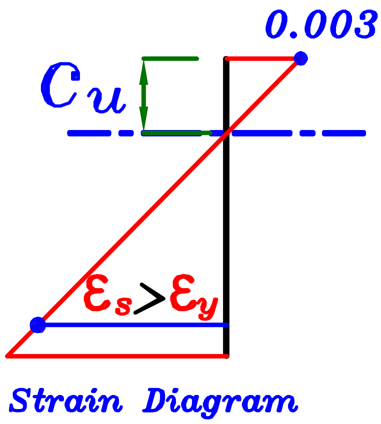
بينما لم يصل الاجهاد علىالخرسانه الى أقصى مقاومه لها F_{cu} .

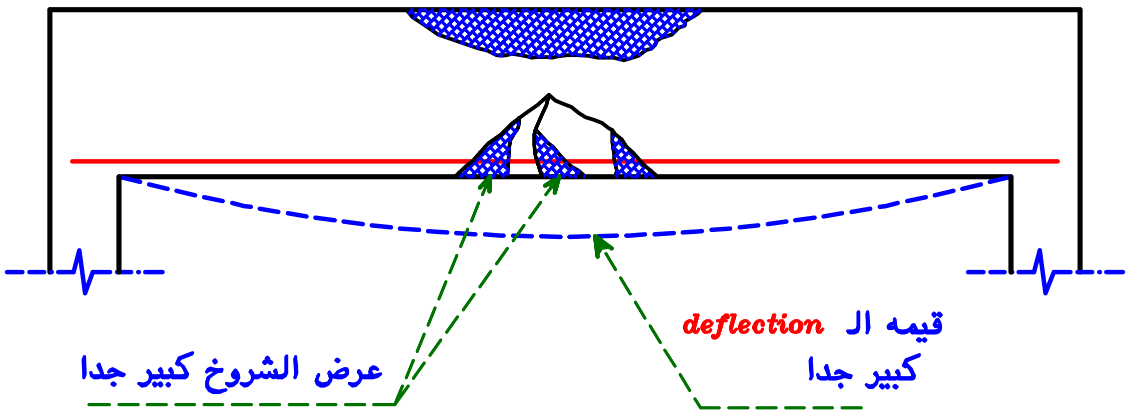
أى يزيد عرض الشروخ كثيرا قبل حدوث الإنهيار

(أى قبل أن تتكسر الخرسانه من جهه الضغط)

و هذا الإنهيار هو المفضل لأنه **إنهيار غير مفاجىء** .

(Ductile Failure)



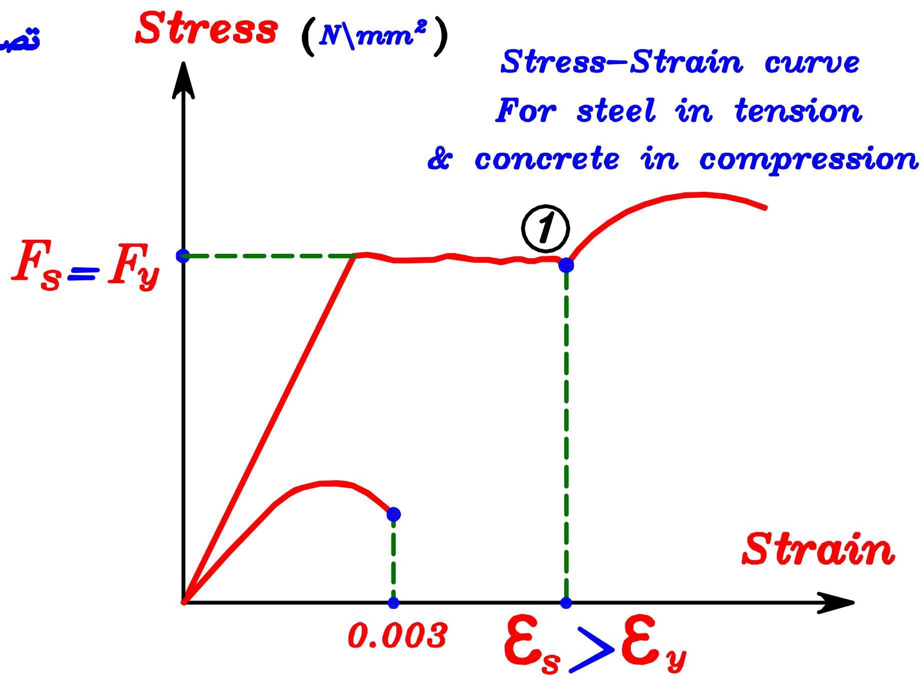


و يسمى *Under Reinforced Section.* لأن كميه الحديد به تكون قليله نسبياً .

F_{cu} تصل الخرسانه الى أقصى إجهاد لها

F_y بعد وصول الحديد إلى

$$\boxed{\begin{aligned} &\varepsilon_s > \varepsilon_y \\ &F_s = F_y \\ &\varepsilon_c = 0.003 \end{aligned}}$$



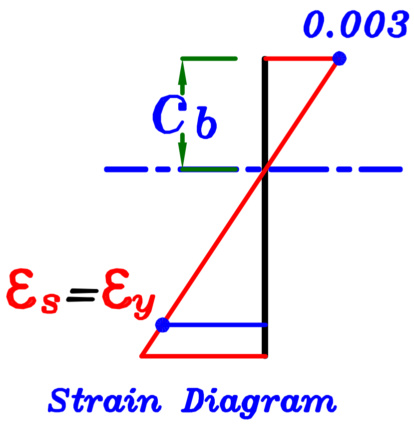
② *Balanced Section.*

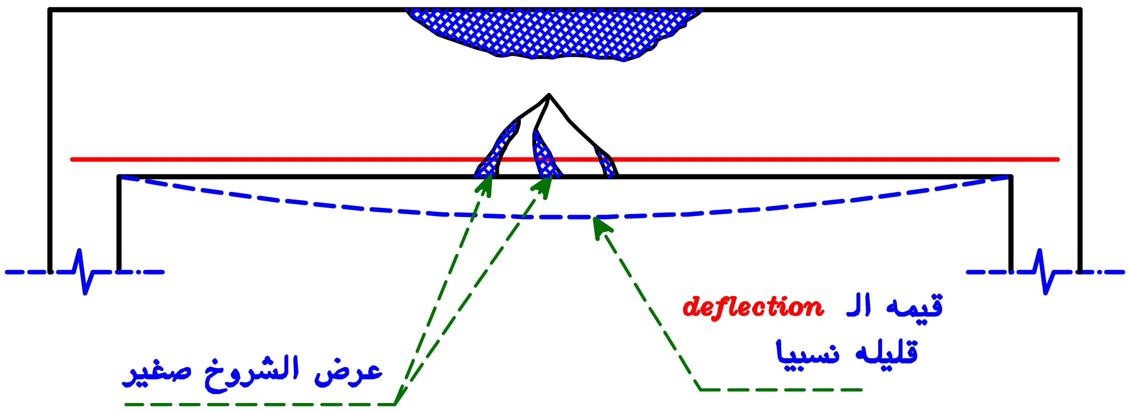
و فيه يصل الاجهاد على الحديد الى أقصى مقاومة له F_y فى نفس الوقت الذى يصل فيه الاجهاد على الخرسانه الى أقصى مقاومه لها F_{cu} .

و يحدث الإنهيار بإنكسار الخرسانه من جهه الضغط .

و هذا الإنهيار غير مفضل لأنه **إنهيار مفاجئ** .

(*Brittle Failure*)



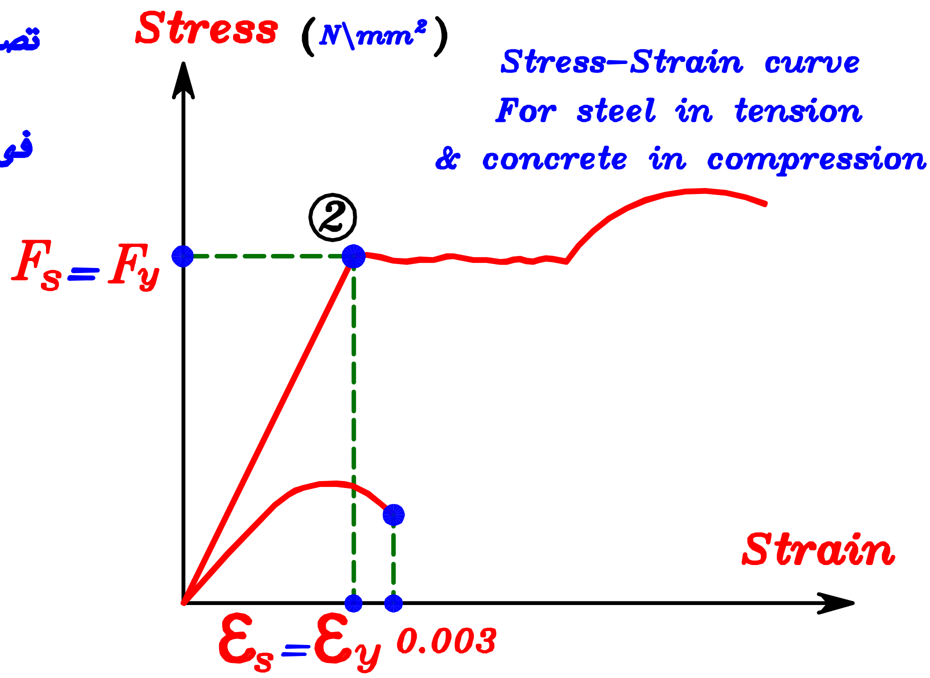


و يسمى *Balanced Section.* لأن الخرسانه و الحديد يصلوا الى مرحله الانهيار فى نفس الوقت تماماً (و هذه حاله نادره الحدوث فى الواقع)

F_{cu} تصل الخرسانه الى أقصى إجهاد لها

F_y فى نفس وقت وصول الحديد إلى

$$\boxed{\begin{aligned} \varepsilon_s &= \varepsilon_y \\ F_s &= F_y \\ \varepsilon_c &= 0.003 \end{aligned}}$$



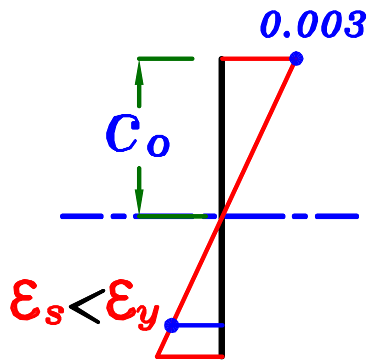
③ Over Reinforced Sections.

و فيه يصل الاجهاد على الخرسانه الى أقصى مقاومه لها F_{cu}
قبل أن يصل الاجهاد على الحديد الى أقصى مقاومه له F_y .

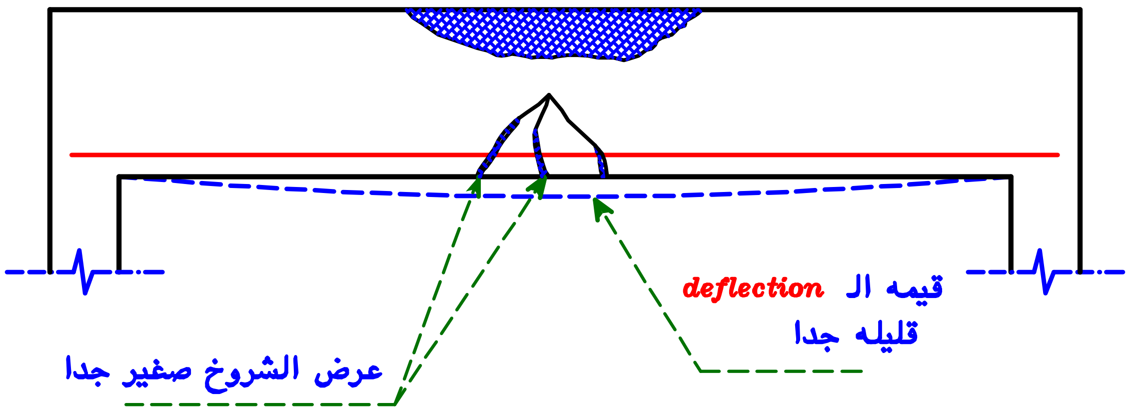
و يكون عرض الشروخ صغير جداً قبل إنهيار الخرسانه فى الضغط .

و هذا النوع من الإنهيار سيئ جدا لأنه لا يعطى أى مؤشر قبل الإنهيار .

(Brittle Failure)



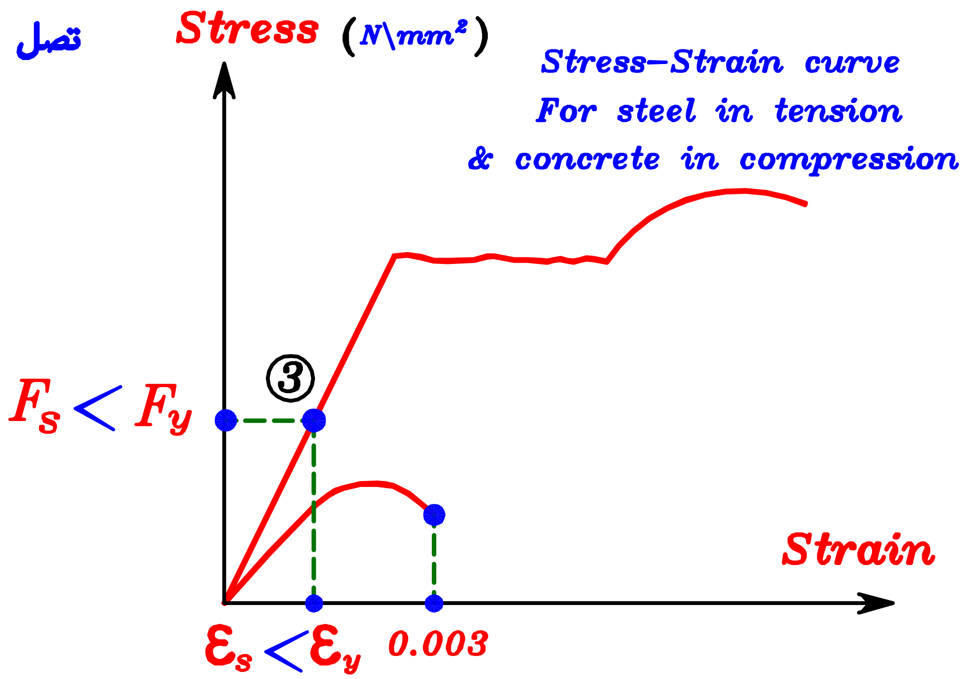
Strain Diagram

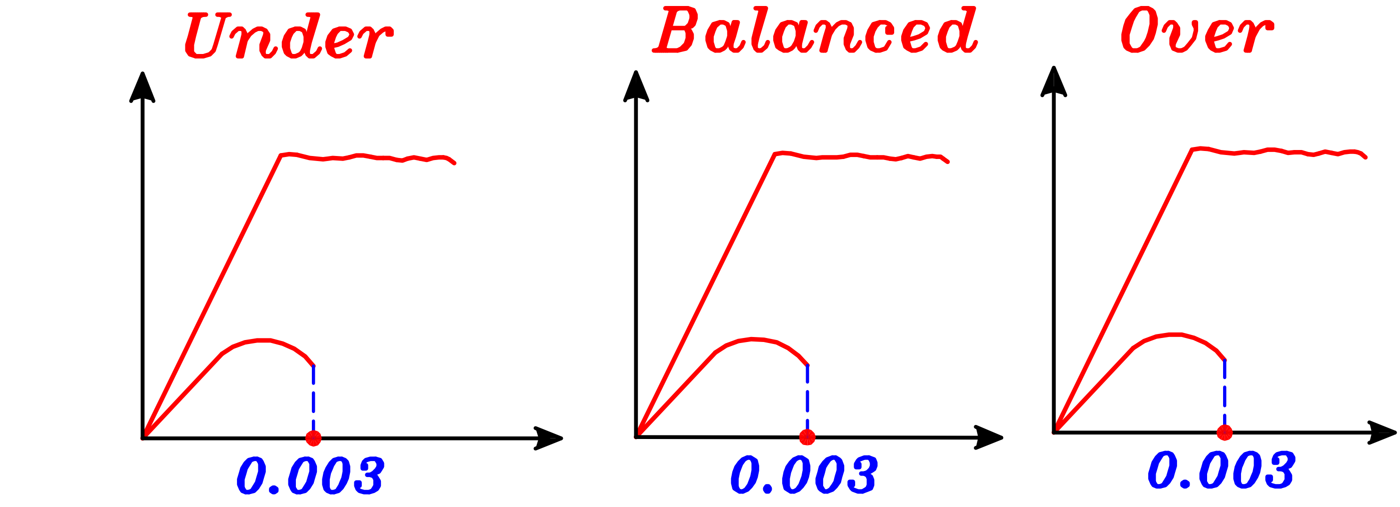


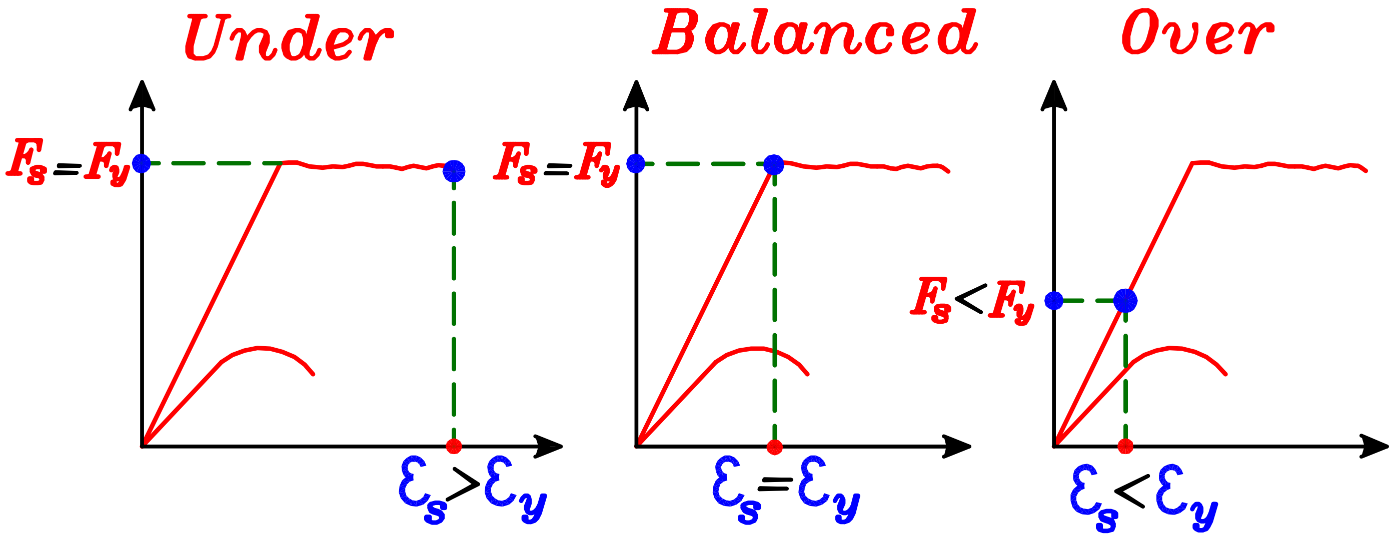
و يسمى *Over Reinforced Section.* لأن كميه الحديد به تكون كبيره .

F_{cu} تصل الخرسانه الى أقصى إجهاد لها
F_y قبل وصول الحديد إلى

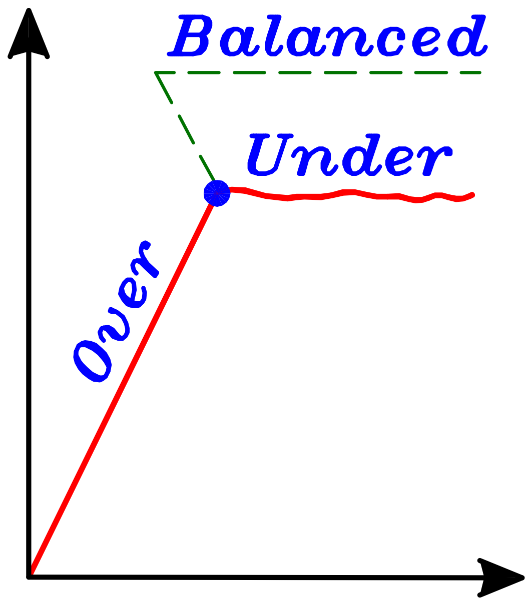
$$\boxed{\begin{array}{l} \varepsilon_s < \varepsilon_y \\ F_s < F_y \\ \varepsilon_c = 0.003 \end{array}}$$

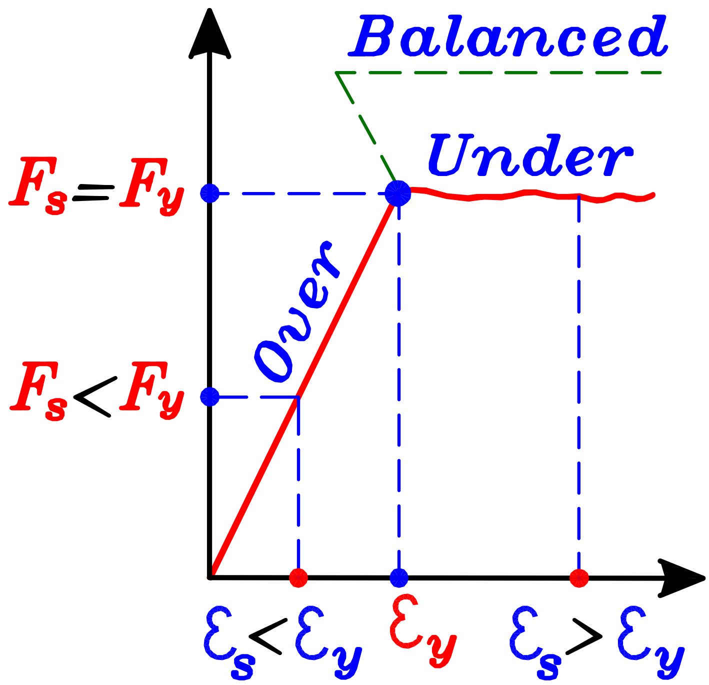



Under
Balanced
Over
0.003
0.003
0.003


Under
Balanced
Over
$F_s = F_y$
$F_s = F_y$
$F_s < F_y$
$\varepsilon_s > \varepsilon_y$
$\varepsilon_s = \varepsilon_y$
$\varepsilon_s < \varepsilon_y$

حفظ

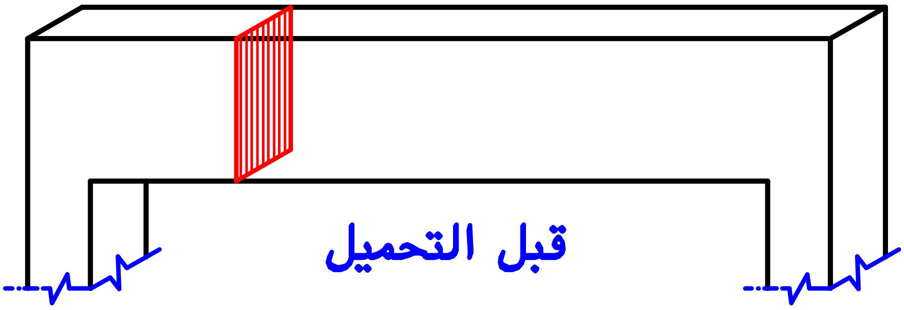

Balanced
Under
Over

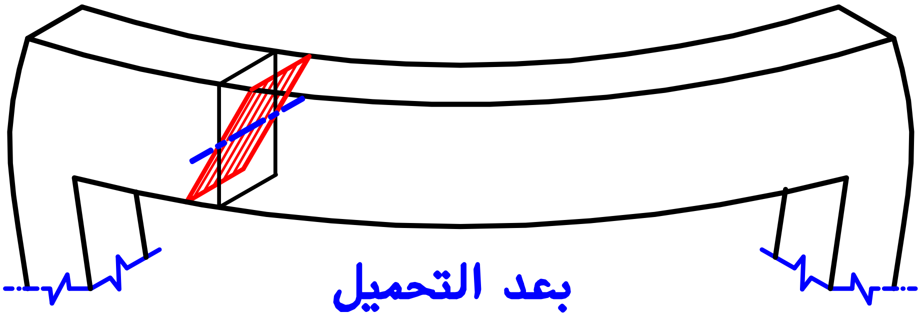

Balanced
Under
Over
$F_s = F_y$
$F_s < F_y$
$\varepsilon_s < \varepsilon_y$
ε_y
$\varepsilon_s > \varepsilon_y$

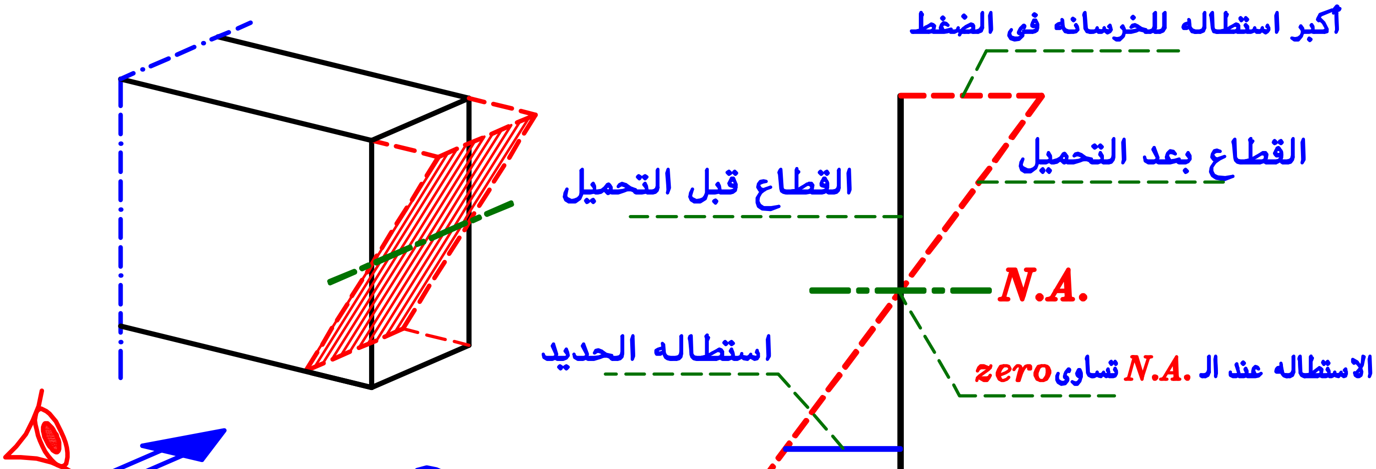
Strain Diagram at Failure.

Elastic Theory

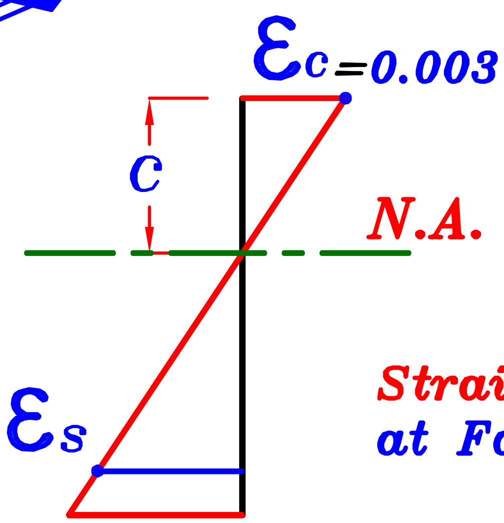
هى نظريه تعتمد على أن شكل القطاع المستوى قبل التحميل يظل مستوى بعد التحميل ٠





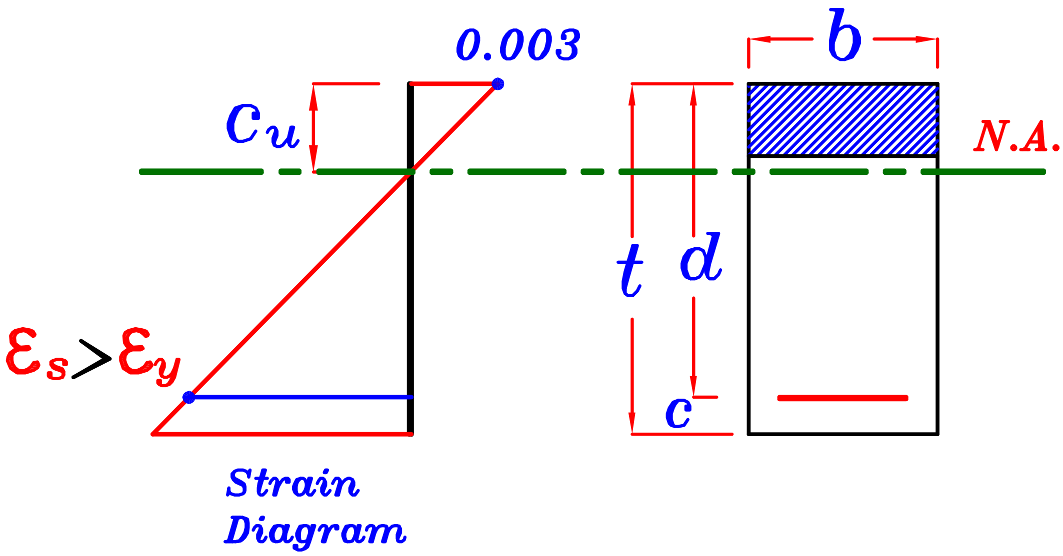


Strain Diagram.

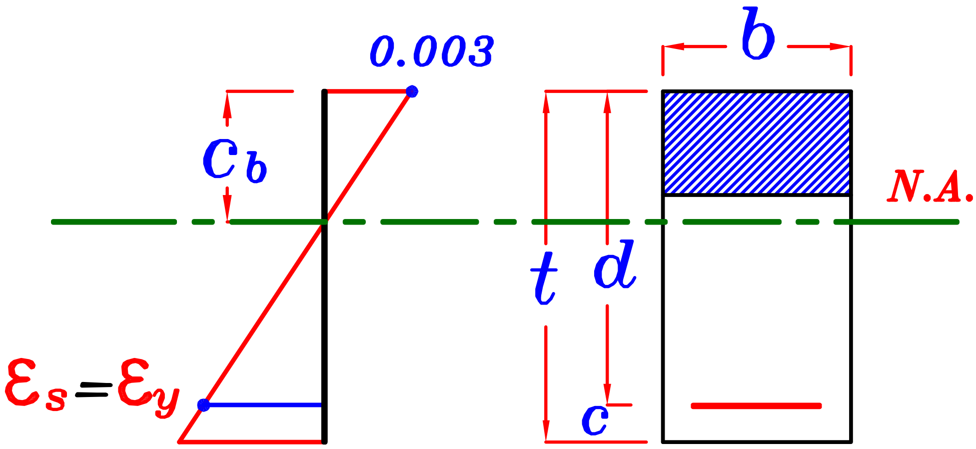


Strain Diagram.
at Faliure

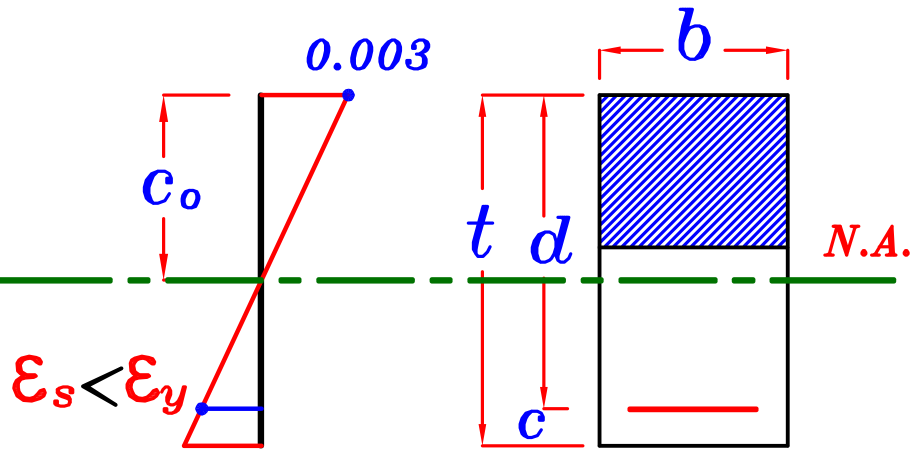
① Under Reinforced Sections.



② Balanced Sections.



③ Over Reinforced Sections.

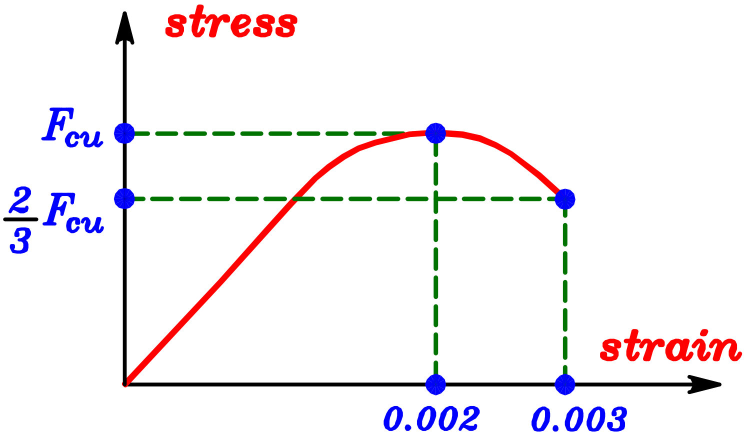


note $\boxed{c_u < c_b < c_o}$

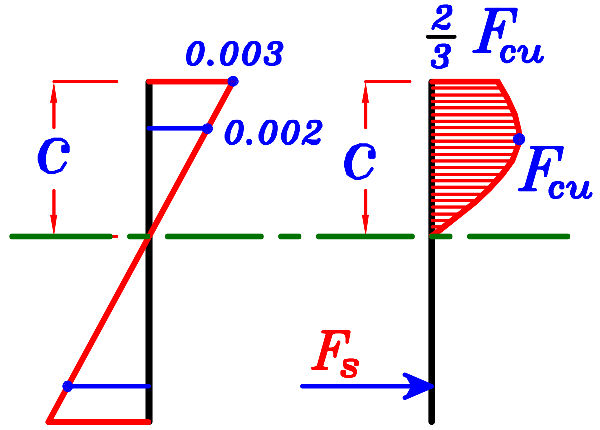
Stress Diagram at Failure.

ممكن استنتاج شكل ال Normal Stress diagram

من شكل كلا من Strian diagram و ال Stress–Strain curve



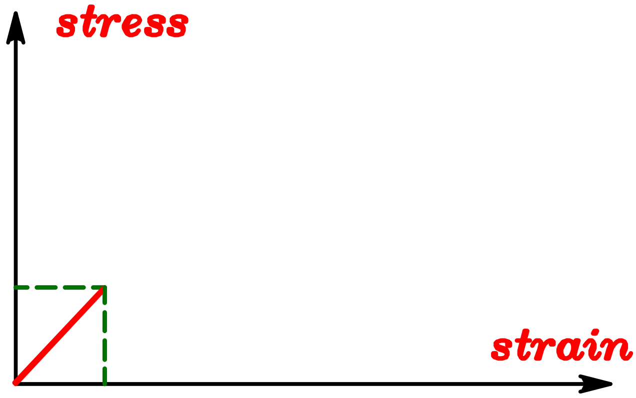
Stress–Strain curve



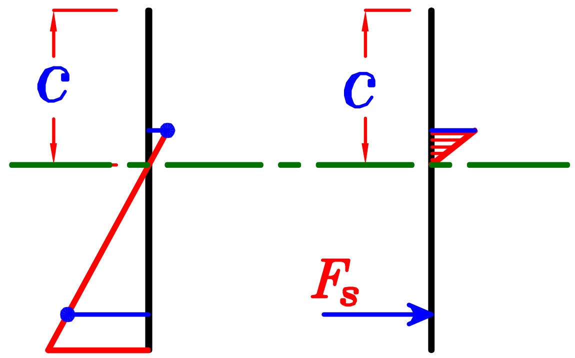
Strain Diagram

Normal Stress Diagram

فى البدايه عندما كان ال Strain قليل كان ال stress قليل و كان فى البدايه خط مستقيم



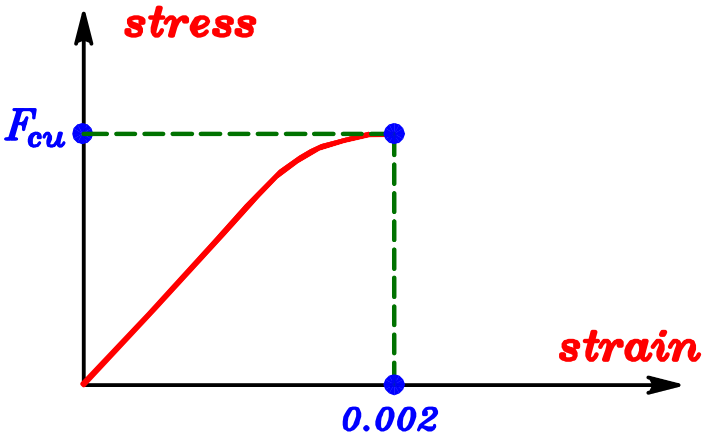
Stress–Strain curve



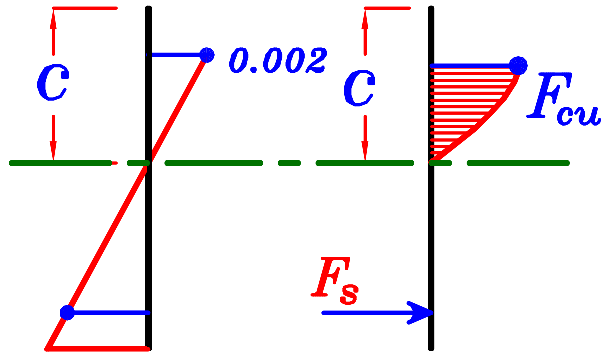
Strain Diagram

Normal Stress Diagram

عند وصول ال Strain الى قيمه 0.002 يكون ال Stress أخذ شكل منحنى و وصل الى قيمه F_{cu}



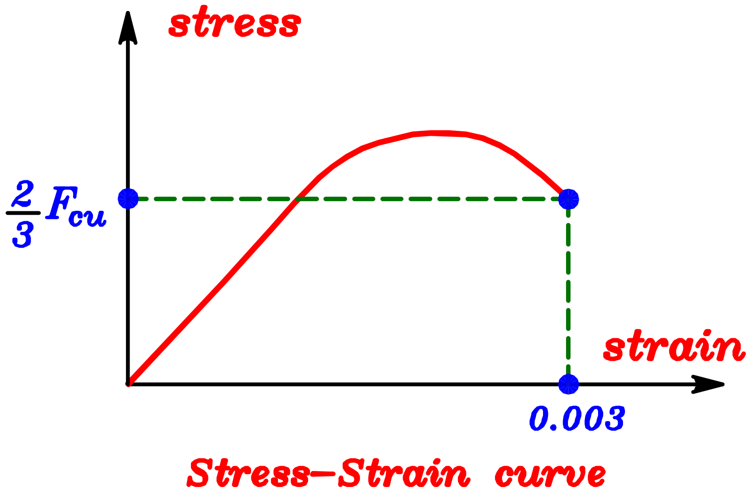
Stress–Strain curve

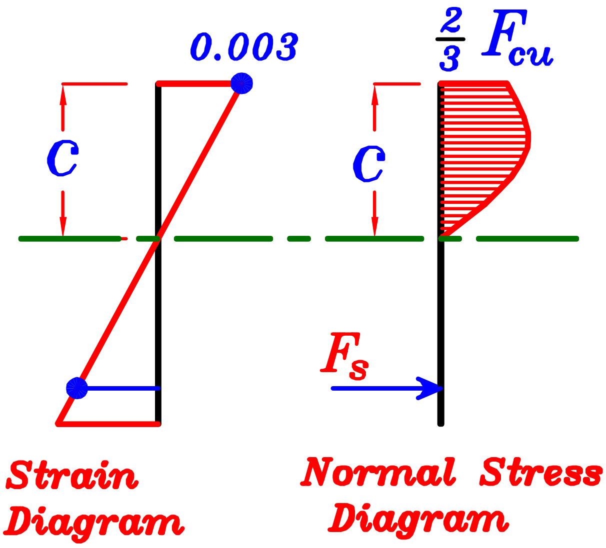


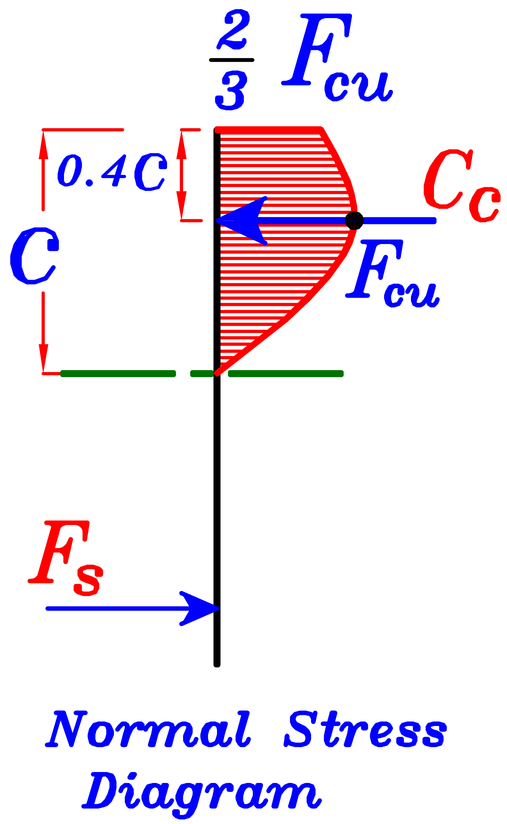
Strain Diagram

Normal Stress Diagram

عند وصول الـ **Strain** الى قيمه 0.003 تبدأ الخرسانه فى الانهيار و يكون الـ **Stress** و صل الى قيمه $\frac{2}{3}F_{cu}$







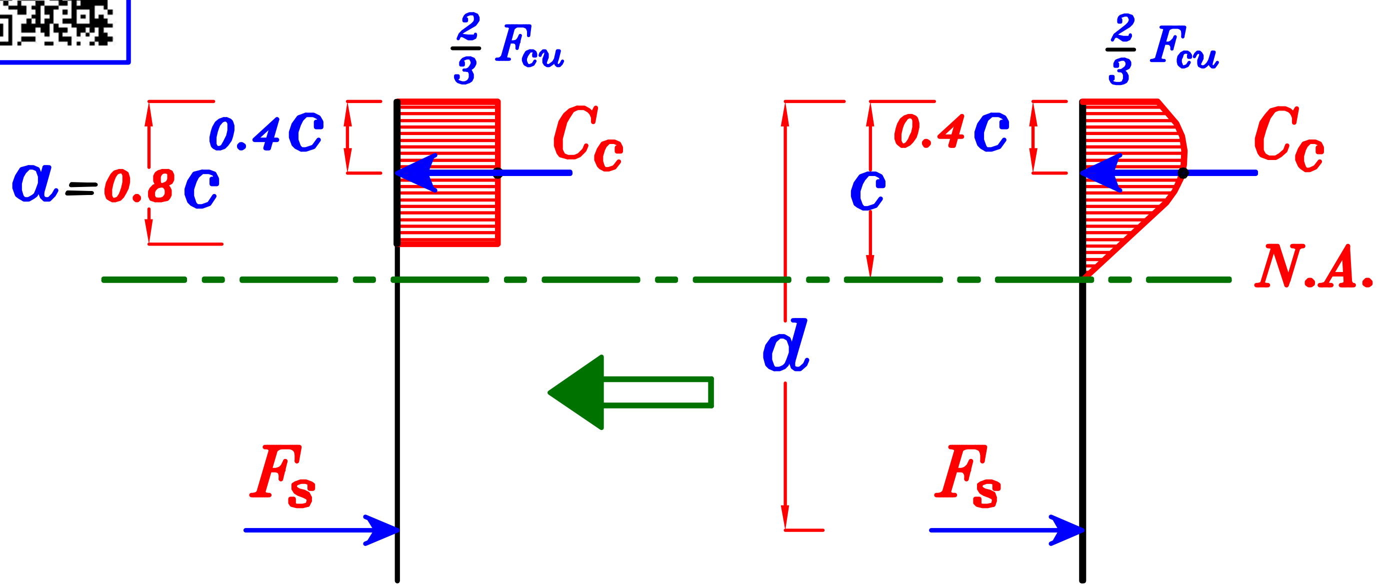
لان شكل الـ **Stress** منحنى لذا فصعب التعامل معه لاننا اذا اردنا حساب مساحه المنحنى أو تحديد مكان المحصله سنحتاج استخدام التكامل .

لذا لتسهيل الحسابات سنلجأ فى الحسابات لـ **Stress** مكافئ يسمى **Equivalent Stress diagram** على شكل مستطيل لكى يكون سهل فى الحسابات

و لكن شرط أن تكون مساحته هى نفس مساحه الـ **Stress** الاصلى

و مكان محصلته هو نفس مكان محصله الـ **Stress** الاصلى

محصله القوى C_c تكون لها نفس القيمه و تؤثر فى نفس المكان



Equivalent Stress Diagram — *Actual Stress Diagram*

لكى تؤثر محصله ال **Equivalent Stress** تؤثر فى نفس مكان المحصله الاصليه أى على بعد $0.4\,C$

اذا سيكون طول ال **Equivalent Stress** يساوى a حيث $(a = 0.8\,C)$

و بتساوى مساحه ال **Equivalent Stress** بمساحه ال **Stress** الاصلى المحسوب بالتكامل

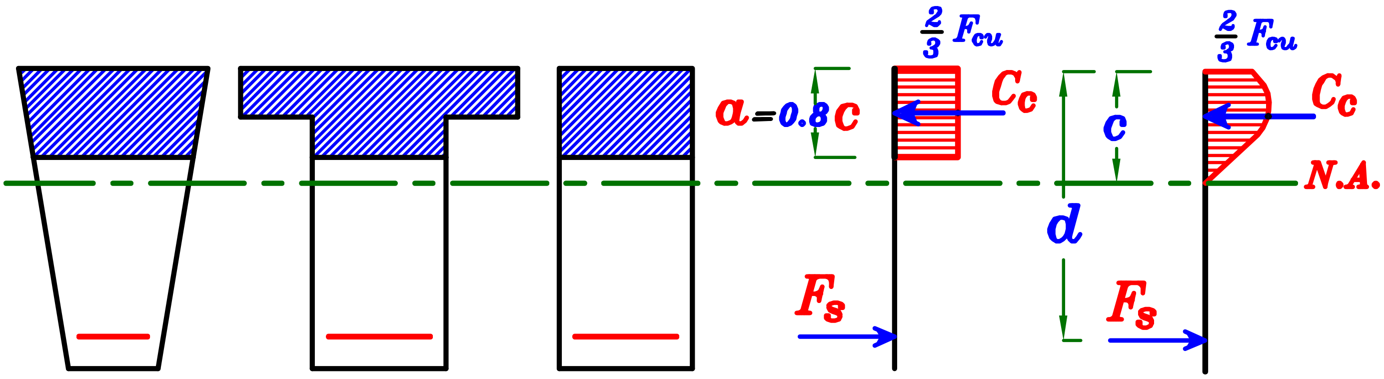
اتضح ان القيمه الثابته لل **Equivalent Stress** تساوى $\frac{2}{3} F_{cu}$

$$\because \quad \boxed{a = 0.8\,C} \qquad \therefore \quad \boxed{C = 1.25\,a}$$

ملحوظه .

شكل ال ***Equivalant Stress*** المستنتج بحيث تكون قيمه و مكان محصله القوى له تساوى نفس قيمه و مكان محصله ال ***Actual Stress*** للقطاعات ***R-Sec, T-Sec., L-Sec. & Trapezoidal Sec.***

تكون قيمه $\boxed{a = 0.8\ C}$

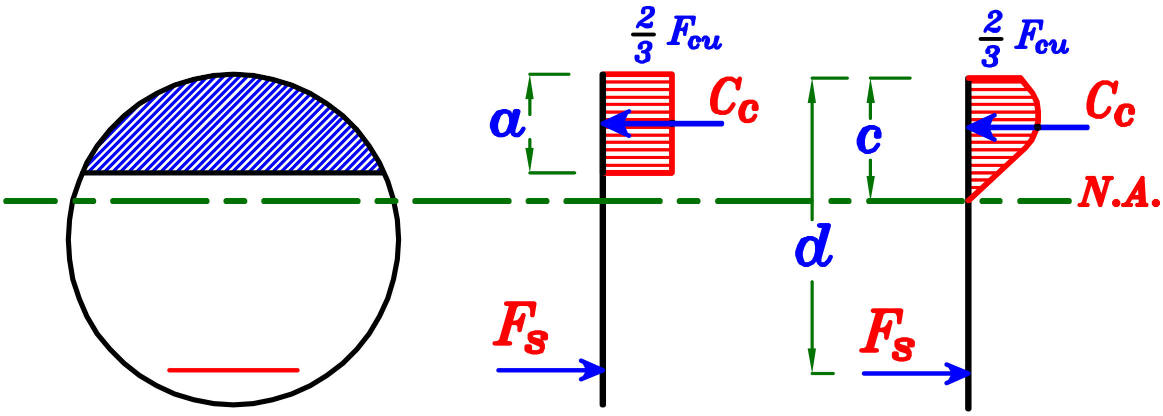


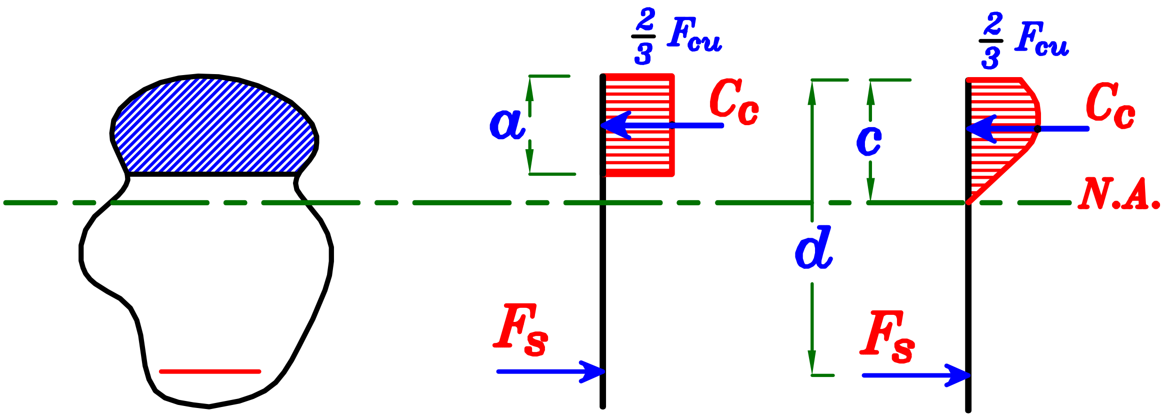
اما اى شكل اخر مثل القطاعات الدائريه او غير منتظمه الشكل

فيجب علينا لتحديد قيمه a التى تجعل قيمه و مكان محصله القوى على الخرسانه

لشكل ال ***Equivalant Stress*** هى نفس قيمه و مكان محصله القوى على الخرسانه

لل ***Actual Stress*** و ذلك عن طريق التكامل . $\boxed{a \neq 0.8\ C}$

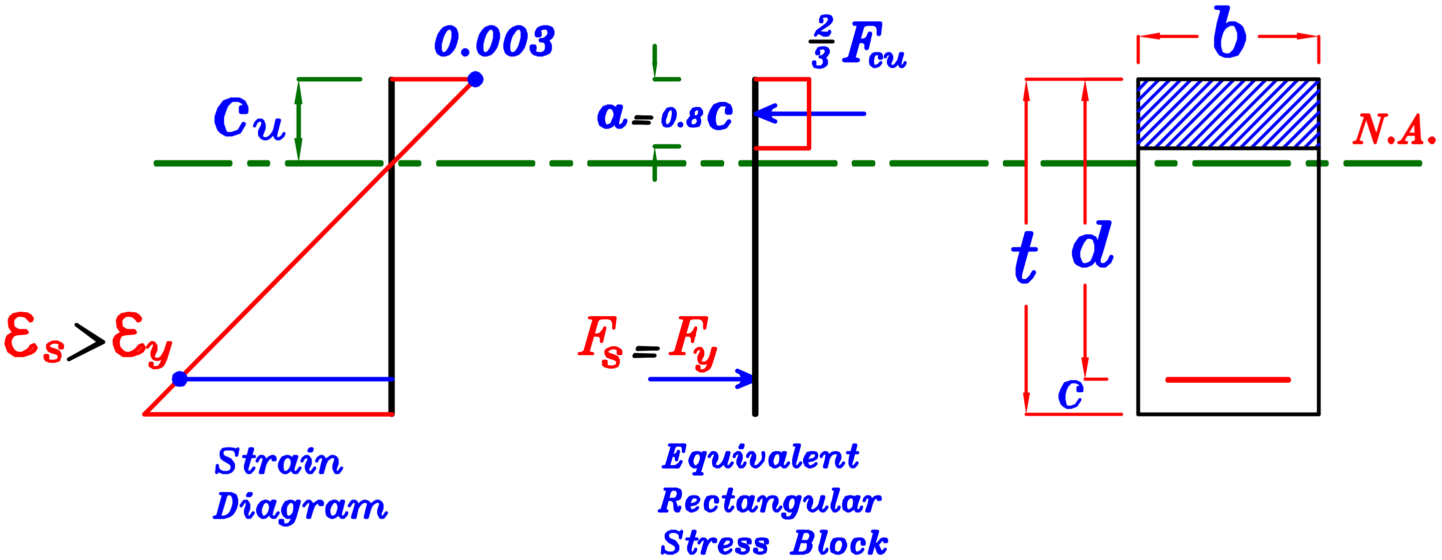




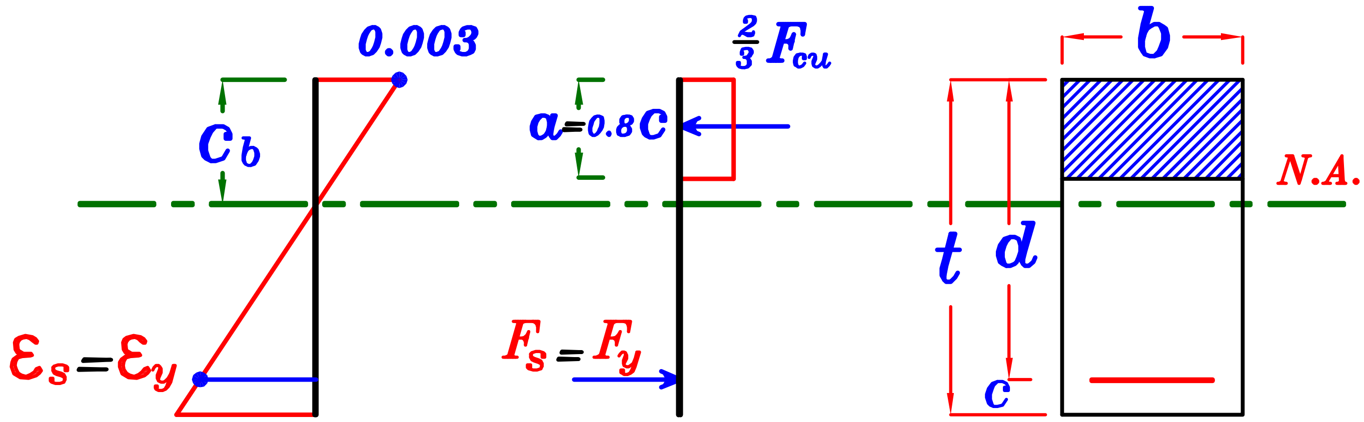
و فى هذا الملف سنتناول دراسه القطاعات المنتظمه فقط

R-Sec , T-Sec. , L-Sec. & Trapezoidal Sec.

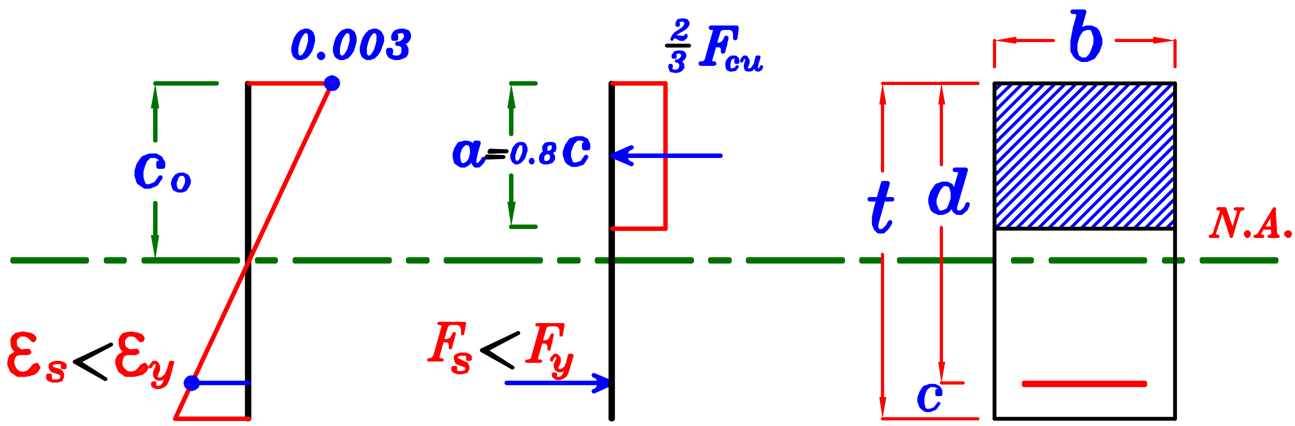
① Under Reinforced Sections.



② Balanced Sections.



③ Over Reinforced Sections.



For Beams at Faliure.

فى مرحله الانهيار لان شكل الاصلى لل **stress** عباره عن منحنى

اذا معادله ال **Normal stress** $\boxed{F = \frac{M\,y}{I}}$ لن تكون صحيحه

و بالتالى كل حسابات القطاع S_{nv} , I_{nv} , I_g , A_s , n لن تكون صحيحه

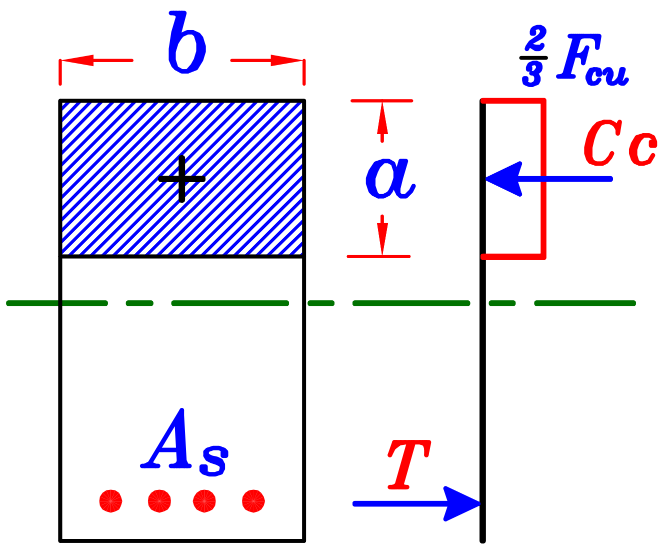
لذا فى مرحله الانهيار لن نستطيع الا استخدام معادلتين فقط.

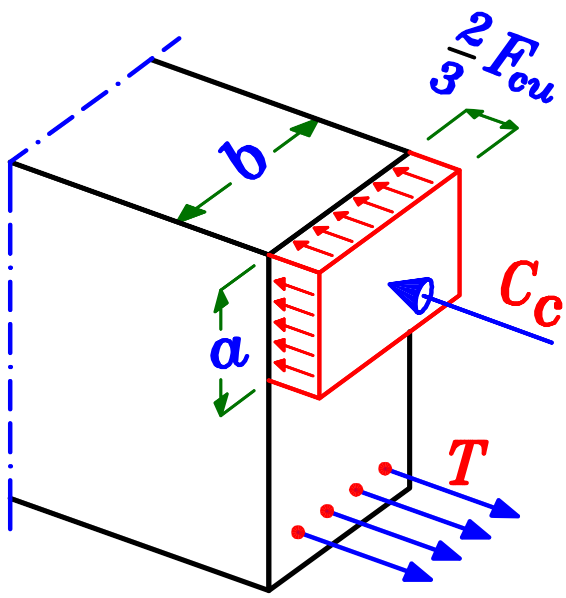
1. Equilibrium Equation. فهم
2. Compatibility Equation. حفظ

Calculations of Normal Forces.

لحساب قيمه أى قوه تؤثر على القطاع سواء ضغط أو شد

$$\boxed{Force = Stress * Area}$$





Compression on Concrete

$$C_c = Stress * Area = \frac{2}{3} F_{cu} * (a * b)$$

Tension

$$T = Stress * Area = F_s * A_s$$

① Equilibrium Equation. معادلة الاتزان

فى أى قطاع لكى يكون متزن

يجب أن يكون مجموع القوى الخارجيه تساوى مجموع القوى الداخليه

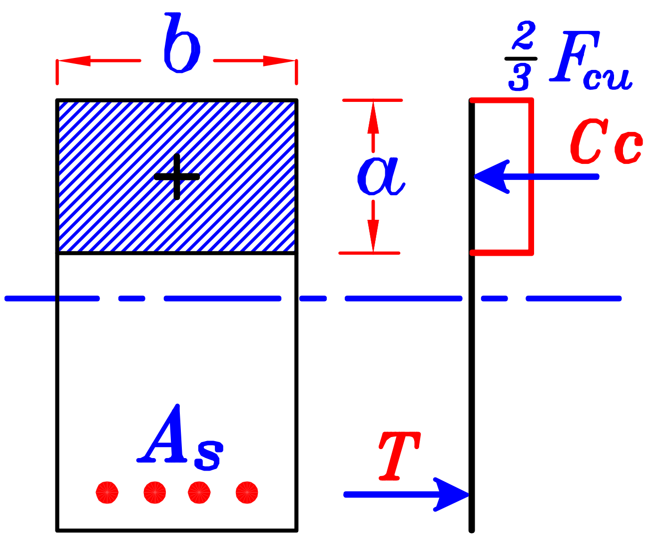
External Normal Force = Zero و لان القوى المحوريه الخارجيه على الكمرات تساوى صفر

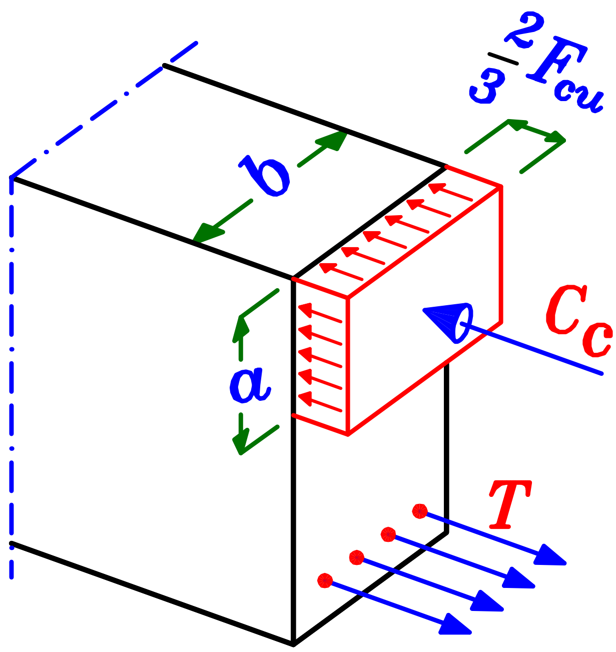
اذا سيكون مجموع القوى المحوريه الداخليه المؤثره على القطاع أيضا تساوى صفر

$\therefore$ Compression Forces + Tension Forces = Zero

$\therefore$ $\boxed{\text{Compression Forces} = \text{Tension Forces}}$

ⓐ Without Compression steel.





$$C_c = Stress * Area = \frac{2}{3}F_{cu} * (a * b)$$

$$T = Stress * Area = F_s * A_s$$

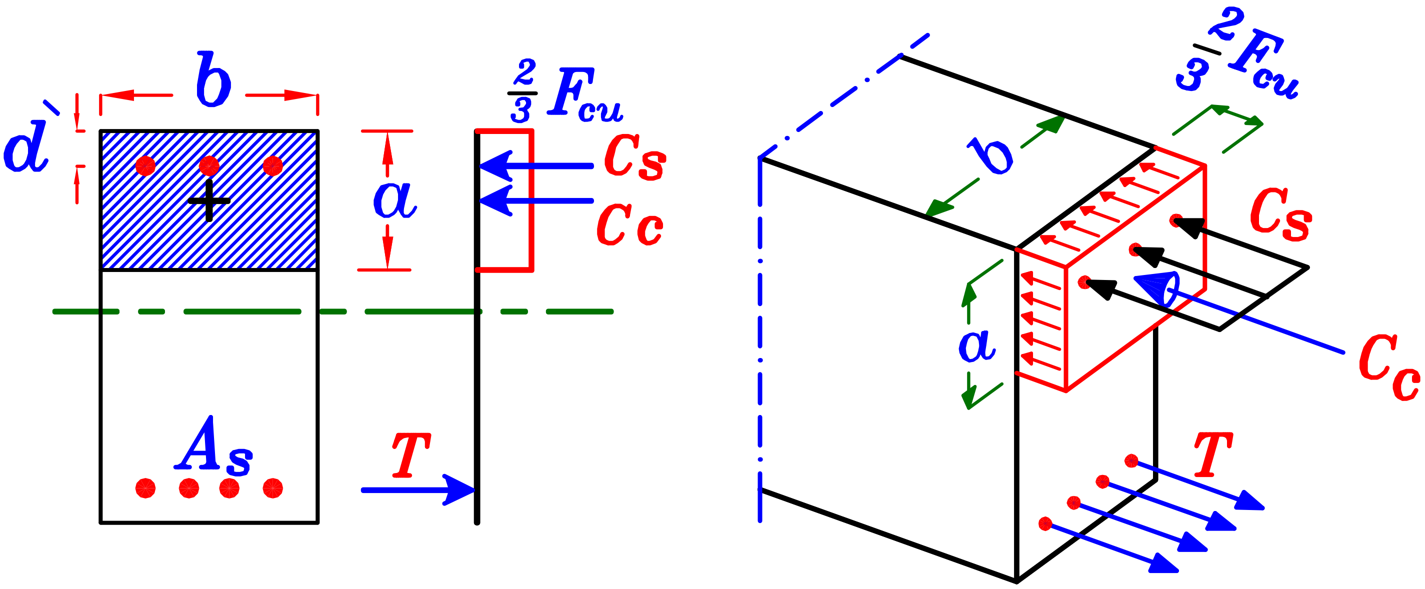
$$\therefore \boxed{\frac{2}{3}F_{cu}\, a\, b = F_s\ A_s}$$

a , F_s مجهولين

For all types of Sections

Under , Balanced & Over

(b) With Compression steel.



$$C_c = \text{Stress} * \text{Area} = \frac{2}{3} F_{cu} * (a * b)$$

Compression on Steel

$$C_s = \text{Stress} * \text{Area} = F_y * A_{s`}$$

نفرض للتسهيل

$$\boxed{F_{s`} = F_y}$$

$$T = \text{Stress} * \text{Area} = F_s * A_s$$

$$\therefore \boxed{\frac{2}{3} F_{cu} a\, b + F_y\, A_{s`} = F_s\, A_s}$$

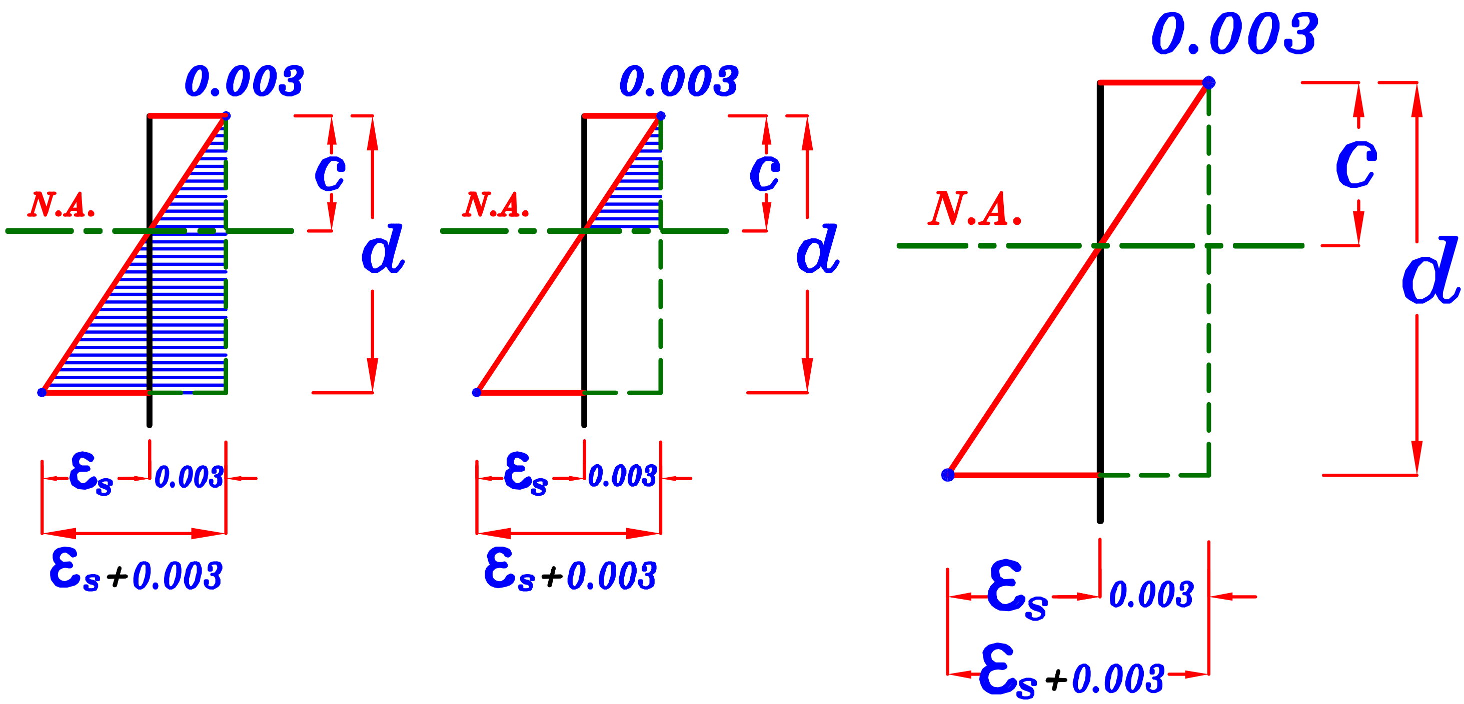
a , F_s مجهولين

For all types of Sections

Under , Balanced & Over

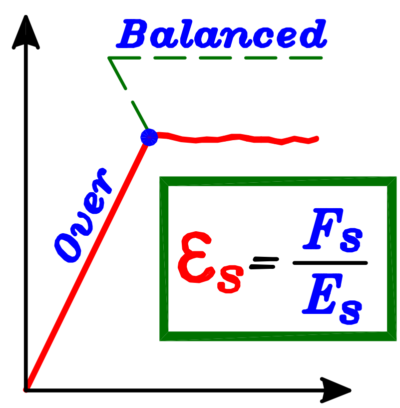
② *Compatibility Equation.* معادله التوافق (التشابه)

من شكل ال *Strain diagram* يتم عمل تشابه مثلثات



$$\frac{c}{d} = \frac{0.003}{0.003 + \varepsilon_s}$$ من تشابه المثلث

$$\because \varepsilon_s = \frac{F_s}{E_s} = \frac{F_s}{2*10^5}$$ *For Balanced & Over only*



$$\therefore \frac{c}{d} = \frac{0.003}{0.003 + \frac{F_s}{2*10^5}} = \frac{600}{600 + F_s}$$

$$\boxed{c = 1.25\, a = \frac{600}{600 + F_s} * d}$$

a , F_s مجهولين

Balanced & Over only

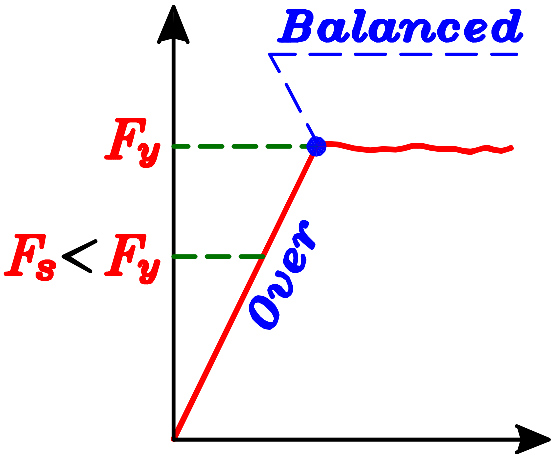
حفظ

Calculation of C_b

From Compatibility Equation.

$$\boxed{C = \frac{600}{600 + F_s} * d}$$

Balanced & Over only



For Balanced section $F_s = F_y$

For Over Reinforced section $F_s < F_y$

$\therefore$ For $C = \dfrac{600}{600 + F_s} * d$

When we take $F_s = F_y$ it will be For Balanced section

$$\therefore \quad \boxed{C_b = \frac{600}{600 + F_y} * d}$$

حفظ

$$\because \quad \boxed{C_u < C_b < C_o}$$

$\therefore$ When $C < C_b \longrightarrow$ The section is Under

When $C = C_b \longrightarrow$ The section is Balanced

When $C > C_b \longrightarrow$ The section is Over

(M_{ult})

Calculation of Ultimate Moment.

هو عزم الإنهيار . أى هو العزم الذى يصل فيه أياً من الحديد
أو الخرسانه إلى الا **max stress or max strain.**

$$\text{max. stress (Concrete)} = F_{cu}$$

$$\text{max. stress (Steel)} = F_y$$

$$\text{max. strain (Concrete)} = \varepsilon_c = 0.003$$

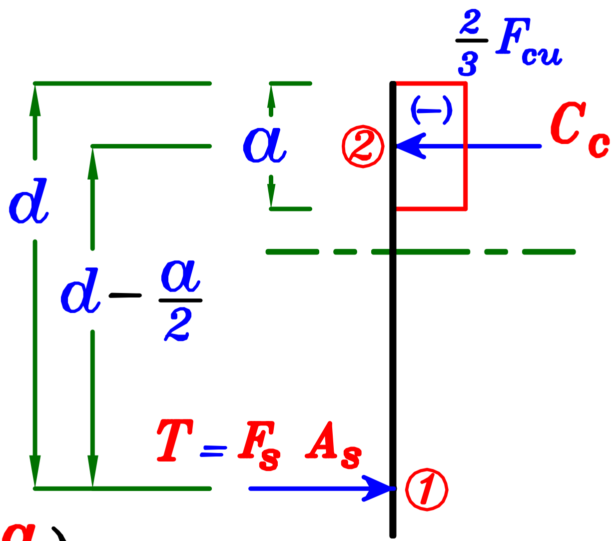
$$\text{max. strain (Steel)} = \varepsilon_y = \frac{F_y}{E_s} = \frac{F_y}{2*10^5}$$

Note $\boxed{\text{When } \varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y}$

How to Determine M_{ult} For a known Section.

$$C_c = \frac{2}{3} F_{cu} * a * b$$

$$T = F_s * A_s$$



M_{ult} = M at point ①
= M at point ②

$$M_{ult} \text{ at point ①} = C_c\left(d-\frac{a}{2}\right)$$

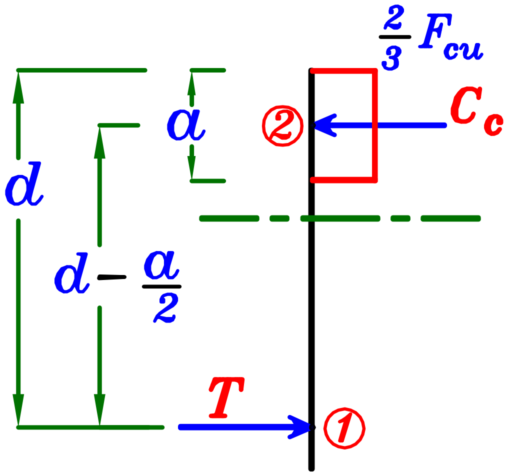
$$= \frac{2}{3} F_{cu}\, a\, b \left(d-\frac{a}{2}\right)$$

$$M_{ult} \text{ at point ②} = T\left(d-\frac{a}{2}\right) = F_s * A_s\left(d-\frac{a}{2}\right)$$

But a, F_s ?? $\therefore$ We have to get « a , F_s » First.

To Calculate M_{ult}

① With Tension Steel only.



① Get $C_b = \dfrac{600}{600 + F_y} * d$

② Use equilibrium equation. $C_c = T$

$$\frac{2}{3} F_{cu} * (a * b) = A_s * F_s \text{ --- } a, F_s = ??$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3} F_{cu} * (a * b) = A_s * F_y \longrightarrow \text{Get } a \longrightarrow \text{Get } C = 1.25\, a$$

③ Check C

* IF $C \leqslant C_b \longrightarrow$ The Section is Under Reinforced or Balanced Sec. and the assumption is right $F_s = F_y$

④ $\therefore$ $$\boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) = A_s F_y \left(d - \frac{a}{2}\right)}$$

* IF $C > C_b \longrightarrow$ The Section is Over Reinforced Sec. and the assumption is wrong $F_s \neq F_y$

$\therefore$ To get the right value oF a, F_s

① From equilibrium eqn.

$$\frac{2}{3} F_{cu}\, a\, b = A_s F_s \text{ ------ ① } \quad a = ?, \; F_s = ?$$

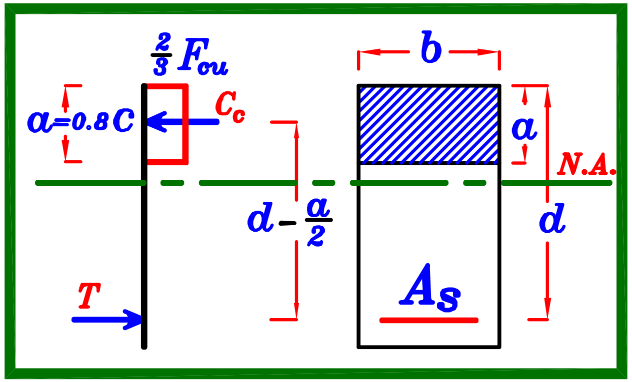
② From compatibility eqn.

$$C = 1.25\, a = \frac{600}{600 + F_s} * d \text{ ------ ② } \quad a = ?, \; F_s = ?$$

From eqns. ①, ② Get a, F_s

④ $\therefore$ $$\boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) = A_s F_s \left(d - \frac{a}{2}\right)}$$

Calculation of M_{ult} For R-sec.
(With Ten. Steel only)



Get $C_b = \frac{600}{600 + F_y} * d$

From equilibrium eqn. $\frac{2}{3} F_{cu} * (a * b) = F_s * A_s$

assume $\varepsilon_s \geq \varepsilon_y \longrightarrow F_s = F_y$ (The section is under reinforced or Balanced Sec.)

$\therefore F_s = F_y \quad \therefore \frac{2}{3} F_{cu} * (a * b) = F_y * A_s \longrightarrow$ Get $a \longrightarrow$ Get $C = 1.25\, a$

IF $\boxed{C}$

IF $C \leq C_b$

- $C < C_b$ — Under Reinforced Section
- $C = C_b$ — Balanced Section

and the assumption is right $F_s = F_y$

$$\therefore \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) = A_s F_y \left(d - \frac{a}{2}\right)}$$

IF $C > C_b$

Over Reinforced Section

and the assumption is wrong $F_s \neq F_y$

To get the right value of a, F_s

$\frac{2}{3} F_{cu}\, a\, b = F_s\, A_s$ --- ① $a = ?\,,\ F_s = ?$

$C = 1.25\, a = \frac{600}{600 + F_s} * d$ --- ② $a = ?\,,\ F_s = ?$

From eqns. ①, ② Get a, F_s

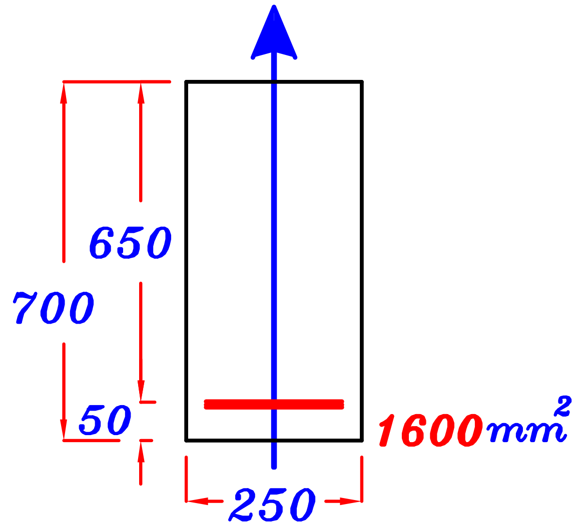
$$\therefore \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) = F_s\, A_s \left(d - \frac{a}{2}\right)}$$

Example.

Data.

$F_{cu} = 25 \ N \backslash mm^2$

st. 360/520



Req.

For the shown Cross-Section

1- Calculate $M_{ult.}$

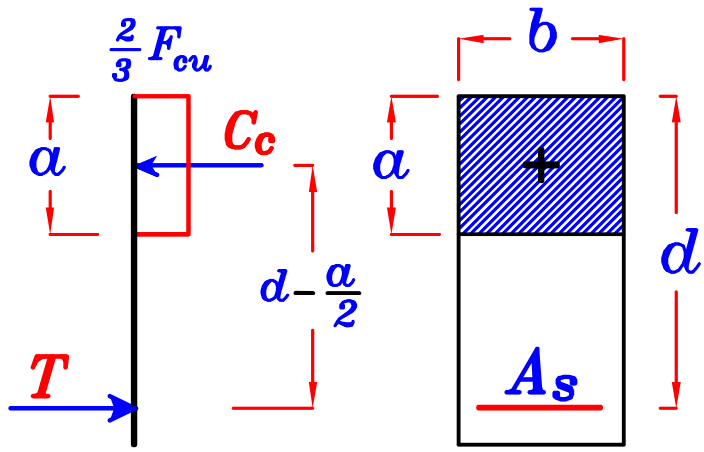
2- Determine which type of Failure will occur For that section.

Solution.

① $$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 650 = 406.25 \ mm$$

$$C_c = Stress * Area = \frac{2}{3} F_{cu} * a * b$$

$$T = Stress * Area = F_s * A_s$$



② From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * b = F_s * A_s$$

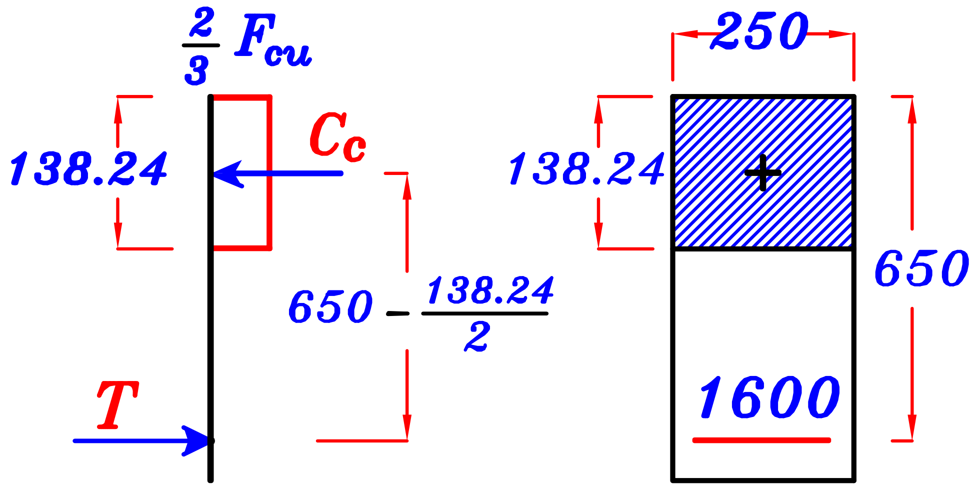
Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3} (25)(a)(250) = (1600)(360) \longrightarrow a = 138.24 \ mm$$

③ $\therefore\ C = 1.25\, a = 1.25 * 138.24 = 172.8\ mm < C_b$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

and the assumption is right $F_s = F_y$



④ By taking the moment about the steel.

$$\therefore\ M_{ult} = C_c * \left(d - \frac{a}{2}\right) = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right)$$

$$M_{ult} = \frac{2}{3}(25)(138.24)(250)\left(650 - \frac{138.24}{2}\right)$$

$$= 334586880\ N.mm = 334.5\ kN.m$$

④ OR By taking the moment about concrete.

$$M_{ult} = T * \left(d - \frac{a}{2}\right) = F_y * A_s \left(d - \frac{a}{2}\right)$$

$$= (360 * 1600)\left(650 - \frac{138.24}{2}\right) = 334586880\ N.mm$$

$$= 334.5\ kN.m$$

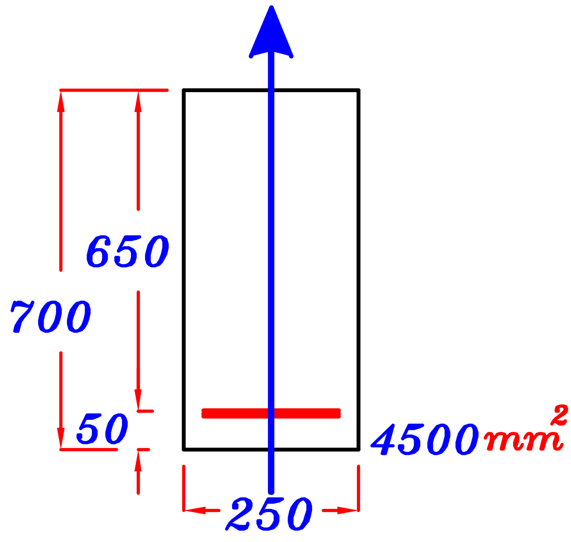
$$\therefore\ \boxed{M_{ult} = 334.5\ kN.m}$$

Example.

Data.

$F_{cu} = 25 \; N \backslash mm^2$

st. 360/520



Req.

For the shown Cross-Section

1- Calculate $M_{ult.}$

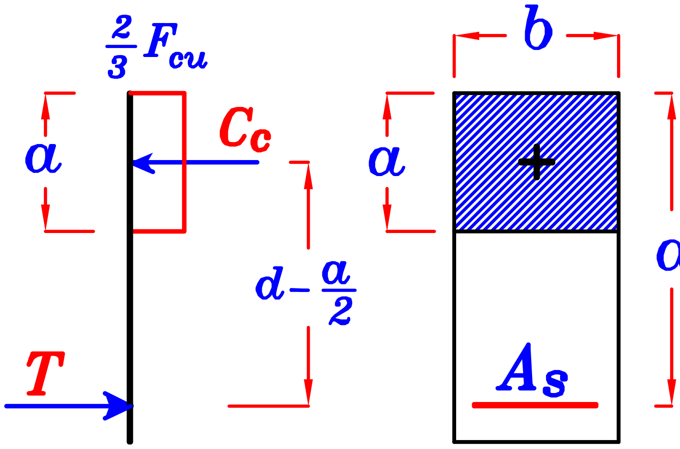
2- Determine which type of Failure will occur For that section.

Solution.

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 650 = 406.25 \; mm$

$C_c = \text{Stress} * \text{Area} = \dfrac{2}{3} F_{cu} * a * b$

$T = \text{Stress} * \text{Area} = F_s * A_s$



② From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * b = F_s * A_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3} (25)(a)(250) = (4500)(360) \longrightarrow a = 388.8 \; mm$$

③ $\therefore\ C = 1.25\,a = 1.25 * 388.8 = 486.0\ mm > C_b$

$\therefore$ $\boxed{\text{The Section is Over Reinforced Sec.}}$

and the assumption is wrong $F_s < F$

To get the right value of a, F_s

$$\therefore\ \frac{2}{3} F_{cu}\, a\, b = A_s\, F_s$$

$$\therefore\ \frac{2}{3} (25)\,(a)\,(250) = (4500)\,(F_s)$$

$$\therefore\ F_s = 0.926\ a \quad \text{---} ① \quad a = ?\ ,\ F_s = ?$$

$$\because\ C = 1.25\,a = \frac{600}{600 + F_s} * d \quad \text{----} ② \quad a = ?\ ,\ F_s = ?$$

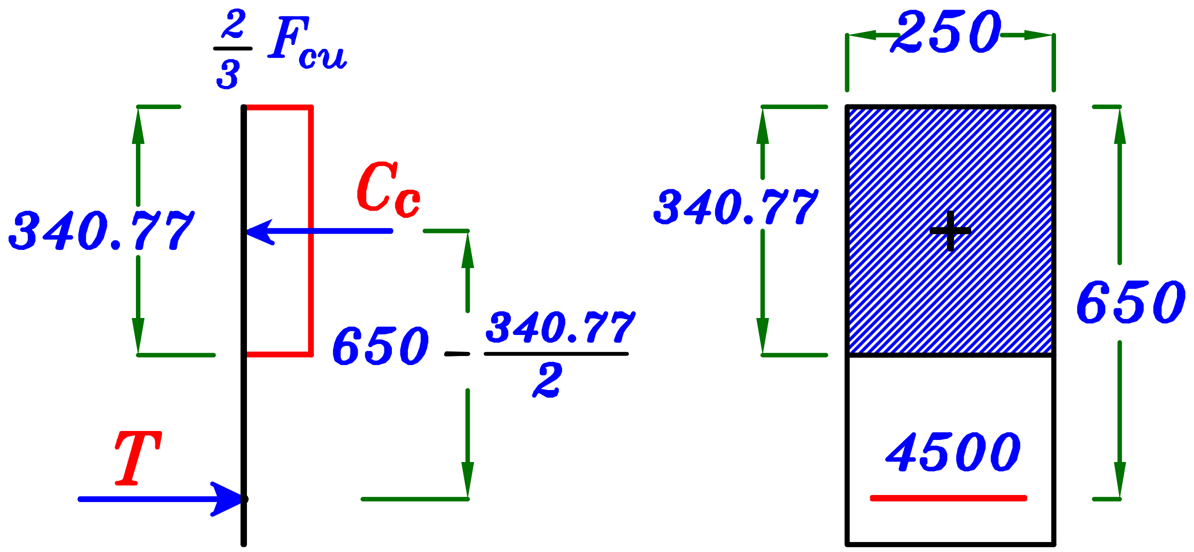
From eqns. ①, ② Get a, F_s

$$\therefore\ 1.25\,a = \frac{600}{600 + 0.926\ a} * 650$$

$$\therefore\ \boxed{a = 340.77\ mm}$$

$$F_s = 0.926\,(340.77) = 315.5\ N\backslash mm^2$$

$$\boxed{F_s = 315.5\ N\backslash mm^2} < F_y$$



④ ***By taking the moment about the steel.***

$$\therefore \quad M_{ult} = C_c * \left(d - \frac{a}{2}\right) = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right)$$

$$M_{ult} = \frac{2}{3} (25)(340.77)(250)\left(650 - \frac{340.77}{2}\right)$$

$$= 680993348.1 \text{ N.mm} = 680.99 \text{ kN.m}$$

④ ***OR By taking the moment about concrete.***

$$M_{ult} = T * \left(d - \frac{a}{2}\right) = F_s * A_s \left(d - \frac{a}{2}\right)$$

$$= (315.5 * 4500)\left(650 - \frac{340.77}{2}\right) = 680933396.3 \text{ N.mm}$$

$$= 680.93 \text{ kN.m}$$

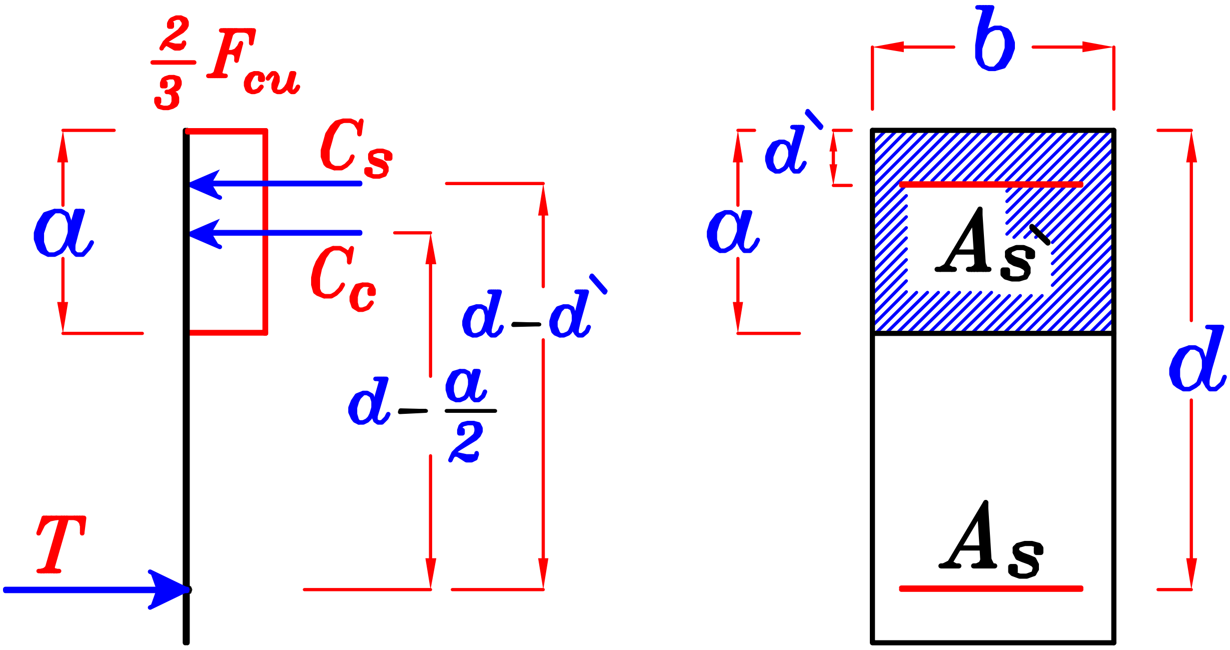
$$\therefore \boxed{M_{ult} = 680.93 \text{ kN.m}}$$

الفرق فى قيمتى العزم ناتج فقط عن التقريب
لكن كلا الاجابتين صحيح .

عند حساب M_{ult} وكان هناك حديد جهه الضغط ($A_{s`}$)

نعمل حل تقريبى للتسهيل بأن نعتبر $F_{s`} = F_y$

و لحساب ال M_{ult} مع وجود ($A_{s`}$) بدقه سنذكرها فى أخر الملف Page No. 175



$$C_c = Stress * Area = \frac{2}{3} F_{cu} * (a\ b)$$

$$C_s = Stress * Area = F_y * A_{s`}$$

By taking the moment about the steel.

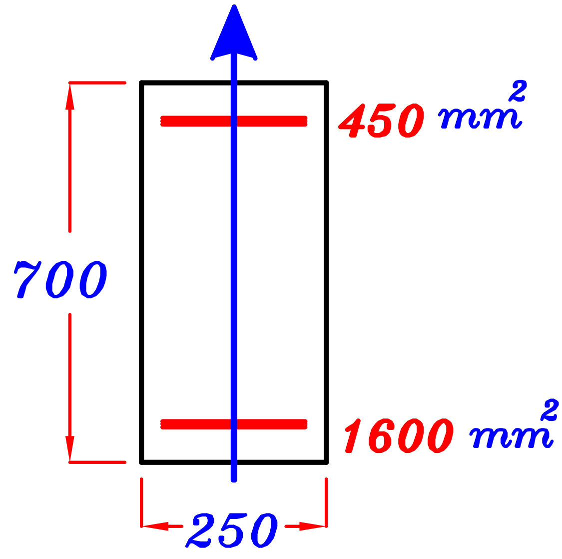
$$\boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + F_y * A_{s`} (d - d`)}$$

Example.

Data.

$F_{cu} = 25 \; N \backslash mm^2$

st. 360/520



Req.

For the shown Cross–Section

1 – Calculate $M_{ult.}$

2 – Determine which type of Failure will occur For that section.

Solution.

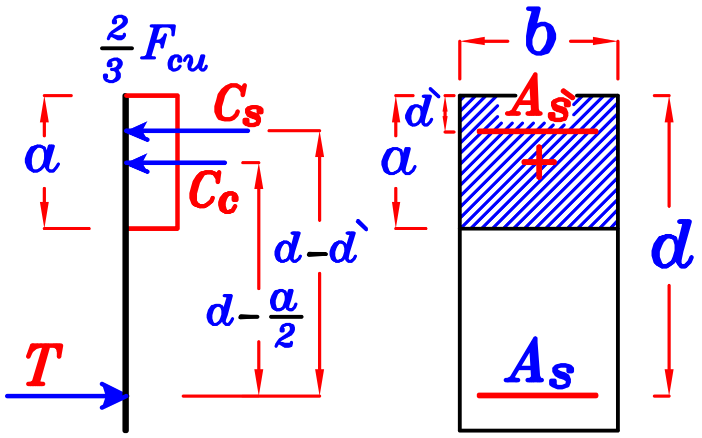
$\because \dfrac{A_{s}`}{A_s} = \dfrac{450}{1600} = 0.28 > 0.2 \quad \therefore$ Use $A_{s}`$

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 650 = 406.25 \text{ mm}$

C_c = Stress * Area = $\frac{2}{3} F_{cu} * a * b$

C_s = Stress * Area = $F_y * A_{s}`$

T = Stress * Area = $F_s * A_s$



② From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * b + F_y * A_{s}` = F_s * A_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

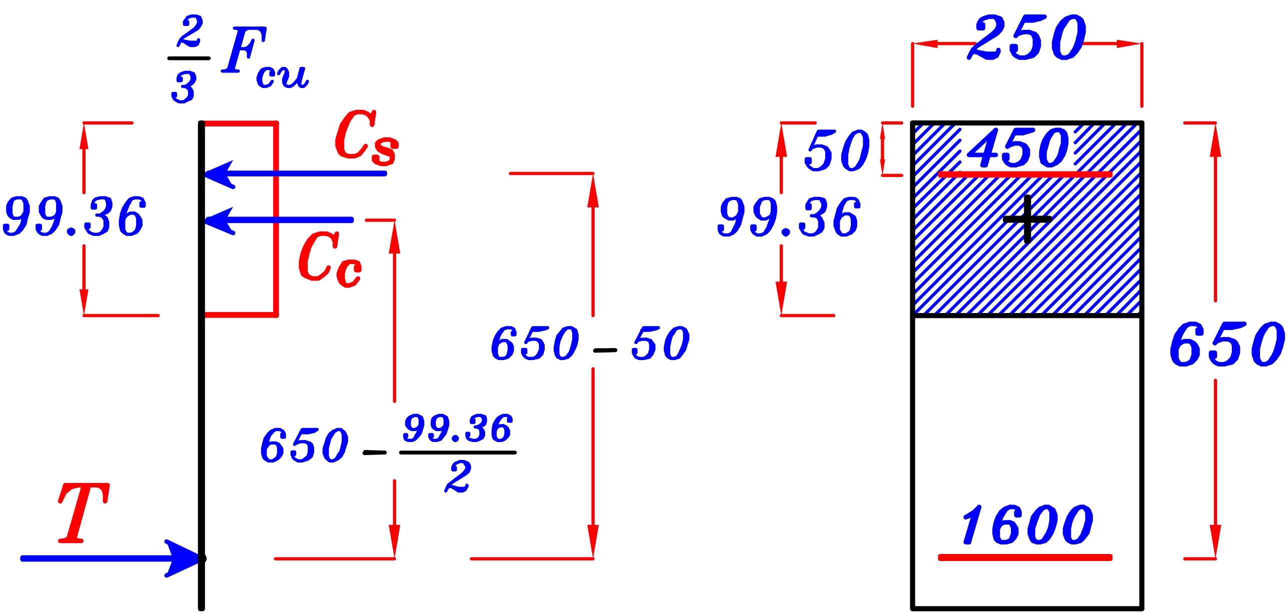
$$\frac{2}{3}(25)(a)(250) + (360)(450) = (360)(1600)$$

$$\boxed{a = 99.36 \text{ mm}}$$

③ $\therefore\ C = 1.25\, a = 124.2\ mm < C_b$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

and the assumption is right $F_s = F_y$



④ By taking the moment about the steel.

$$M_{ult} = C_c * (d - \frac{a}{2}) + C_s * (d - d')$$

$$M_{ult} = \frac{2}{3} F_{cu}\, a\, b\, (d - \frac{a}{2}) + F_y * A_s' (d - d')$$

$$M_{ult} = \frac{2}{3} (25)(99.36)(250)\left(650 - \frac{99.36}{2}\right) + 360 * 450\,(650 - 50)$$

$$= 345732480\ N.mm = 345.7\ kN.m$$

$\therefore$ $\boxed{M_{ult} = 345.7\ kN.m}$

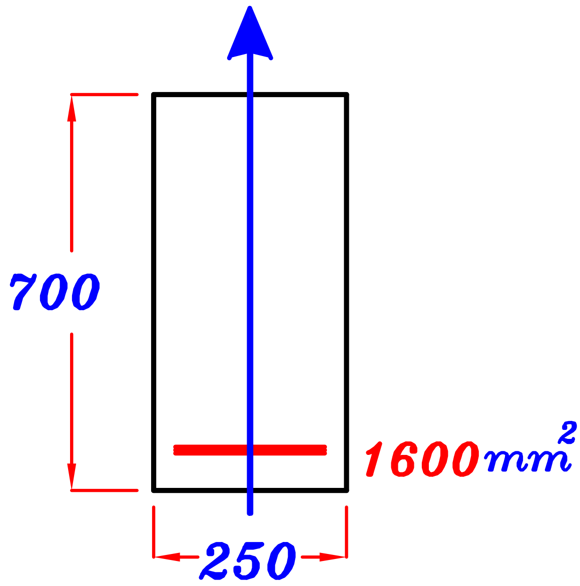
Note.

في حاله وجود حديد جهه ال $(A_{s`})$

فان الزياده الحادثه في قيمه M_{ult} لن تكون كبيره

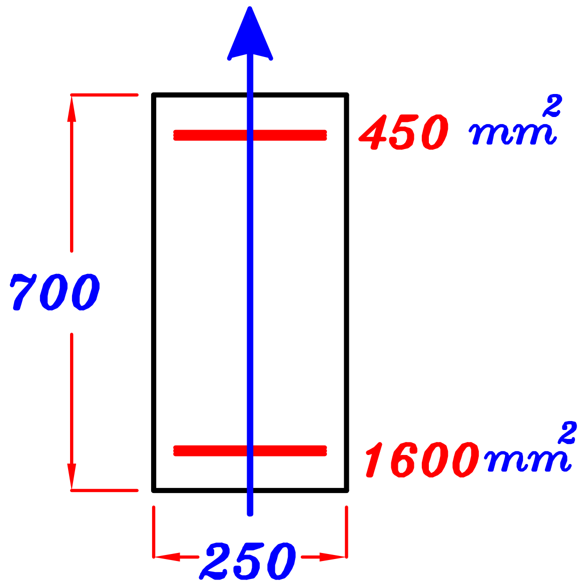
Example Page 69

$$M_{ult} = 334.5 \text{ kN.m}$$



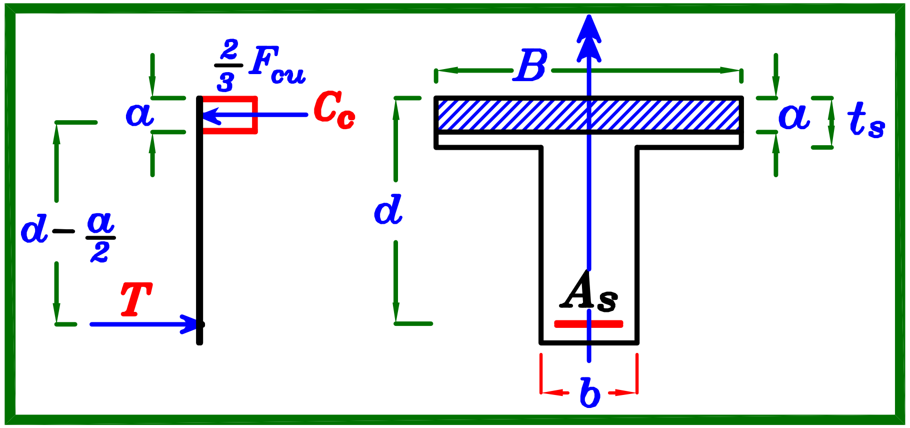
Example Page 75

$$M_{ult} = 345.7 \text{ kN.m}$$



Calculation of M_{ult} For T-sec. (With Ten. Steel only)

Get $C_b = \frac{600}{600+F_y} * d$



Assume $a \leq t_s$

From equilibrium eqn. $\frac{2}{3}F_{cu} * (a*B) = F_s * A_s$

assume $F_s = F_y$ (The section is under reinforced or Balanced Sec.)

$\therefore \frac{2}{3}F_{cu} * (a*B) = F_y * A_s \longrightarrow$ Get $a \longrightarrow$ Get $C = 1.25\,a$

IF $a \leq t_s \longrightarrow C < C_b$ the First & second assumptions are right.

$$\therefore \boxed{M_{ult} = \frac{2}{3}F_{cu}\,a\,B\left(d-\frac{a}{2}\right) = F_y\,A_s\left(d-\frac{a}{2}\right)}$$

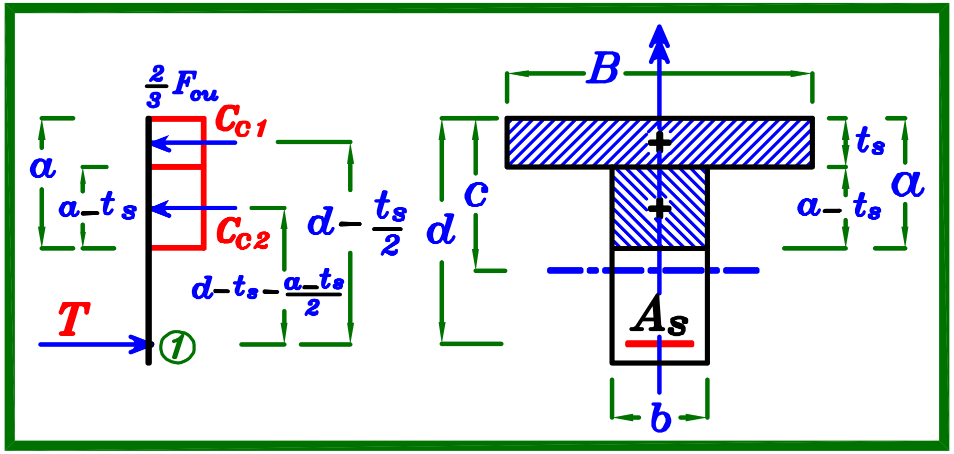
IF $a > t_s$ The First assumption is wrong.

From equilibrium eqn. $C_{c1} + C_{c2} = T$

$\frac{2}{3}F_{cu} * t_s * B + \frac{2}{3}F_{cu} * (a-t_s) * b = F_s * A_s$

assume $F_s = F_y$ (The section is under reinforced or Balanced Sec.)

Get $a > t_s \longrightarrow$ Get $C = 1.25\,a$



IF $\boxed{C}$

IF $C \leq C_b$ right assumption

$M_{ult} = C_{c1}\left(d-\frac{t_s}{2}\right) + C_{c2}\left(d-t_s-\frac{a-t_s}{2}\right)$

$$\boxed{M_{ult} = \left(\frac{2}{3}F_{cu}*t_s*B\right)\left(d-\frac{t_s}{2}\right) + \left(\frac{2}{3}F_{cu}*(a-t_s)*b\right)\left(d-t_s-\frac{a-t_s}{2}\right)}$$

IF $C > C_b$ wrong assumption

To get the right value of a, F_s

$\frac{2}{3}F_{cu}*t_s*B + \frac{2}{3}F_{cu}*(a-t_s)*b = A_s*F_s$ --- ① $a = ?$, $F_s = ?$

$C = 1.25\,a = \frac{600}{600+F_s} * d$ --- ② $a = ?$, $F_s = ?$

From eqns. ①, ② Get a, F_s

$$\boxed{M_{ult} = \left(\frac{2}{3}F_{cu}*t_s*B\right)\left(d-\frac{t_s}{2}\right) + \left(\frac{2}{3}F_{cu}*(a-t_s)*b\right)\left(d-t_s-\frac{a-t_s}{2}\right)}$$

Example.

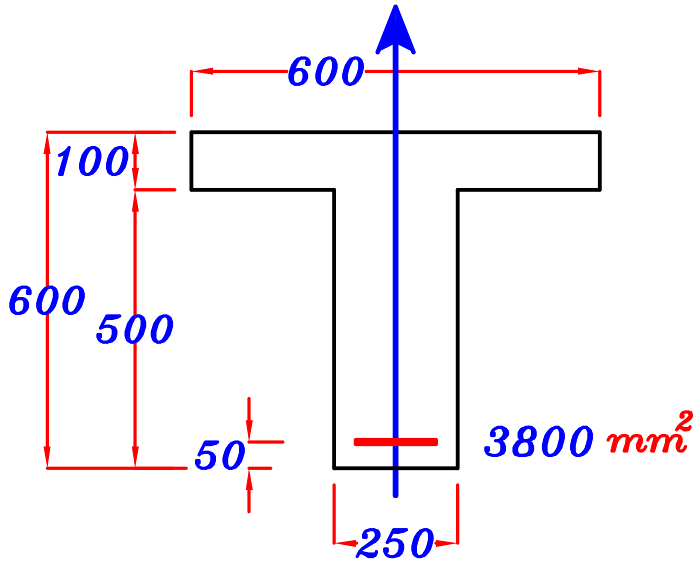
Data.

$F_{cu} = 25 \ N\backslash mm^2$

st. 360/520

Req.

For the shown Cross-Section
Calculate $M_{ult.}$

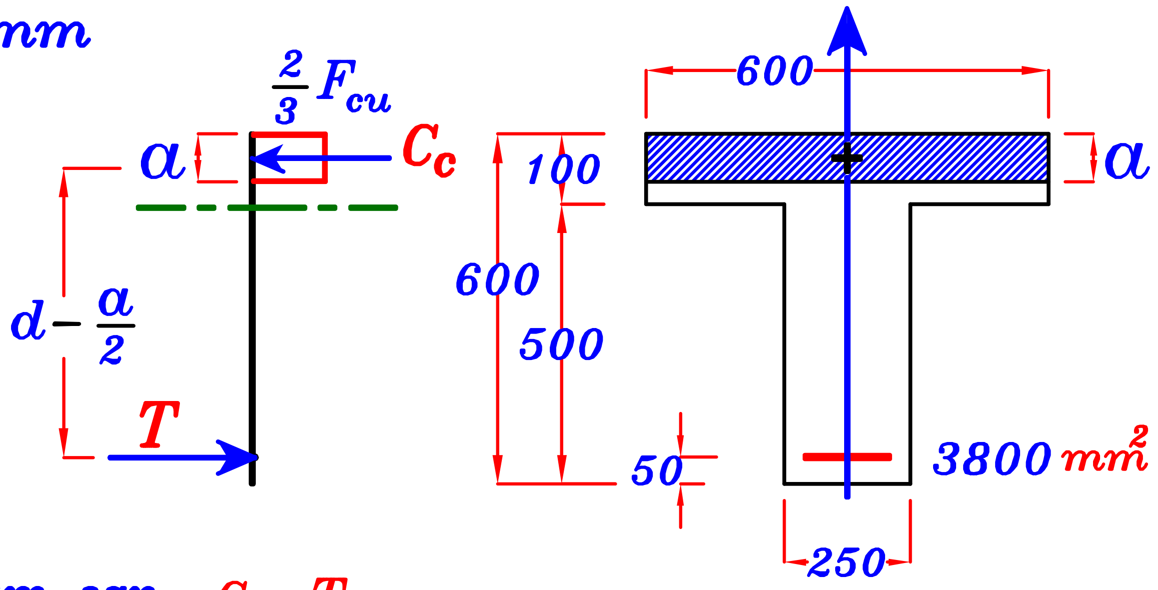


Solution.

① $$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 550 = 343.75 \ mm$$

② Assume $a \leqslant t_s$

$a < 100 \ mm$



From equilibrium eqn. $C_c = T$

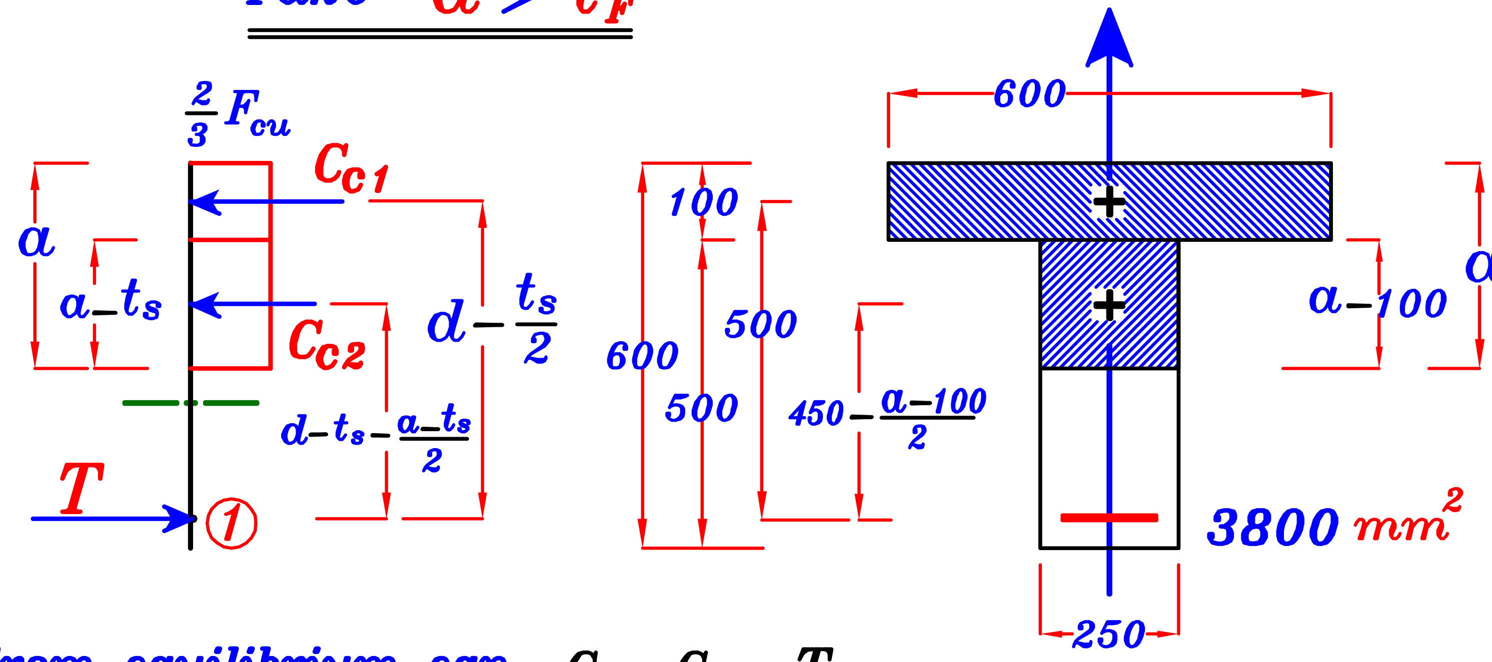
$$\frac{2}{3} F_{cu} * a * B = F_s * A_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3} (25)(a)(600) = (360)(3800) \longrightarrow a = 136.8 \ mm > t_s$$

$a > t_F$ wrong assumption $\therefore$ Take $a > t_s$

$\therefore$ **Take $a > t_F$**



From equilibrium eqn. $C_{c1} + C_{c2} = T$

$$\frac{2}{3} F_{cu} * t_s * B + \frac{2}{3} F_{cu} * (a - t_s) * b = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (*under reinforced or Balanced Sec.*)

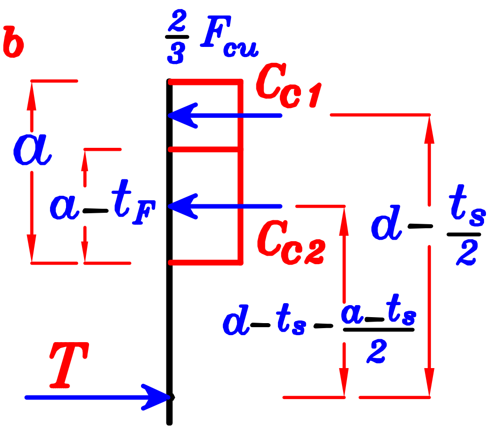
$$\frac{2}{3}(25)(100)(600) + \frac{2}{3}(25)(a - 100)(250) = (3800)(360)$$

$\longrightarrow a = 188.32\ mm > t_s$ **right assumption**

$\therefore C = 1.25\ a = 1.25 * 188.32 = 235.4\ mm < C_b$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

and the assumption is right $F_s = F_y$



$$M_{ult} = C_{c1}\left(d - \frac{t_s}{2}\right) + C_{c2}\left(d - t_s - \frac{a - t_s}{2}\right)$$

$$M_{ult} = \left(\frac{2}{3} F_{cu} * t_s * B\right)\left(d - \frac{t_s}{2}\right) + \left(\frac{2}{3} F_{cu} * (a - t_s) * b\right)\left(d - t_s - \frac{a - t_s}{2}\right)$$

$$= \frac{2}{3}(25)(100)(600)\left(550 - \frac{100}{2}\right) + \frac{2}{3}(25)(188.32 - 100)(250)\left(550 - 100 - \frac{188.32 - 100}{2}\right)$$

$$= 649349120\ N.mm = 649.34\ kN.m$$

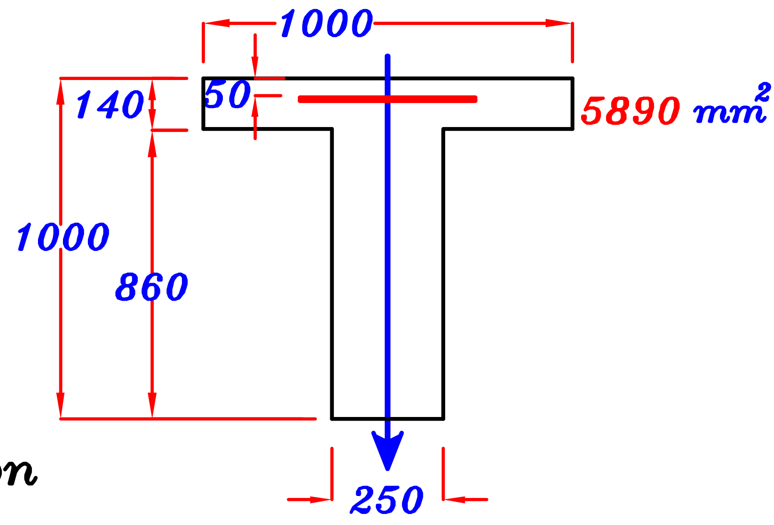
$$\therefore \boxed{M_{ult} = 649.34\ kN.m}$$

Example.

Data.

$F_{cu} = 25 \ N\backslash mm^2$

st. $360/520$



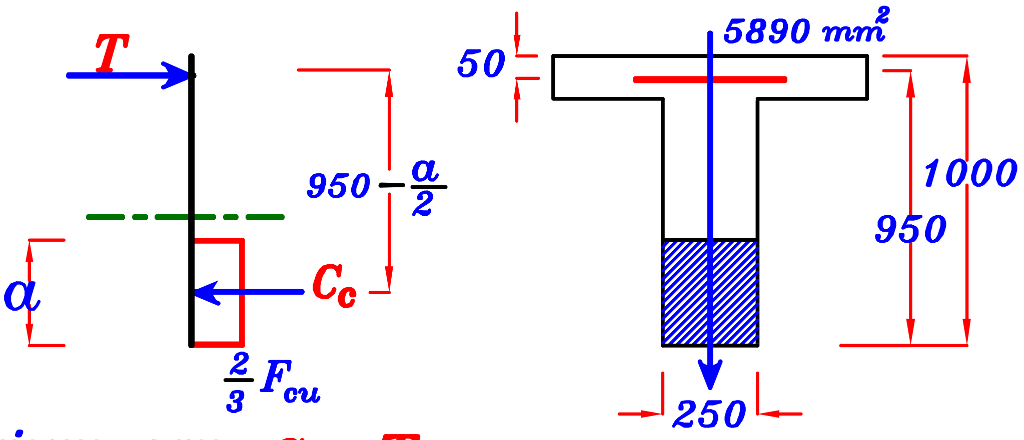
Req.

For the shown Cross-Section

Calculate $M_{ult.}$

Solution.

① $$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 950 = 593.75 \ mm$$



② From equilibrium eq$\underline{n}$. $C_c = T$

$$\frac{2}{3} F_{cu} * a * b = F_s * A_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3} (25)(a)(250) = (360)(5890) \longrightarrow a = 508.9 \ mm$$

$$\therefore C = 1.25 \ a = 1.25 * 508.9 = 636.1 \ mm > C_b$$

$\therefore$ **The Section is Over Reinforced Sec.**

and the assumption is wrong $F_s < F_y$

To get the right value of a, F_s

$\therefore \frac{2}{3} F_{cu} \ a \ b = F_s \ A_s \qquad \therefore \frac{2}{3} (25)(a)(250) = (F_s)(5890)$

$\therefore F_s = 0.707 \ a \quad$ --- ① $\quad a = ?, \ F_s = ?$

$$\because \quad C = 1.25\,a = \frac{600}{600 + F_s} * d \quad --- ② \quad a = ?\,,\ F_s = ?$$

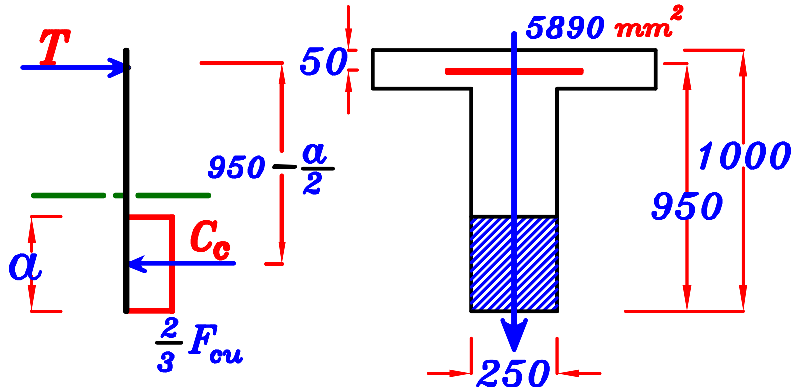
From eqns. ①, ② Get a, F_s

$$\therefore \quad 1.25\,a = \frac{600}{600 + 0.707\,a} * 950$$

$$\therefore \quad \boxed{a = 483.98 \text{ mm}}$$

$$F_s = 0.707\,(483.98) = 342.17 \ N/mm^2$$

$$\boxed{F_s = 342.17 \ N\backslash mm^2} < F_y$$



$$\therefore \quad M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right)$$

$$M_{ult} = \frac{2}{3}(25)(483.98)(250)\left(950 - \frac{483.98}{2}\right) = 1427761166 \ N.mm$$

$$= 1427.76 \ kN.m$$

$$\therefore \quad \boxed{M_{ult} = 1427.76 \ kN.m}$$

or

$$M_{ult} = A_s F_s \left(d - \frac{a}{2}\right)$$

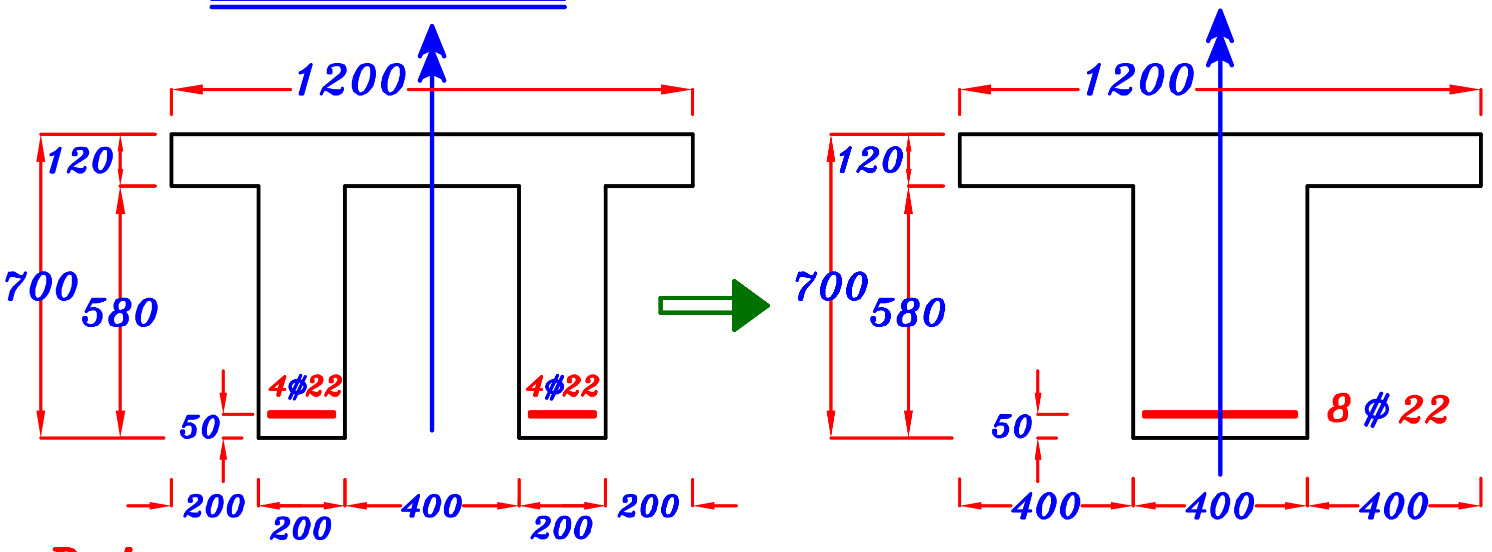
$$M_{ult} = (5890)\,(342.17)\left(950 - \frac{483.98}{2}\right) = 1426910114 \ N.mm$$

$$1426.91 \ kN.m$$

$$\therefore \quad \boxed{M_{ult} = 1426.91 \ kN.m}$$

الفرق فى قيمتى العزم ناتج فقط عن التقريب
لكن كلا الاجابتين صحيح .

Example.



Data. $F_{cu} = 25 \ N\backslash mm^2$ st. 360/520

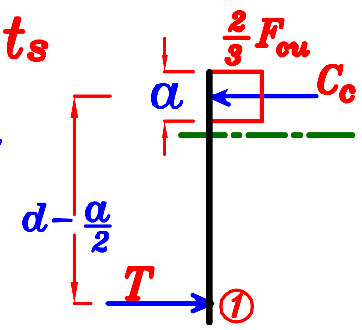
Req. For the shown Cross-Section Calculate Factor of Safty.

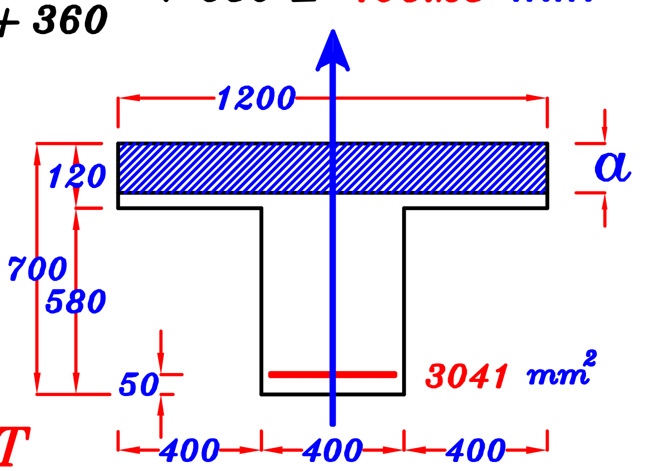
Solution. $A_s = 8\phi 22 = 8\left[\frac{\pi * 22^2}{4}\right] = 3041 \ mm^2$

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 650 = 406.25 \ mm$

② Assume $a \leqslant t_s$

$a < 120 \ mm$





③ From equilibrium eqn. $C_c = T$

$\frac{2}{3} F_{cu} * a * B = A_s * F_s$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$\frac{2}{3}(25)(a)(1200) = (3041)(360) \longrightarrow a = 54.74 \ mm < t_s \quad \therefore$ O.K.

$\therefore \ C = 1.25 \ a = 1.25 * 54.74 = 68.42 \ mm < C_b$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$ and the assumption is right $F_s = F_y$

$\therefore \ M_{ult} = \frac{2}{3} F_{cu} \, a \, B \left(d - \frac{a}{2}\right)$

$M_{ult} = \frac{2}{3}(25)(54.74)(1200)\left(650 - \frac{54.74}{2}\right) = 681655324 \ N.mm = 681.65 \ kN.m$

$\therefore \ \boxed{M_{ult} = 681.65 \ kN.m}$

Calculate M_w

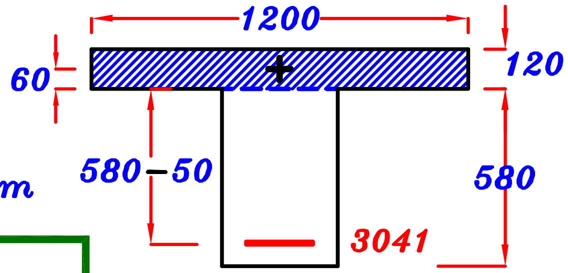
$F_{cu} = 25 \ N\backslash mm^2 \longrightarrow F_{cb} = 9.5 \ N\backslash mm^2$

$F_y = 360 \ N\backslash mm^2 \longrightarrow F_s = 200 \ N\backslash mm^2$

$S_{nv.}(above) = 120 * 1200 * (60) = 8640000 \ mm^3$

$S_{nv.}(under) = 15 * 3041 * (580 - 50) = 24175950 \ mm^3$

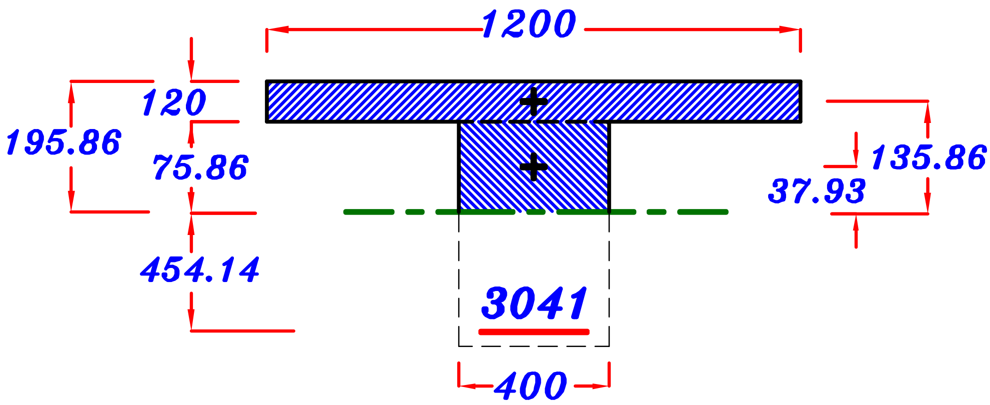
$\because S_{nv.}(under) > S_{nv.}(above) \quad \therefore Z > 120 mm$



① Get Z by taking $\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$

$$(1200)(120)(Z - 60) + (400)(Z - 120)\left(\frac{Z - 120}{2}\right) = (15)(3041)(650 - Z)$$

$$\boxed{Z = 195.86 mm}$$



② $$I_{nv} = \frac{1200(120)^3}{12} + (1200)(120)(135.86)^2 + \frac{400(75.86)^3}{3} + (15)(3041)(454.14)^2 = 12296731390 \ mm^4$$

③ $$M_{wc} = \frac{F_{cb} * I_{nv}}{Z} \quad \text{--------- not as } T\_Sec.$$

$$= \frac{9.5 * 12296731390}{195.86} = 596441071.1 \ N.mm = 596.44 \ kN.m$$

④ $$M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \frac{\left(\frac{200}{15}\right) * 12296731390}{650 - 195.86} = 361026156 \ N.mm$$

$$= 361.02 \ kN.m$$

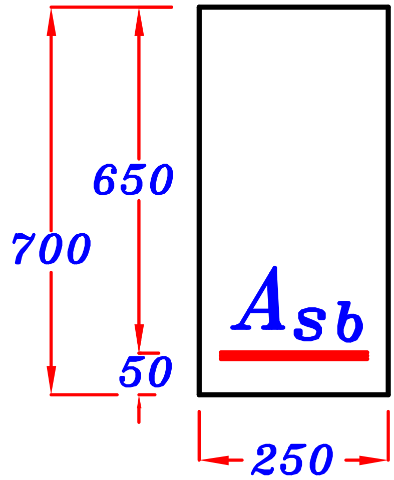
⑤ $$\boxed{M_w = 361.02 \ kN.m}$$

$$\boxed{\text{Factor of Safty} = \frac{M_{ult}}{M_w} = \frac{681.65}{361.02} = 1.89}$$

Example.

Data. $F_{cu} = 25 \; N\backslash mm^2$

st. 360/520



Req. Calculate A_{sb} (A_s balanced)

To make the sec. is balanced Sec.

and then get M_b (M_{ult} For balanced sec)

Solution.

For Balanced Sec. $C = C_b$, $a = a_b = 0.8\, C_b$, $F_s = F_y$

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 650 = 406.25 \; mm$

② $a = a_b = 0.8\, C_b = 0.8 * 406.25 = 325 \; mm$

③ From equilibrium eqn. $C_c = T$

$\frac{2}{3} F_{cu} * (a_b * b) = A_{sb} * F_y$

$\frac{2}{3} (25)(325)(250) = A_{sb} (360)$ $\therefore$ $\boxed{A_{sb} = 3761.5 \; mm^2}$

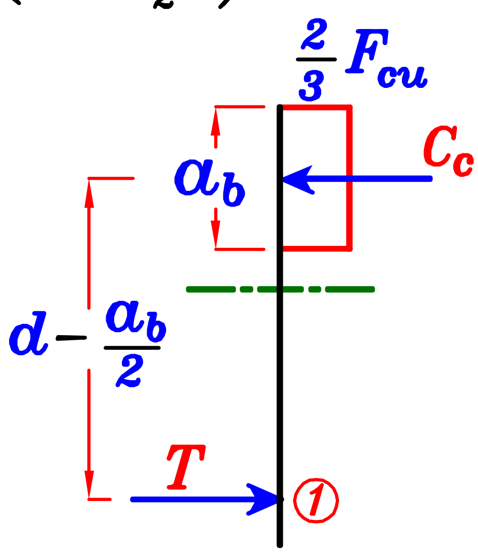
$\therefore M_b = \frac{2}{3} F_{cu}\, a_b\, b \left(d - \frac{a_b}{2}\right) = \frac{2}{3} (25)(325)(250)\left(650 - \frac{325}{2}\right)$

$M_b = 660156250 \; N.mm = 660.15 \; kN.m$

or $M_b = A_{sb}\, F_y \left(d - \frac{a_b}{2}\right)$

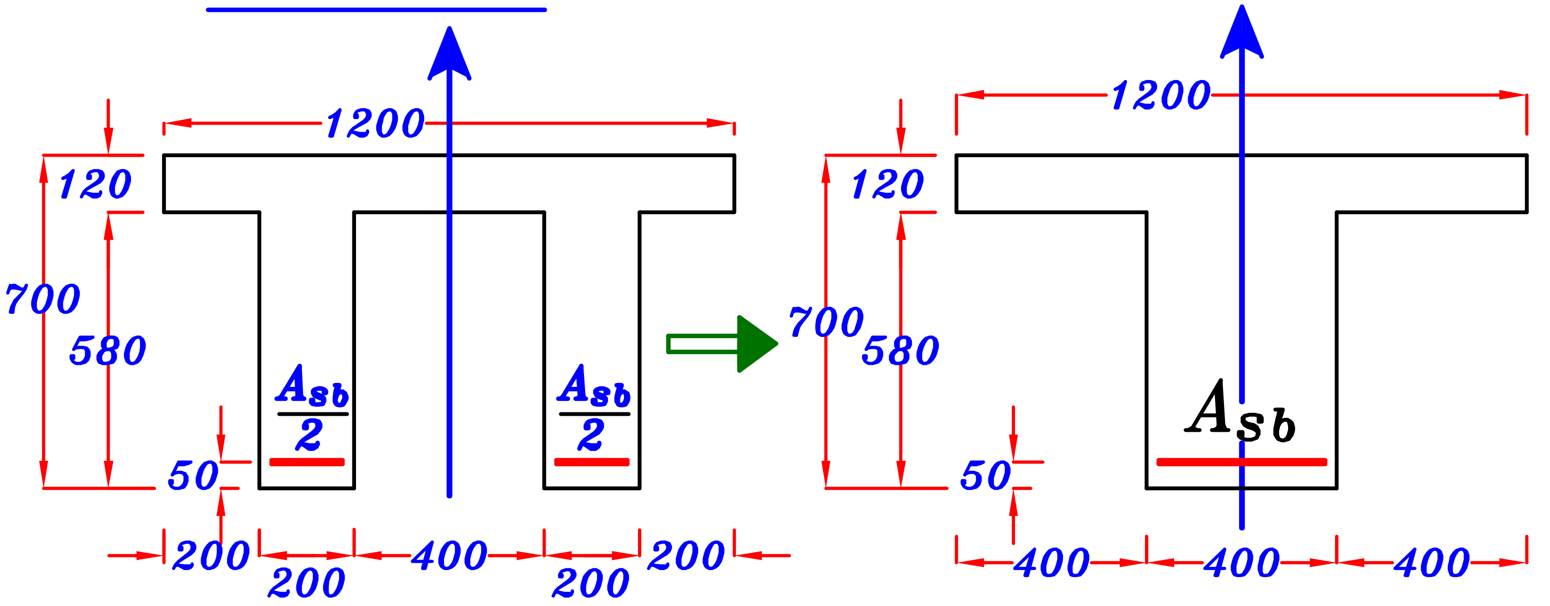
$M_b = 3761.5 \,(360)\left(650 - \frac{325}{2}\right)$

$M_b = 660156250 \; N.mm = 660.15 \; kN.m$



$\therefore$ $\boxed{M_b = 660.15 \; kN.m}$

Example.



Data. $F_{cu} = 25\ N\backslash mm^2$ st. 360/520

Req. Calculate A_{sb} To make the sec. is balanced Sec. and then get M_b

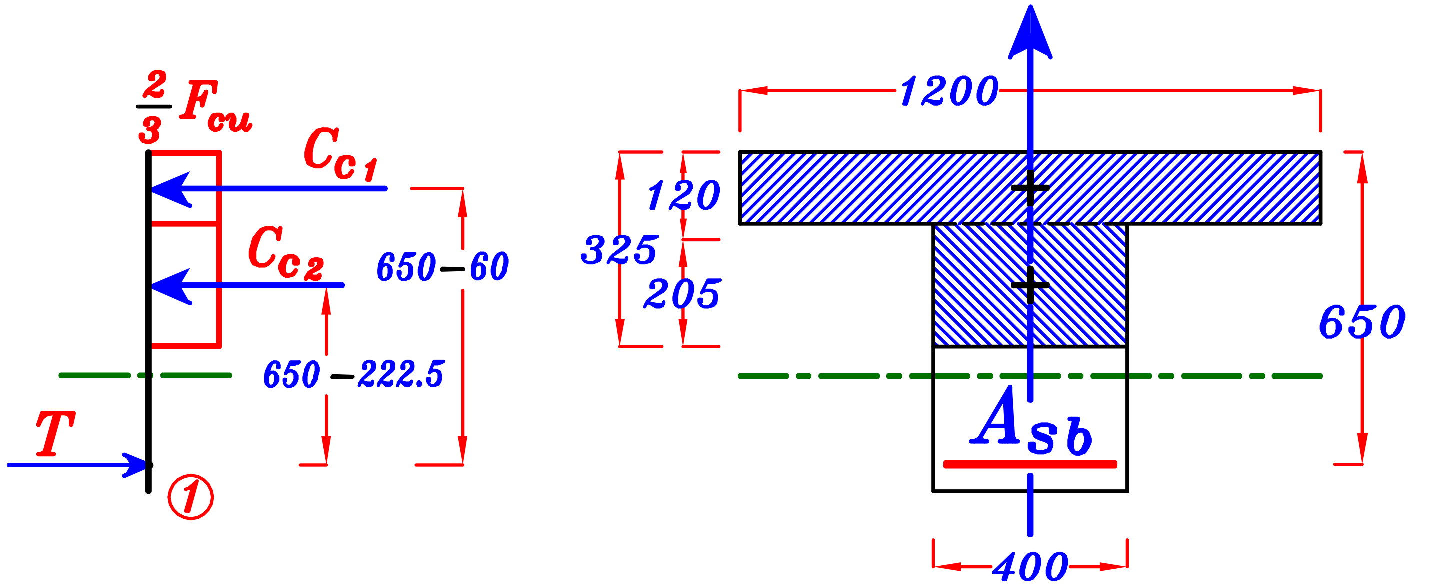
---

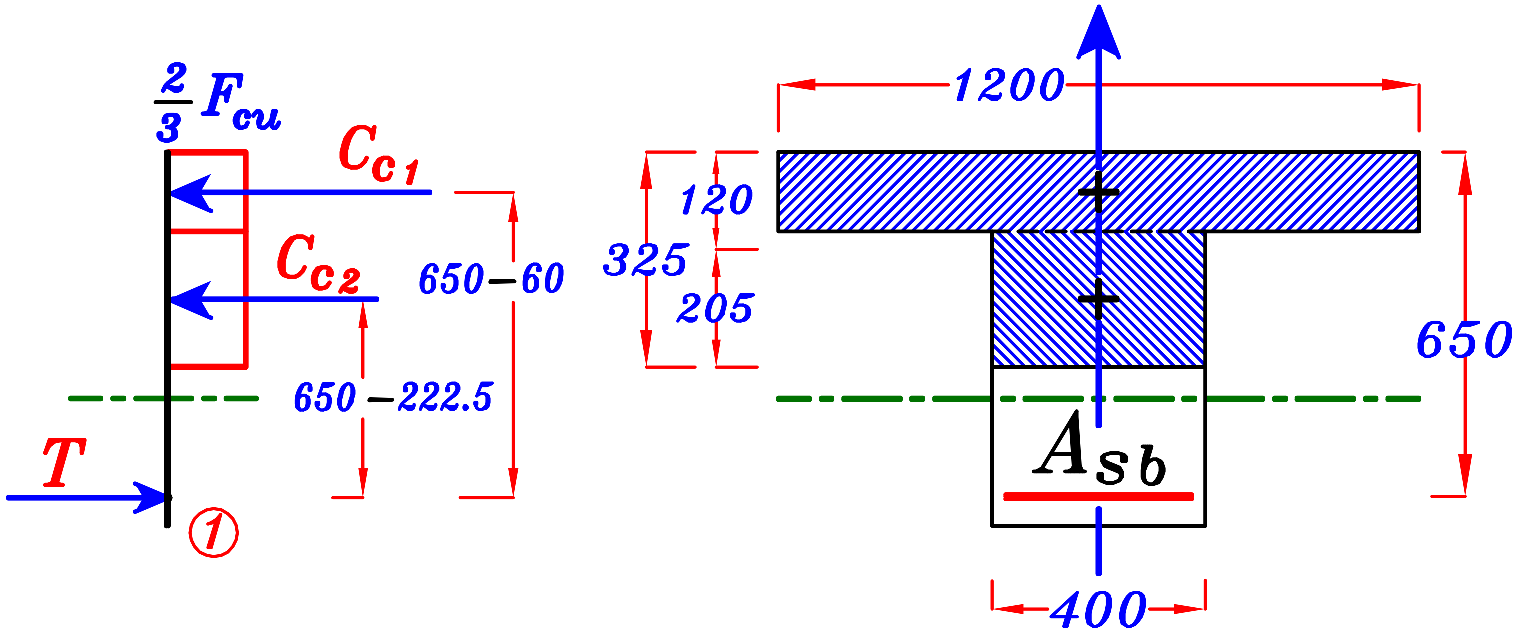
Solution.

For Balanced Sec. $C = C_b$, $a = a_b = 0.8\ C_b$, $F_s = F_y$

① $$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 650 = 406.25\ mm$$

② $$a = a_b = 0.8\ C_b = 0.8 * 406.25 = 325\ mm > t_s$$





③ **From equilibrium eqn.** $C_{c1} + C_{c2} = T$

$$\frac{2}{3} F_{cu} * t_F * B + \frac{2}{3} F_{cu} * (a_b - t_s) * b = F_y * A_{sb}$$

$$\frac{2}{3} (25)(120)(1200) + \frac{2}{3} (25)(325 - 120)(400) = (360)\ A_{sb}$$

$$\therefore \boxed{A_{sb} = 10463 \ mm^2}$$

④ $\therefore M_{ult} = \frac{2}{3} F_{cu}\, t_s\, B \left(d - \frac{t_s}{2}\right) + \frac{2}{3} F_{cu} (a_b - t_s)\, b \left(d - t_s - \frac{a - t_s}{2}\right)$

$$M_{ult} = \frac{2}{3}(25)(120)(1200)\left(650 - \frac{120}{2}\right) + \frac{2}{3}(25)(325-120)(400)\left(650 - 120 - \frac{325-120}{2}\right)$$

$$= 2000250000 \ N.mm = 2000.25 \ kN.m$$

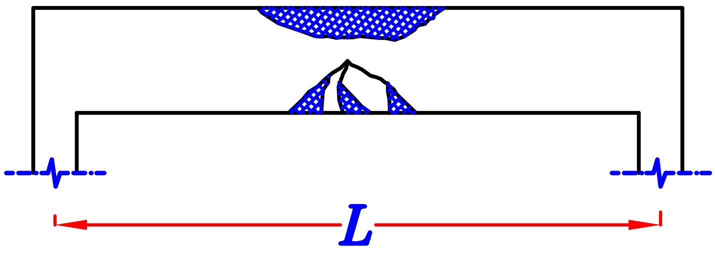
$$\therefore \boxed{M_b = M_{ult} = 2000.25 \ kN.m}$$

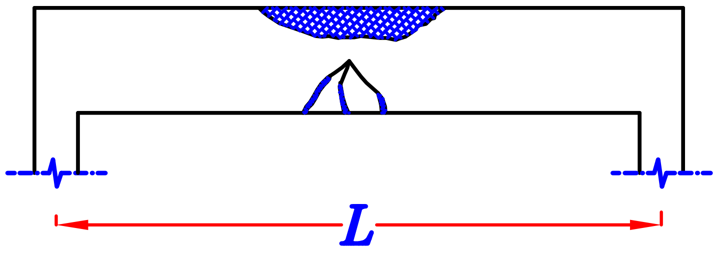
Example.

① Explain the type of Failure of each beam.

② IF the cross-sec. of each beam is (300 * 700), Fined the expected range of area of steel reinforcement For each beam.

$F_{cu} = 25 \quad N\backslash mm^2 \qquad st. \ 360/250$

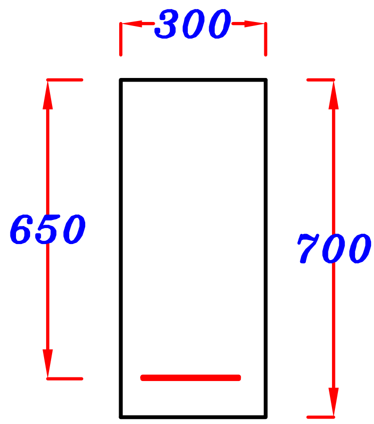




① Under Reinforced Sec. (Tension Failure) — Over Reinforced Sec. (Compression Failure)

②

$$C_b = \frac{600}{600 + F_y} * d$$



$$= \frac{600}{600 + 360} * 650 = 406.25 \ mm$$

$$a = a_b = 0.8 \ C_b = 0.8 * 406.25 = 325 \ mm$$

From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * b = A_s * F_s$$

For balanced Sec. $a = a_b = 325 \ mm$, $F_s = F_y$, $A_s = A_{sb}$

$$\therefore \ \frac{2}{3} F_{cu} * a_b * b = A_{sb} * F_y$$

$$\frac{2}{3} (25)(325)(300) = A_{sb} (360) \longrightarrow \therefore \boxed{A_{sb} = 4513.8 \ mm^2}$$

$\therefore$ For Under Reinforced Sec. $A_s < 4513.8 \ mm^2$

$\therefore$ For Over Reinforced Sec. $A_s > 4513.8 \ mm^2$

$\left(M_{U.L.}\right)$

Introduction of Ultimate Limit Moment

هو العزم الذى تم عليه تصميم القطاع بطريقه Ultimate Limits Design Method

و للتصميم بهذه الطريقه يجب الاخذ فى الاعتبار قيم Factor Of Safety

Factors Of Safety For Limit State Design Method.

* F.O.S. For Loads.

F.O.S. For Dead Load. = 1.4
F.O.S. For Live Load. = 1.6
} To increase the Load.

F.O.S. For Dead Load. = 0.9
F.O.S. For Live Load. = zero
} To decrease the Load.

Load (To Increase) = 1.4 D.L. + 1.6 L.L.

= 1.5 (D.L.+ L.L.) IF L.L. $\geqslant$ 0.75 D.L.

Load (To Decrease) = 0.9 D.L. + 0.0 L.L.

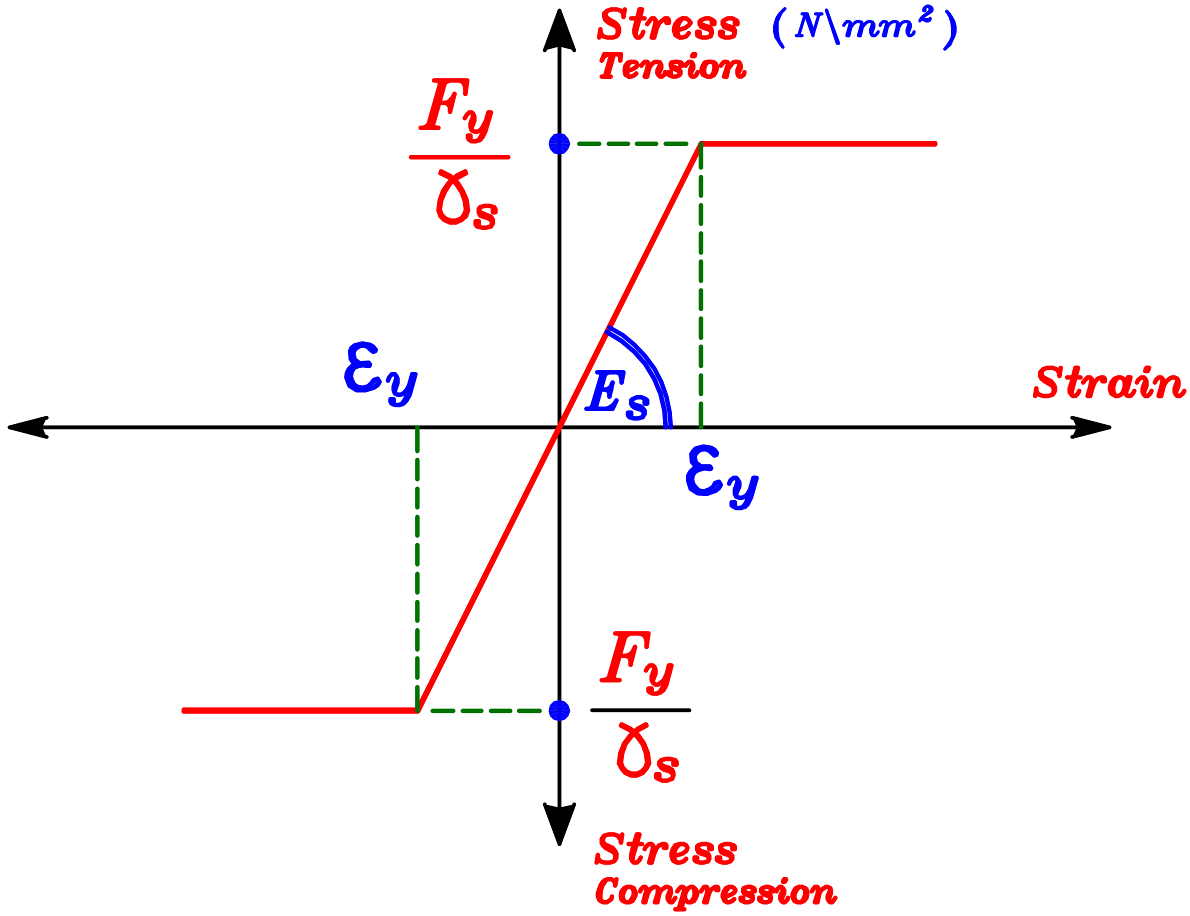
* F.O.S. For Materials.

Case of bending moment only (M) or Tension only (T)
or Axial tension & bending moment $(M+T)$
or Shear (Q) only or Torsion only (M_t) or Shear & Torsion $(Q+M_t)$

$$\boxed{\gamma_c = 1.5 \quad , \quad \gamma_s = 1.15}$$

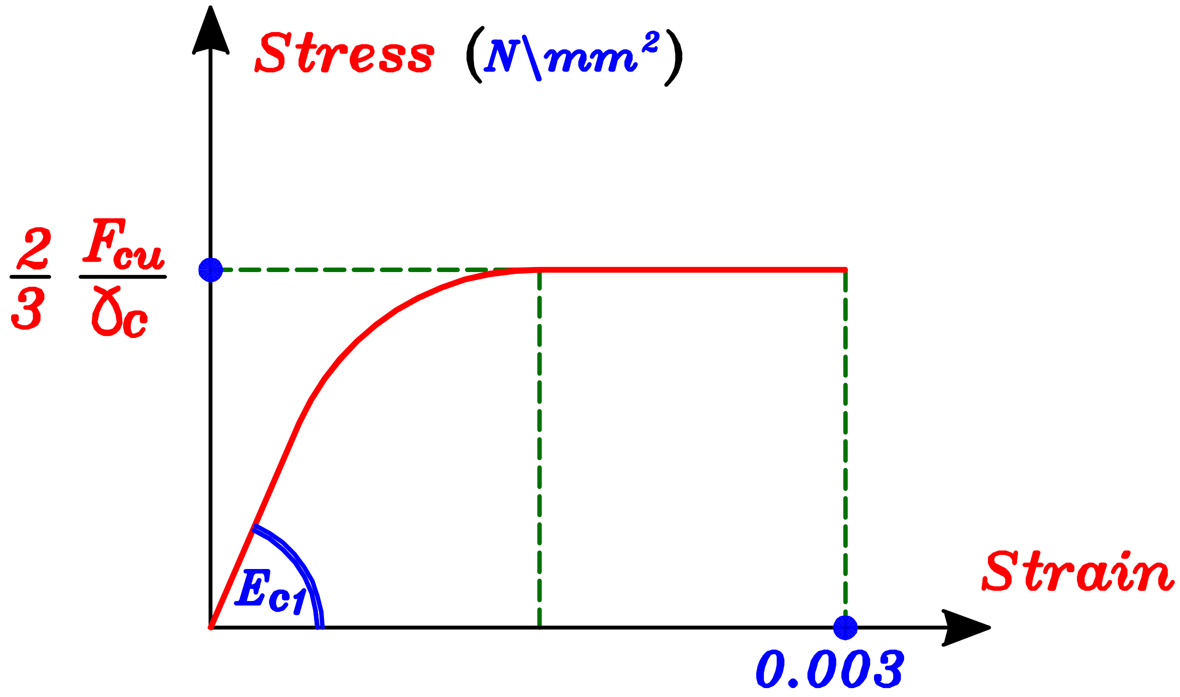
$\therefore$ Allowable stress For concrete. $= \left(\dfrac{F_{cu}}{\gamma_c}\right)$

Allowable stress For steel. $= \left(\dfrac{F_y}{\gamma_s}\right)$



Idealized Stress–Strain Curve For Steel.

المنحنى الاعتبارى للاجهاد و الانفعال للحديد .



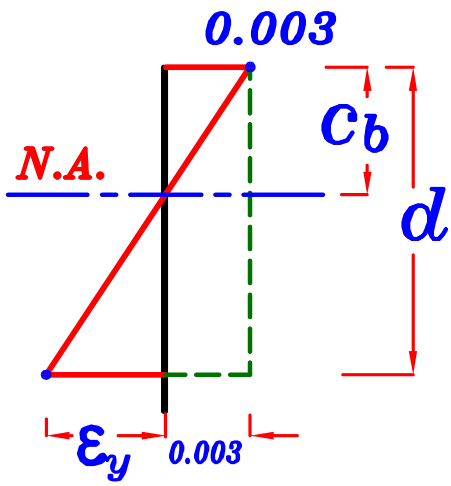
Idealized Stress–Strain Curve For Concrete.

المنحنى الاعتبارى للاجهاد و الانفعال للخرسانه

و يجب أن يكون القطاع Under Reinforced sec.

Properties of Under Reinforced Section.

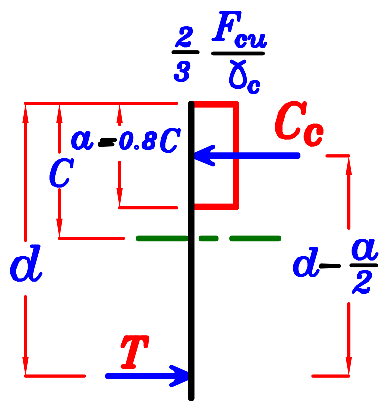
$$\because \; C_b = \frac{600}{600 + (F_y \backslash \gamma_s)} * d$$



For under Reinforced section $C \leqslant C_b$

① $C \leqslant C_{max.}$ where: $\boxed{C_{max} = \frac{2}{3} C_b}$

$$\therefore \; \boxed{C_{max} = \frac{2}{3}\left[\frac{600}{600 + (F_y \backslash \gamma_s)} * d\right]}$$



> IF $C > C_{max.}$ $\longrightarrow$ over reinforced sec. نعتبر كأن القطاع
> و هذا لا ينفع فى التصميم

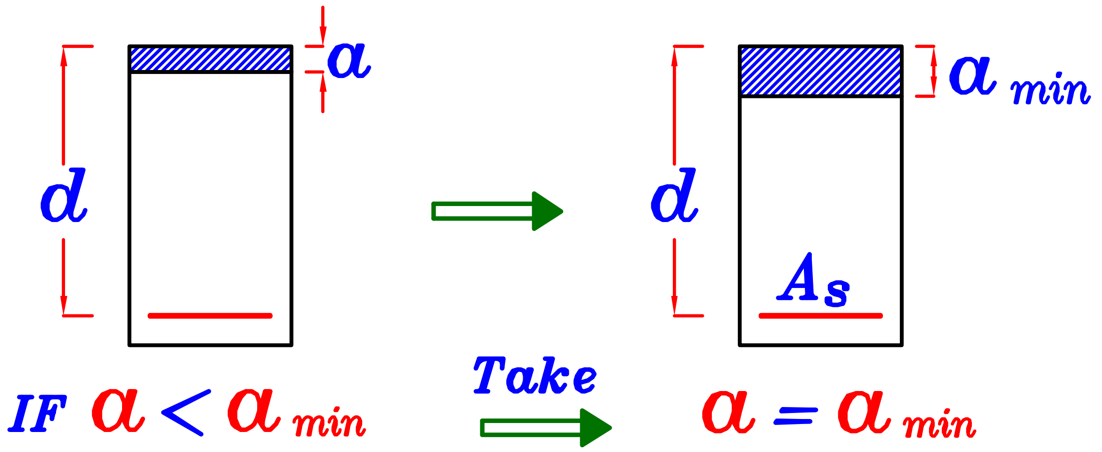
② $a \leqslant a_{max.}$ $\boxed{a_{max.} = 0.8\, C_{max.}}$

$$\therefore \; \boxed{a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)} * d\right]}$$

> IF $a > a_{max.}$ $\longrightarrow$ over reinforced sec. نعتبر كأن القطاع
> و هذا لا ينفع فى التصميم

③ $a \geqslant a_{min}$ $\boxed{a_{min} = 0.1\,d}$

عند التصميم يجب عمل check على a بحيث لا تكون أقل من a_{min}



عن طريق زياده كميه الحديد

Egyptian Code Page (4–6) Table (4–1)

جدول (٤-١) معامل الحد الأقصى لمقاومة العزوم R_{max} ونسبة صلب التسليح القصوى μ_{max}
ونسبة العمق الأقصى لمحور الخمول إلى العمق الفعال c_{max}/d للقطاعات المسلحة جهة الشد فقط

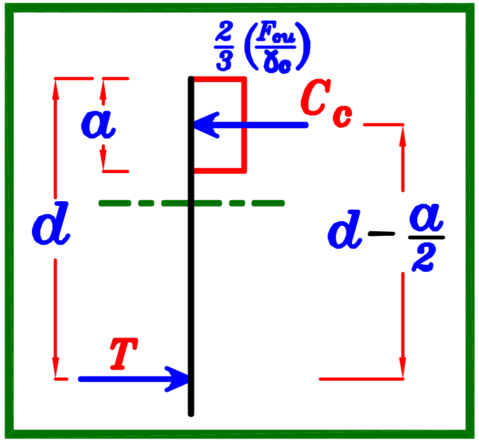
رتبة الصلب*	c_{max}/d	μ_{max}	R_{max}
240/350	0.50	$8.56\times10^{-4}\ f_{cu}$	0.214
280/450	0.48	$7.00\times10^{-4}\ f_{cu}$	0.208
360/520	0.44	$5.00\times10^{-4}\ f_{cu}$	0.194
400/600	0.42	$4.31\times10^{-4}\ f_{cu}$	0.187
450/520**	0.40	$3.65\times10^{-4}\ f_{cu}$	0.180

* طبقاً للجدول (٢-١) وحيث f_{cu} بوحدات ن/مم٢ .

** خاصة لصلب الشبك مع استيفاء ما جاء بالبند (٤-٢-١-١-٣) .

Calculation of $M_{U.L.}$ (With Ten. Steel Only)

Calculate $a_{max} = 0.8\left(\frac{2}{3}\right) C_b = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600+(F_y \backslash \gamma_s)}\right] * d$



From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b = F_s * A_s$

assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$\therefore \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b = \frac{F_y}{\gamma_s} * A_s \longrightarrow$ Get a

IF $\boxed{a}$

IF $a \leqslant 0.1\,d$

take $a = 0.1\,d$

يجب أخذ العزم عند الخرسانه

$\therefore M_{U.L.} = A_s \frac{F_y}{\gamma_s}\left(d-\frac{a}{2}\right)$

$\therefore M_{U.L.} = A_s \frac{F_y}{\gamma_s}\left(d-\frac{0.1d}{2}\right)$

$\therefore M_{U.L.} = A_s F_y\, d \,\frac{1}{1.15}\left(1-\frac{0.1}{2}\right)$

$$\therefore \boxed{M_{U.L.} = 0.826\, A_s\, F_y\, d}$$

$0.1\,d < a < a_{max.}$

Right Assumption $F_s = \frac{F_y}{\gamma_s}$

يؤخذ العزم عند الحديد أو الخرسانه

$$\therefore \boxed{\begin{aligned} M_{U.L.} &= \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a\, b\left(d-\frac{a}{2}\right) \\ M_{U.L.} &= A_s * \frac{F_y}{\gamma_s}\left(d-\frac{a}{2}\right)\end{aligned}}$$

IF $a > a_{max.}$

Wrong Assumption $F_s \neq \frac{F_y}{\gamma_s}$

Take $a = a_{max.}$

يجب أخذ العزم عند الحديد

$$\therefore \boxed{\begin{aligned} M_{U.L.} &= \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a_{max.}\, b\left(d-\frac{a_{max.}}{2}\right) \\ M_{U.L.} &= R_{max.}\frac{F_{cu}}{\gamma_c}\, b\, d^2\end{aligned}}$$

Example.

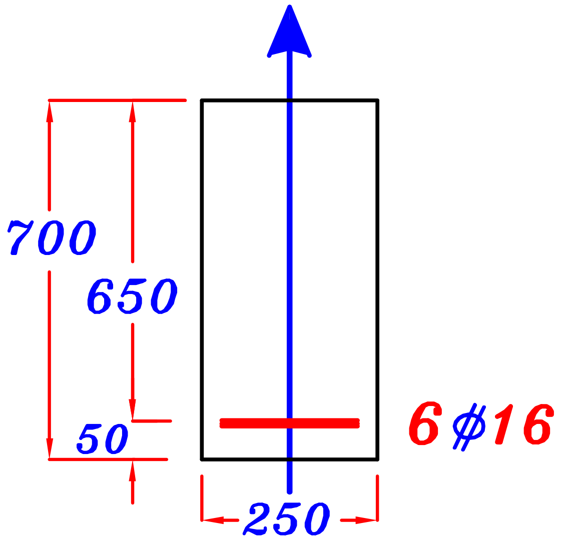
Data.

$F_{cu} = 25 \ N\backslash mm^2$

$st. \ 360/520$

Req.

Calculate $M_{U.L.}$



Solution.

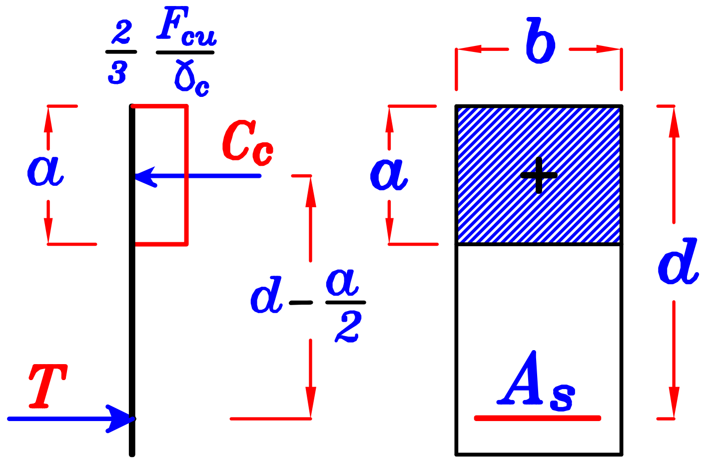
$$A_s = 6\phi16 = 6\left[\frac{\pi * 16^2}{4}\right] = 1206 \ mm^2$$

$$a_{min} = 0.1\ d = 0.1 * 650 = 65 \ mm$$

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)}\right] * d = 0.35\ d = 0.35 * 650 = 227.5 \ mm$$

$$C_c = Stress * Area = \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b$$

$$T = Stress * Area = F_s * A_s$$



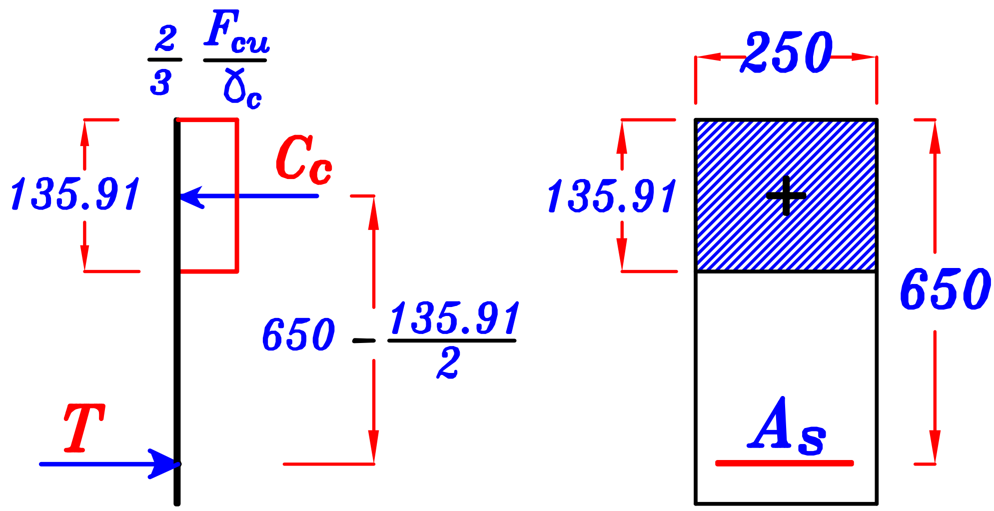
From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b = A_s * F_s$ ---- a , F_s

assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$$\therefore \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b = \frac{F_y}{\gamma_s} * A_s$$

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)(250) = (1206)\left(\frac{360}{1.15}\right) \longrightarrow \boxed{a = 135.91 \ mm}$$

$\therefore 0.1\,d < a < a_{max.}$ Right assumption $F_s = \frac{F_y}{\gamma_s}$



By taking the moment about the steel.

$$M_{U.L.} = \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a\, b\left(d - \frac{a}{2}\right)$$

$$M_{U.L.} = \frac{2}{3}\left(\frac{25}{1.5}\right)(135.91)(250)\left(650 - \frac{135.91}{2}\right)$$

$$= 219738155.4 \text{ N.mm} = 219.73 \text{ kN.m}$$

OR take the moment about the concrete.

$$M_{U.L.} = A_s\frac{F_y}{\gamma_s}\left(d - \frac{a}{2}\right)$$

$$M_{U.L.} = 1206\left(\frac{360}{1.15}\right)\left(650 - \frac{135.91}{2}\right)$$

$$= 219739701.9 \text{ N.mm} = 219.74 \text{ kN.m}$$

$$\boxed{M_{U.L.} = 219.73 \text{ kN.m}}$$

الفرق فى قيمتى العزم ناتج فقط عن التقريب
لكن كلا الاجابتين صحيح .

Example.

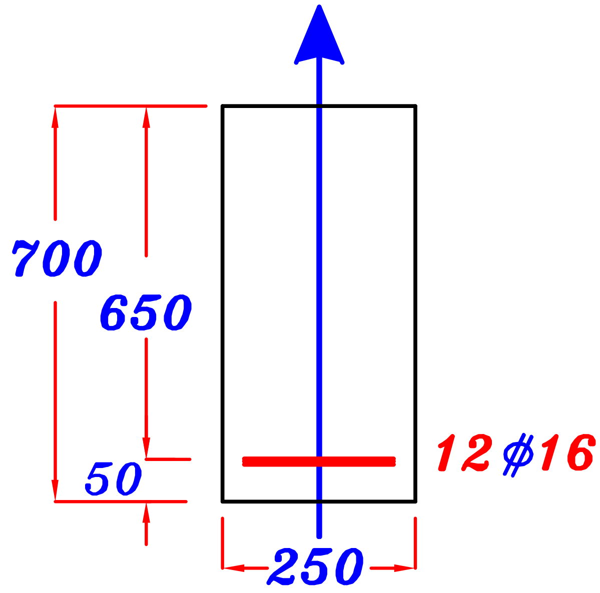
Data.

$F_{cu} = 25\ N\backslash mm^2$

st. $360/520$

Req.

Calculate $M_{U.L.}$



Solution.

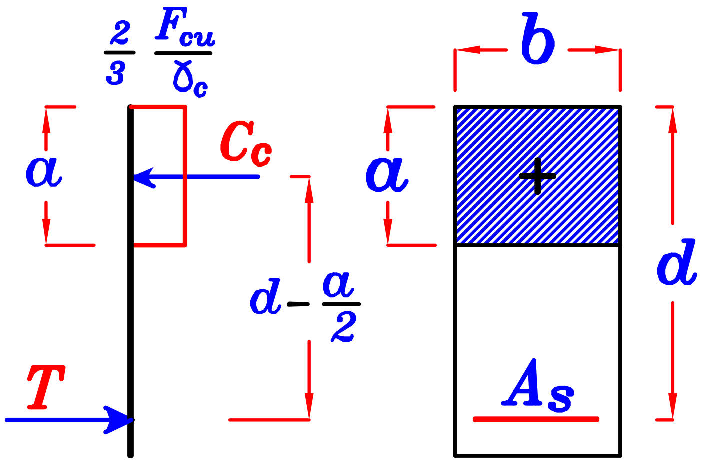
$$A_s = 12\phi16 = 12\left[\frac{\pi * 16^2}{4}\right] = 2412\ mm^2$$

$$a_{min} = 0.1\ d = 0.1 * 650 = 65\ mm$$

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y\backslash \gamma_s)}\right] * d = 0.35\ d = 0.35 * 650 = 227.5\ mm$$

$$C_c = Stress * Area = \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b$$

$$T = Stress * Area = F_s * A_s$$



From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b = F_s * A_s$ ---- a , F_s

assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$$\therefore \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b = \frac{F_y}{\gamma_s} * A_s$$

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)(250) = \left(\frac{360}{1.15}\right)(2412) \longrightarrow \boxed{a = 271.82 mm}$$

$\therefore \quad a > a_{max.} \longrightarrow$ Take $a = a_{max.}$

$$M_{U.L.} = \frac{2}{3} \frac{F_{cu}}{\gamma_c} a_{max.} b \left(d - \frac{a_{max.}}{2}\right)$$

$$M_{U.L.} = \frac{2}{3} \left(\frac{25}{1.5}\right) (227.5)(250) \left(650 - \frac{227.5}{2}\right)$$

$$= 338880208.3 \text{ N.mm} = 338.88 \text{ kN.m}$$

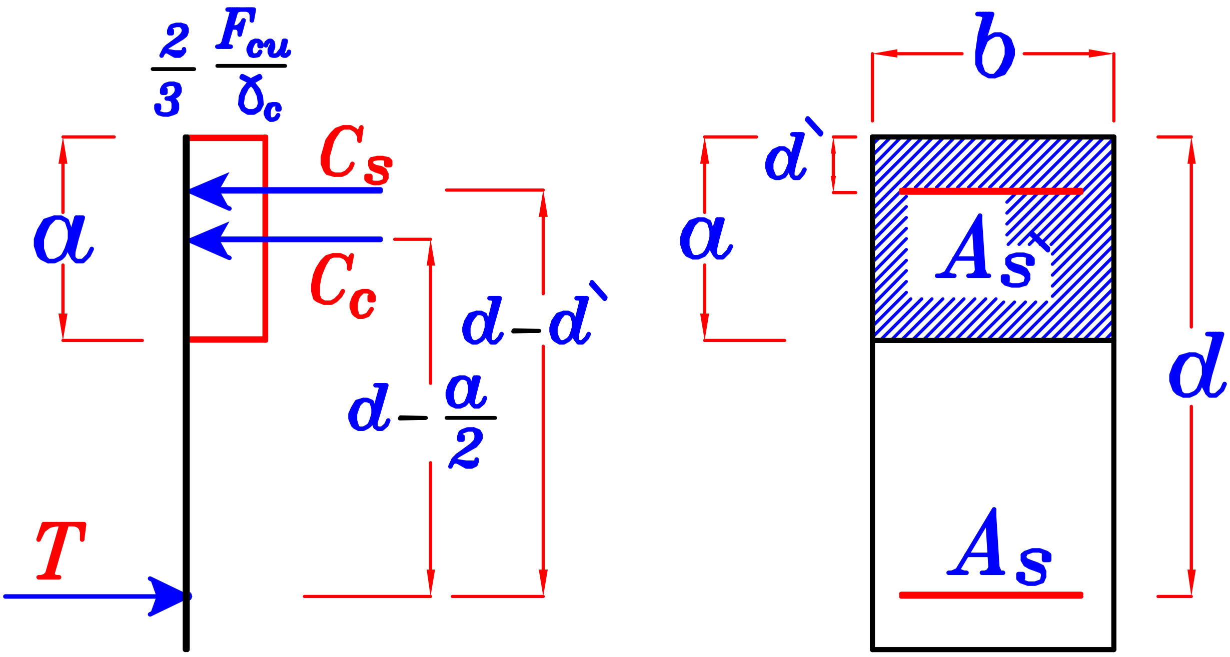
$$\boxed{M_{U.L.} = 338.88 \text{ kN.m}}$$

Approximate Calculation of ($M_{U.L.}$) with Comp. Steel.

عند حساب $M_{U.L.}$ وكان هناك حديد جهه الضغط ($A_{s`}$)

نعمل حل تقريبى للتسهيل بأن نعتبر $F_{s`} = \dfrac{F_y}{\gamma_s}$

و لحساب ال $M_{U.L.}$ مع وجود ($A_{s`}$) بدقه سنذكرها فى أخر الملف *Page No. 175*



$$C_c = \text{Stress} * \text{Area} = \frac{2}{3}\frac{F_{cu}}{\gamma_c} * (a\ b)$$

$$C_s = \text{Stress} * \text{Area} = \frac{F_y}{\gamma_s} * A_{s`}$$

By taking the moment about the steel.

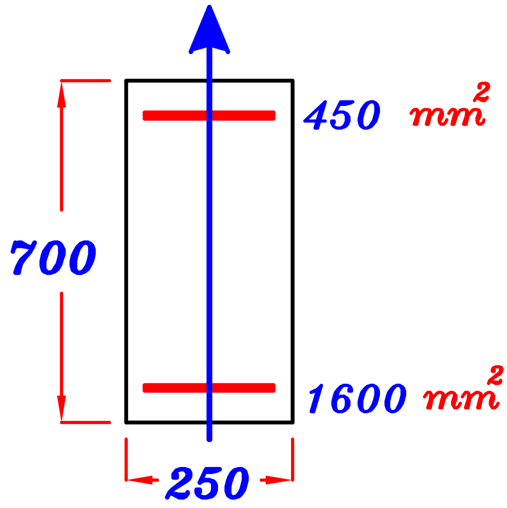
$$\boxed{M_{ult} = \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a\, b\left(d - \frac{a}{2}\right) + \frac{F_y}{\gamma_s} * A_{s`}\,(d-d`)}$$

Example.

Data.

$F_{cu} = 25\ N\backslash mm^2$ st. 360/520

Req. Calculate $M_{U.L.}$



Solution. $\because \frac{A_{s`}}{A_s} = \frac{450}{1600} = 0.28 > 0.2 \quad \therefore$ Use $A_{s`}$

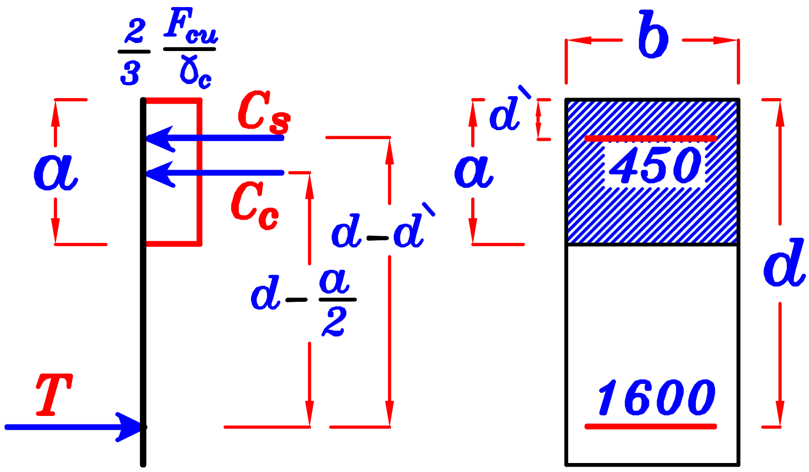
$$a_{min} = 0.1\ d = 0.1 * 650 = 65\ mm$$

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y\backslash \gamma_s)}\right] * d = 0.35\ d = 0.35 * 650 = 227.5\ mm$$

$$C_c = Stress * Area = \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b$$

$$C_s = Stress * Area = \frac{F_y}{\gamma_s} * A_{s`}$$

$$T = Stress * Area = F_s * A_s$$



From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b + \frac{F_y}{\gamma_s} * A_{s`} = F_s * A_s$

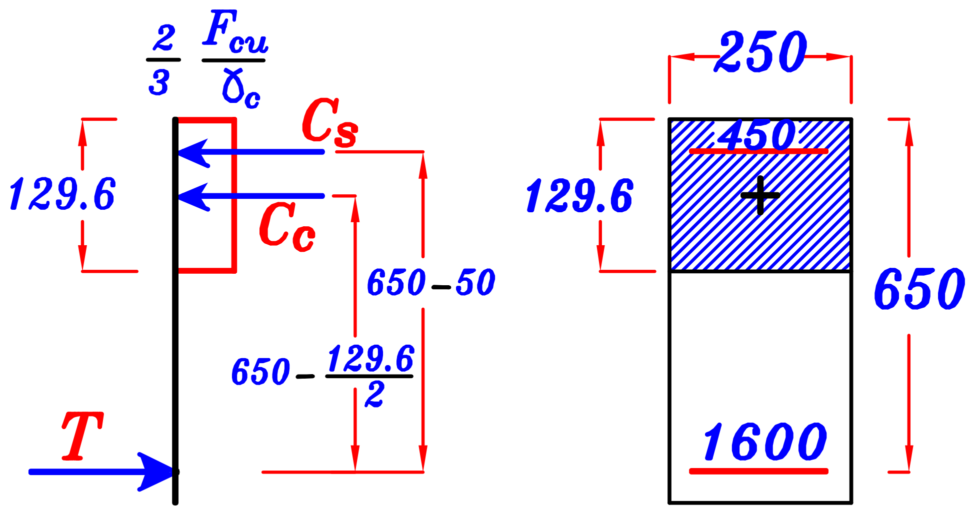
assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$$\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b + \frac{F_y}{\gamma_s} * A_{s`} = \frac{F_y}{\gamma_s} * A_s$$

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)\ (250) + \left(\frac{360}{1.15}\right)(450) = \left(\frac{360}{1.15}\right)(1600)$$

$$\boxed{a = 129.6\ mm}$$

$\therefore\ 0.1\,d < a < a_{max.}$ Right assumption $F_s = \dfrac{F_y}{\gamma_s}$



By taking the moment about the steel.

$$M_{U.L.} = \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a\, b\left(d - \frac{a}{2}\right) + \frac{F_y}{\gamma_s} * A_{s`}\,(d - d`)$$

$$M_{U.L.} = \frac{2}{3}\left(\frac{25}{1.5}\right)(129.6)(250)\left(650 - \frac{129.6}{2}\right) + \left(\frac{360}{1.15}\right)(450)\,(650 - 50)$$

$$M_{U.L.} = 295193739 \ \text{N.mm} = 295.19 \ \text{kN.m}$$

$$\boxed{M_{U.L.} = 295.19 \ \text{kN.m}}$$

Example.

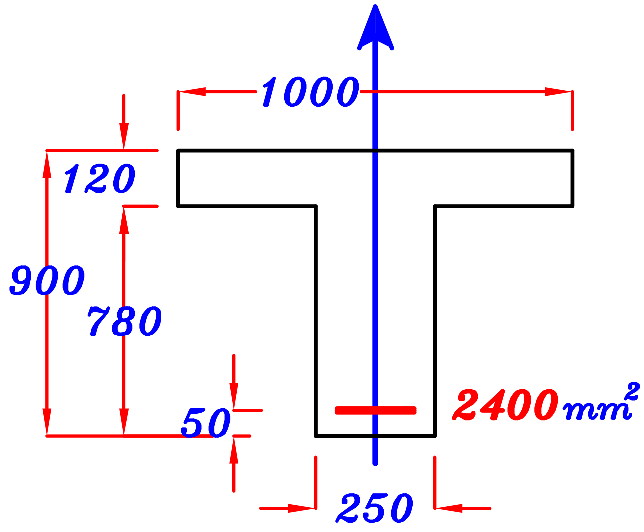
Data.

$F_{cu} = 25\ N\backslash mm^2$

st. 360/520

Req.

Calculate $M_{U.L.}$



Solution.

$a_{min} = 0.1\ d = 0.1 * 850 = 85\ mm$

$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y\backslash \gamma_s)}\right] * d = 0.35\ d = 0.35 * 850 = 297.5\ mm$

assume $a \leqslant t_s \quad a < 120\ mm$

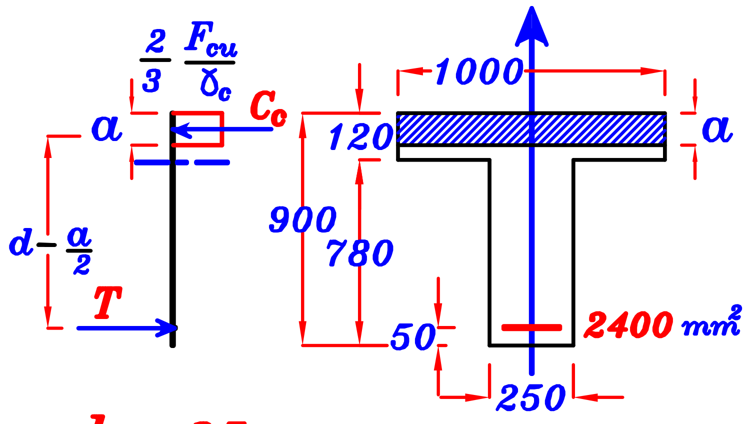
From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * B = F_s * A_s$ ---- a , F_s

assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$\therefore \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * B = \frac{F_y}{\gamma_s} * A_s$

$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)(1000) = \left(\frac{360}{1.15}\right)(2400)$

$\longrightarrow a = 67.6\ mm < t_s \quad \therefore$ o.k.



, $a < 0.1\,d \quad \therefore$ take $a = 0.1\,d = 85\ mm$

$M_{U.L.} = \frac{F_y}{\gamma_s} A_s\left(d - \frac{a}{2}\right) = \left(\frac{360}{1.15}\right) 2400 \left(850 - \frac{85}{2}\right)$

$= 606678260.9\ N.mm = 606.67\ kN.m$

$$\boxed{M_{U.L.} = 606.67\ kN.m}$$

Example.

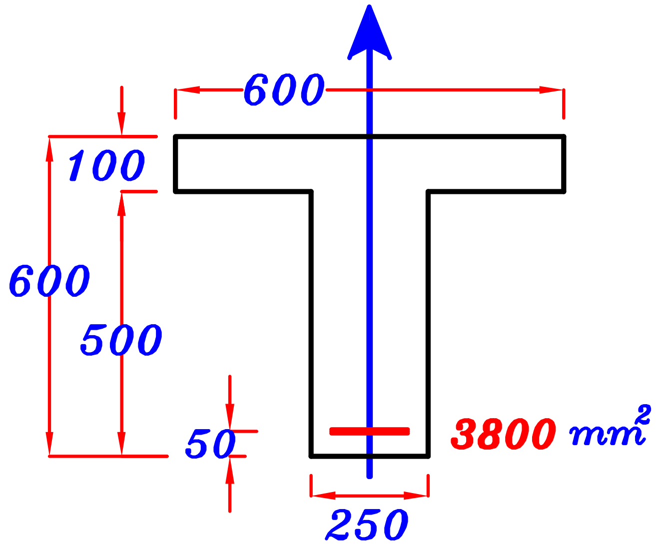
Data.

$F_{cu} = 25\ N\backslash mm^2$

st. 360/520

Req.

Calculate $M_{U.L.}$

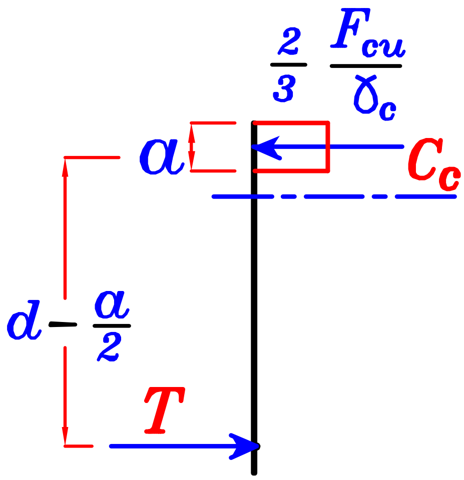


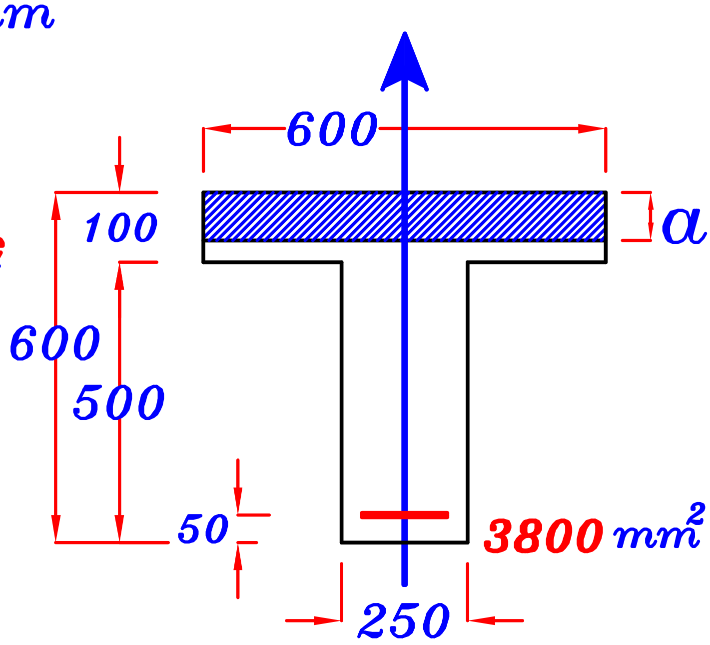
Solution.

$$a_{min} = 0.1\ d = 0.1 * 550 = 55\ mm$$

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{6000}{6000 + (F_y \backslash \gamma_s)}\right] * d = 0.35\ d = 0.35 * 550 = 192.5\ mm$$

assume $a \leqslant t_s$ $\quad a < 100\ mm$





From equilibrium eqn. $C_c = T$

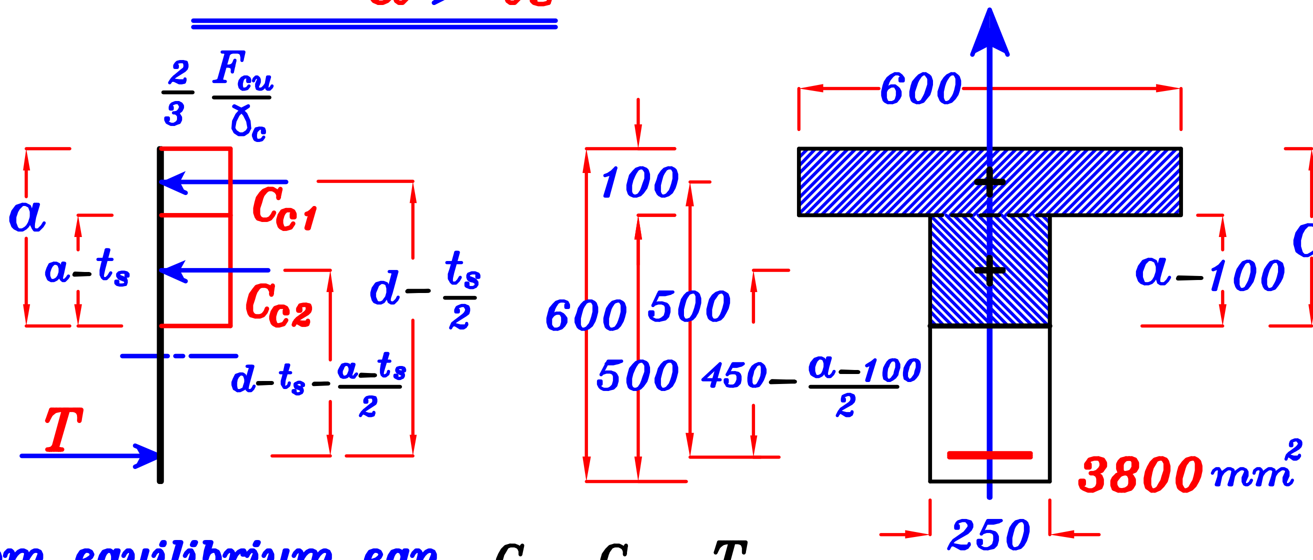
$$\frac{2}{3}\ \frac{F_{cu}}{\gamma_c} * a * B = F_s * A_s \text{ ---- } a\ ,\ F_s$$

Assume $F_s = \frac{F_y}{\gamma_s} \longrightarrow$ (under reinforced Sec.)

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)(600) = \left(\frac{360}{1.15}\right)(3800) \longrightarrow a = 178.4\ mm > t_s$$

$a > t_s$ wrong assumption $\quad \therefore$ Take $\quad a > t_s$

$\therefore$ Take $a > t_s$



From equilibrium eqn. $C_{c1}+C_{c2}=T$

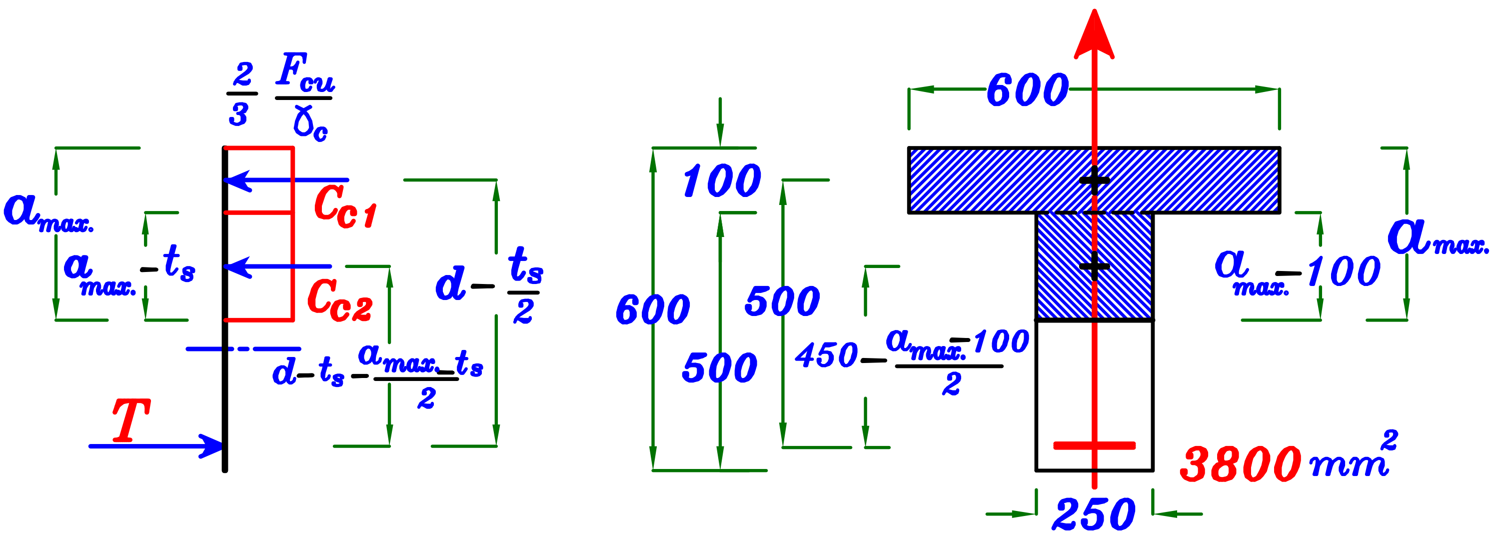
$$\frac{2}{3}\frac{F_{cu}}{\gamma_c}*t_s*B+\frac{2}{3}\frac{F_{cu}}{\gamma_c}*(a-t_s)*b=F_s*A_s$$

Assume $F_s=\frac{F_y}{\gamma_s}$ $\longrightarrow$ (under reinforced Sec.)

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(100)(600)+\frac{2}{3}\left(\frac{25}{1.5}\right)(a-100)(250)=\left(\frac{360}{1.15}\right)(3800)$$

$\longrightarrow a = 288.24$ mm

$\therefore a > a_{max.} \longrightarrow$ Take $a = a_{max.} = 192.5$ mm



$$M_{U.L.}=\left(\frac{2}{3}\frac{F_{cu}}{\gamma_c}*t_s*B\right)\left(d-\frac{t_s}{2}\right)+\left(\frac{2}{3}\frac{F_{cu}}{\gamma_c}*(a_{max.}-t_s)*b\right)\left(d-t_s-\frac{a_{max.}-t_s}{2}\right)$$

$$M_{U.L.}=\frac{2}{3}\left(\frac{25}{1.5}\right)(100)(600)\left(550-\frac{100}{2}\right)+\frac{2}{3}\left(\frac{25}{1.5}\right)(192.5-100)(250)\left(550-100-\frac{192.5-100}{2}\right)$$

$$M_{U.L.}=437074652.8 \text{ N.mm} = 437.07 \text{ kN.m}$$

$$\therefore \boxed{M_{U.L.}=437.07 \text{ kN.m}}$$

Example.

For the section it is required to calculate :

a – The Cracking Moment. $(M_{cr.})$

b – The Working Moment. (M_{w})

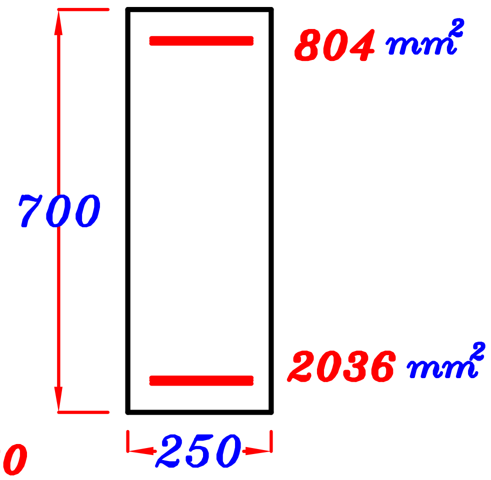
c – The Failure Moment. (M_{ult})

d – The Ultimate Limit Moment. $(M_{U.L.})$

e – The Factor Of Safety For Loads.

f – The Factor Of Safety For Material.

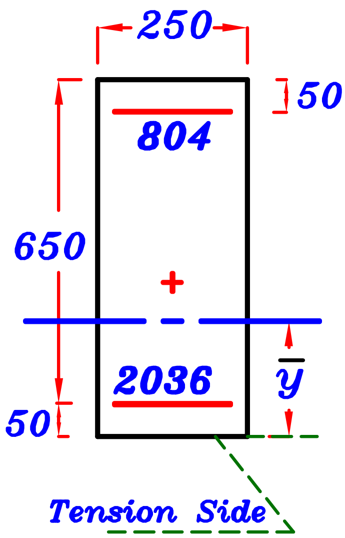
g – The Global Factor Of Safety.



Data : $F_{cu} = 25 \ kN\backslash m^2$, st. 360/520

---

a – M_{cr}



① $$n = \frac{E_s}{E_c} = \frac{2*10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$$

② $$A_v = b*t + (n-1)A_s + (n-1)A_s'$$

$$A_v = 250*700 + (10-1)(2036) + (10-1)(804) = 200560 \ mm^2$$

③ $$\bar{y}_t = \frac{250*700*350 + (10-1)(2036)(50) + (10-1)(804)(650)}{200560} = 333.4 \ mm$$

④ $$I_{gross} = \frac{250*700^3}{12} + 250*700(350-333.4)^2 + (10-1)(2036)(333.4-50)^2$$

$$+ (10-1)(804)(650-333.4)^2 = 9391063167 \ mm^4$$

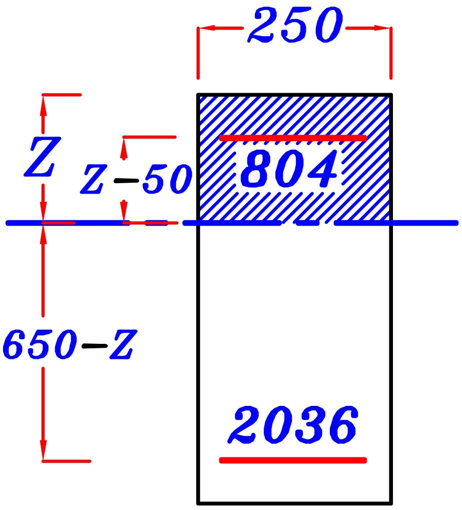
⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0 \ N\backslash mm^2$$

⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.0 * 9391063167}{333.4} = 84502668 \ mm.N$$

$$= 84.5 \ kN.m$$

$$\boxed{M_{cr} = 84.5 \ kN.m}$$

b – M_w



Allowable stresses

$F_{cu} = 25 \ N\backslash mm^2 \longrightarrow F_c = 9.5 \ N\backslash mm^2$

$F_y = 360 \ N\backslash mm^2 \longrightarrow F_s = 200 \ N\backslash mm^2$

① Take $n = 15$

② Get Z by taking $\boxed{S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}}$

$$b(Z)\left(\frac{Z}{2}\right) + (n-1)A_{s`}(Z-d`) = n\,A_s(d-Z)$$

$$250(Z)\left(\frac{Z}{2}\right) + (14)(804)(Z-50) = (15)(2036)(650-Z)$$

$$\boxed{Z = 270.1 \ mm}$$

③ Get $I_{nv} = \dfrac{bZ^3}{3} + (n-1)A_{s`}(Z-d`)^2 + n\,A_s(d-Z)^2$

$$I_{nv} = \frac{250(270.1)^3}{3} + (14)(804)(270.1-50)^2 + (15)(2036)(650-270.1)^2$$

$$= 6595014217 \ mm^4$$

④ $M_{wc} = \dfrac{F_c * I_{nv}}{Z} = \dfrac{9.5 * 6595014217}{270.1} = 231960885 \ N.mm$

$= 231.9 \ kN.mm$

⑤ $M_{ws} = \dfrac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z} = \dfrac{\left(\frac{200}{15}\right) * 6595014217}{650-270.1} = 231464919 \ N.mm$

$= 231.46 \ kN.m$

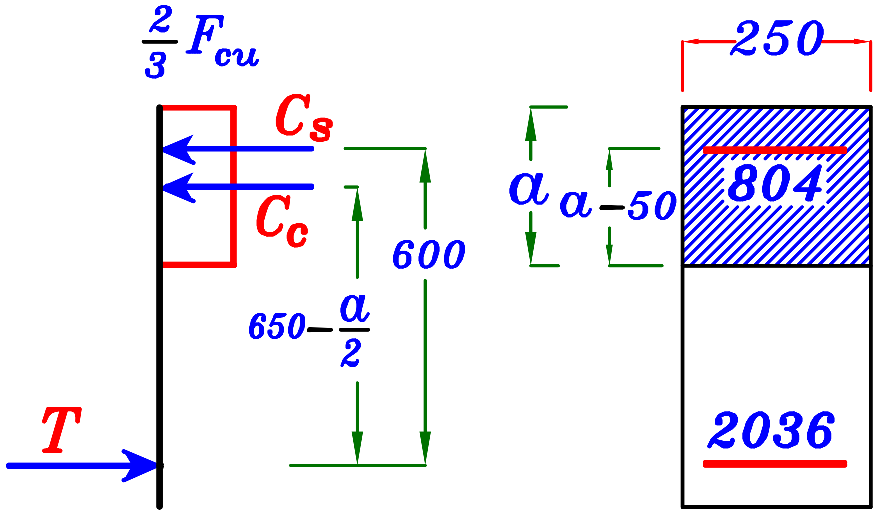
⑥ $\boxed{M_w = 231.46 \ kN.m}$

c – Mult.

ملحوظه : للتسهيل نأخذ ال stress على حديد الضغط يساوى F_y

$d = 650\ mm$

$d` = 50\ mm$



① $C_b = \frac{600}{600+F_y} * d = \frac{600}{600+360} * 650 = 406.25\ mm$

② From equilibrium eqn. $C_c + C_s = T$

$$\frac{2}{3} F_{cu} * (a * b) + F_y * A_{s`} = F_s * A_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(25)(a)(250) + (360)(804) = (360)(2036)$$

$\longrightarrow a = 106.4\ mm \longrightarrow C = 1.25\ a = 1.25 * 106.4 = 133.0\ mm < C_b$

$\therefore$ **The Section is Under Reinforced Sec.**

and the assumption is right $F_s = F_y$

$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, b\left(d - \frac{a}{2}\right) + F_y\, A_{s`}(d - d`)$$

$$= \frac{2}{3}(25)(106.4)(250)\left(650 - \frac{106.4}{2}\right) + (360)(804)(650-50)$$

$$= 438245333\ N.mm = 438.2\ kN.m$$

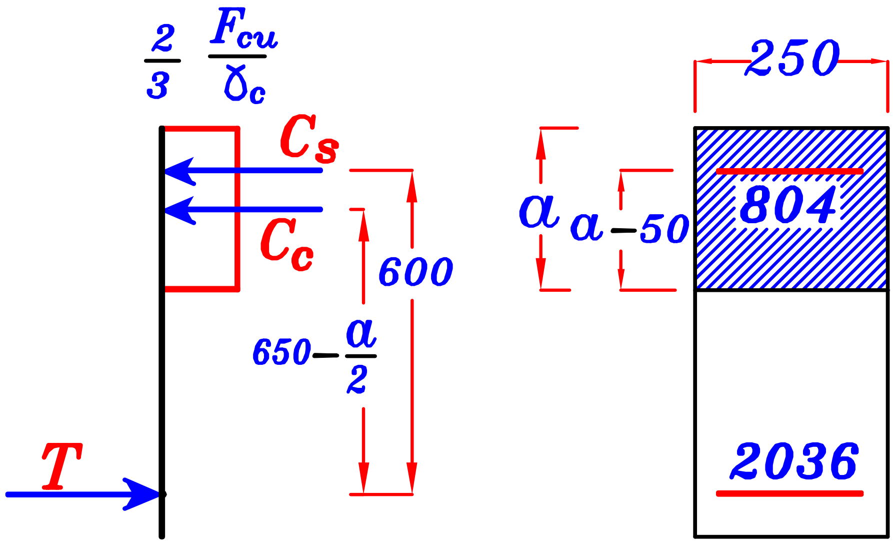
$$\boxed{M_{ult} = 438.2\ kN.m}$$

$d - M_{U.L.}$

ملحوظه : للتسهيل نأخذ ال stress على حديد الضغط يساوى $\frac{F_y}{\gamma_s}$

$d = 650\ mm$

$d` = 50\ mm$



$$a_{min} = 0.1\ d = 0.1 * 650 = 65\ mm$$

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600+(F_y\backslash\gamma_s)}\right] * d = 0.35\,d = 0.35 * 650 = 227.5\ mm$$

From equilibrium eqn. $C_c + C_s = T$

$$\frac{2}{3}\ \frac{F_{cu}}{\gamma_c} * (a * b) + \frac{F_y}{\gamma_s} * A_{s`} = F_s * A_s$$

Assume $F_s = \frac{F_y}{\gamma_s}$ $\longrightarrow$ (under reinforced)

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)(250) + \left(\frac{360}{1.15}\right)(804) = \left(\frac{360}{1.15}\right)(2036)$$

$\longrightarrow a = 138.8\ mm$ $\quad\therefore 0.1\,d < a < a_{max}$

Right assumption $F_s = \frac{F_y}{\gamma_s}$

$$\therefore M_{U.L.} = \frac{2}{3}\ \frac{F_{cu}}{\gamma_c}\ a\ b\left(d-\frac{a}{2}\right) + \frac{F_y}{\gamma_s}\ A_{s`}\,(d-d`)$$

$$\therefore M_{U.L.} = \frac{2}{3}\left(\frac{25}{1.5}\right)(138.8)(250)\left(650-\frac{138.8}{2}\right) + \left(\frac{360}{1.15}\right)(804)(650-50)$$

$$= 374865729\ N.mm = 374.8\ kN.m$$

$$\boxed{M_{U.L.} = 374.8\ kN.m}$$

e - The Factor Of Safety For Loads.

$$= \left(\frac{M_{U.L.}}{M_w}\right) = \frac{374.8}{231.46} = 1.62$$

F - The Factor Of Safety For Material.

$$= \left(\frac{M_{ult}}{M_{U.L.}}\right) = \frac{438.2}{374.8} = 1.17$$

g - The Global Factor Of Safety.

$$= \left(\frac{M_{ult}}{M_w}\right) = \frac{438.2}{231.46} = 1.89$$

طرق التصميم .

يوجد طريقتان لتصميم الكمرات

1- Working Stress Design Method.

2- Ultimate Limits Design Method.

و تعريف الـ $M_{w\,all}$ هو العزم الذى صمم عليه القطاع بطريقه
Working Stress Design Method.
او هو العزم الذى لو أثر على القطاع سيجعل القطاع just safe فى طريقه
Working Stress Design Method.

و تعريف الـ $M_{U.L.\,all}$ هو العزم الذى صمم عليه القطاع بطريقه
Ultimate Limits Design Method.
او هو العزم الذى لو أثر على القطاع سيجعل القطاع just safe فى طريقه
Ultimate Limits Design Method.

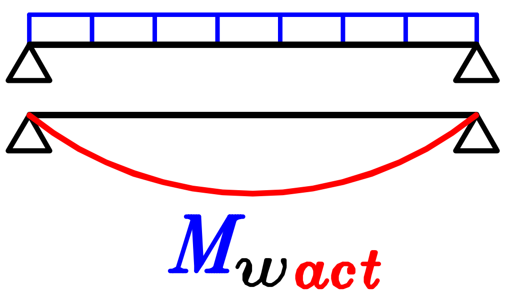
Types of Actual moments. انواع الاحمال التى من الممكن ان تؤثر على القطاع

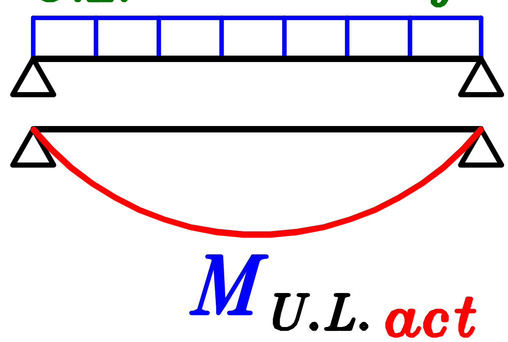
1- Working Loads. و هى الاحمال العاديه المؤثره على الكمره

2- Ultimate Limit Loads.

و هى الاحمال المؤثره على الكمره لكن بعد ضرب قيمتها فى Factor of Safty For Loads

$w_{working}$ $\qquad$ $w_{U.L.} \simeq w_{working} * 1.5$





Factor of Safty. F.O.S.

For a given section

1 – The Factor Of Safety For Loads $= \left(\dfrac{M_{U.L.}}{M_w}\right)$

2 – The Factor Of Safety For Material $= \left(\dfrac{M_{ult}}{M_{U.L.}}\right)$

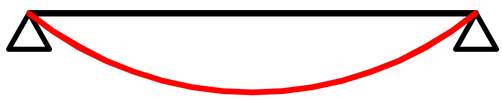
3 – The Global Factor Of Safety.

$$= F.O.S._{Loads} * F.O.S._{Material}$$

$$= \left(\frac{\cancel{M_{U.L.}}}{M_w}\right) * \left(\frac{M_{ult}}{\cancel{M_{U.L.}}}\right) = \left(\frac{M_{ult}}{M_w}\right)$$

---

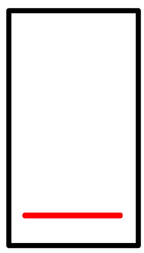
1 – In Case that $M_{w\,act}$ is given



$M_{w\,act}$

$$\therefore \quad F.O.S. = \frac{M_{ult}}{M_{w\,act}}$$

2 – In Case that $M_{w\,act}$ is not given



$M_{w\,all}$

$$\therefore \quad F.O.S. = \frac{M_{ult}}{M_{w\,all}}$$

IF asked to get the value of unknown in working case or allowable case $\xrightarrow{use}$ M_w

IF asked to get the value of unknown in Design

$\xrightarrow{use}$ M_w *IF asked by using* ***Working Stress Design Method.***

$\xrightarrow{use}$ $M_{U.L.}$ *IF asked by using* ***Ultimate Limits Design Method.***

For water structures allowable moment is Cracking moment

Just safe فى المنشأت المعرضه مباشره الى الماء يكون القطاع

اذا كان العزم المؤثر عليه هو M_{cr}

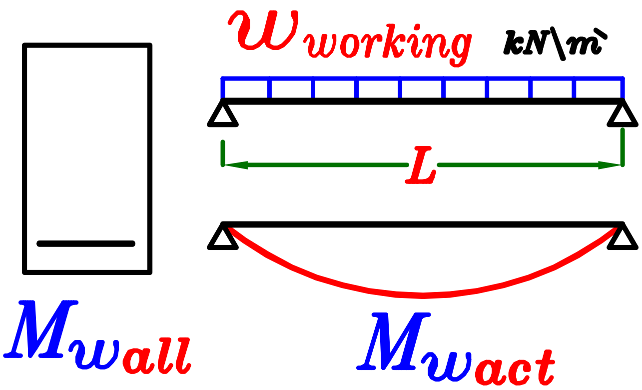
Check Safty.

عندما يتم طلب تحديد اذا كان القطاع المعطى *Safe* أم لا .

1– Check Safty with Working Method.

a – IF $M_{w\,act} \leqslant M_{w\,all} \longrightarrow$ ***Safe***

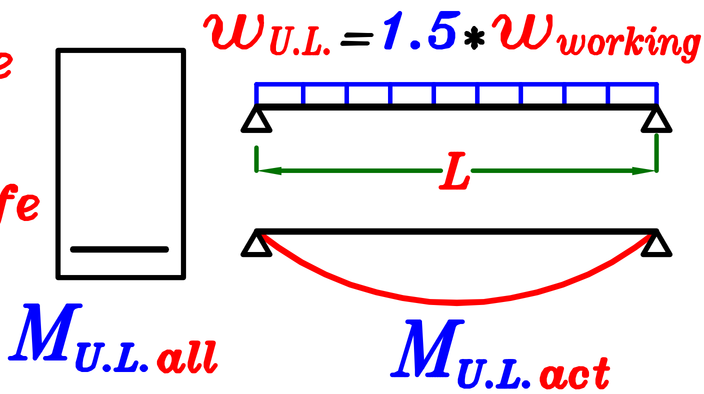
b – IF $M_{w\,act} > M_{w\,all} \longrightarrow$ ***Unsafe***



2– Check Safty with Ultimate Limit Method.

a – IF $M_{U.L.\,act} \leqslant M_{U.L.\,all} \longrightarrow$ ***Safe***

b – IF $M_{U.L.\,act} > M_{U.L.\,all} \longrightarrow$ ***Unsafe***



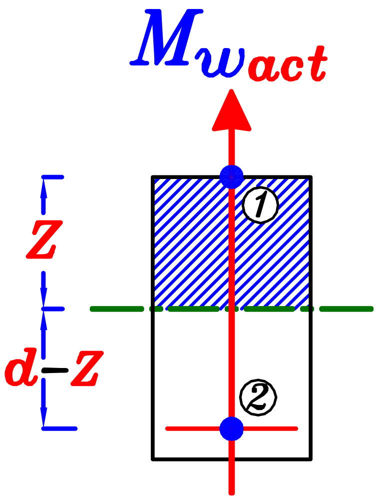
Check Stresses.

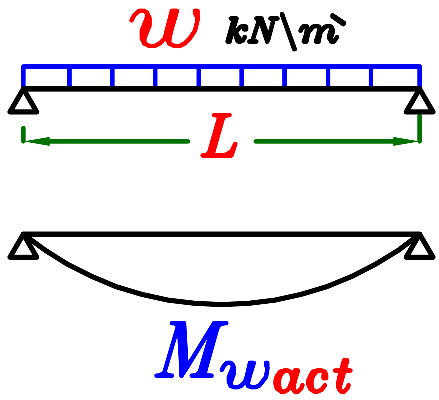
عندما يتم طلب تحديد اذا كان القطاع المعطى Safe أم لا .

عن طريق مقارنه الـ Actual Stresses with Allawable Stresses

فى هذه الحاله يتم المقارنه بطريقه الـ working method

For R–Sections.





Actual Stress on Concrete

$$\boxed{F_1 = \frac{M_{w\,act} * Z}{I_{nv}}}$$

Actual Stress on Steel

$$\boxed{F_2 = n\,\frac{M_{w\,act} * (d-Z)}{I_{nv}}}$$

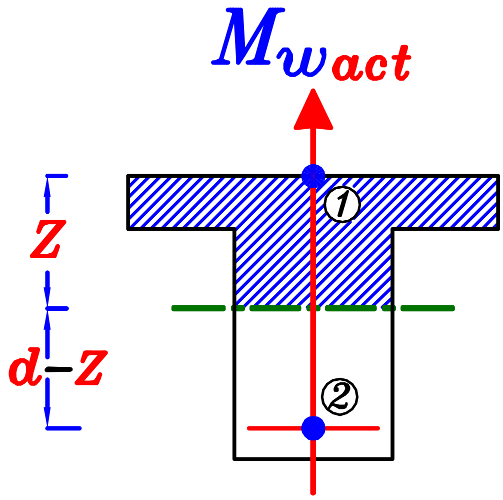
Allowable Stress on Concrete $\longrightarrow$ F_c From Tables

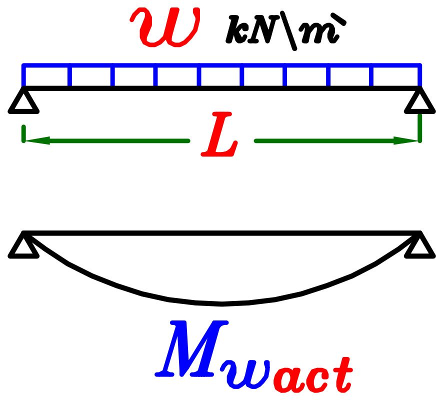
Allowable Stress on Steel $\longrightarrow$ F_s From Tables

IF Values of Actual Stresses on both concrete and steel are less than allowable stresses then the beam will be Safe.

IF the value of any of the Actual Stresses is more than the value of allowable stress then the beam will be Unsafe.

For T−Sec. or L−Sec.





Actual Stress on Concrete

$$\boxed{F_1 = \frac{M_{w\,act} * Z}{I_{nv}}}$$

Actual Stress on Steel

$$\boxed{F_2 = n\,\frac{M_{w\,act} * (d-Z)}{I_{nv}}}$$

Allowable Stress on Concrete $\longrightarrow$ $\frac{2}{3} F_c$ From Tables

For T−Sec. & L−Sec.

Allowable Stress on Steel $\longrightarrow$ F_s From Tables

IF Values of Actual Stresses on both concrete and steel are less than allowable stresses then the beam will be Safe.

IF the value of any of the Actual Stresses is more than the value of allowable stress then the beam will be Unsafe.

Example.

For the reinforced concrete cross-section shown in the Figure
It is required to calculate :

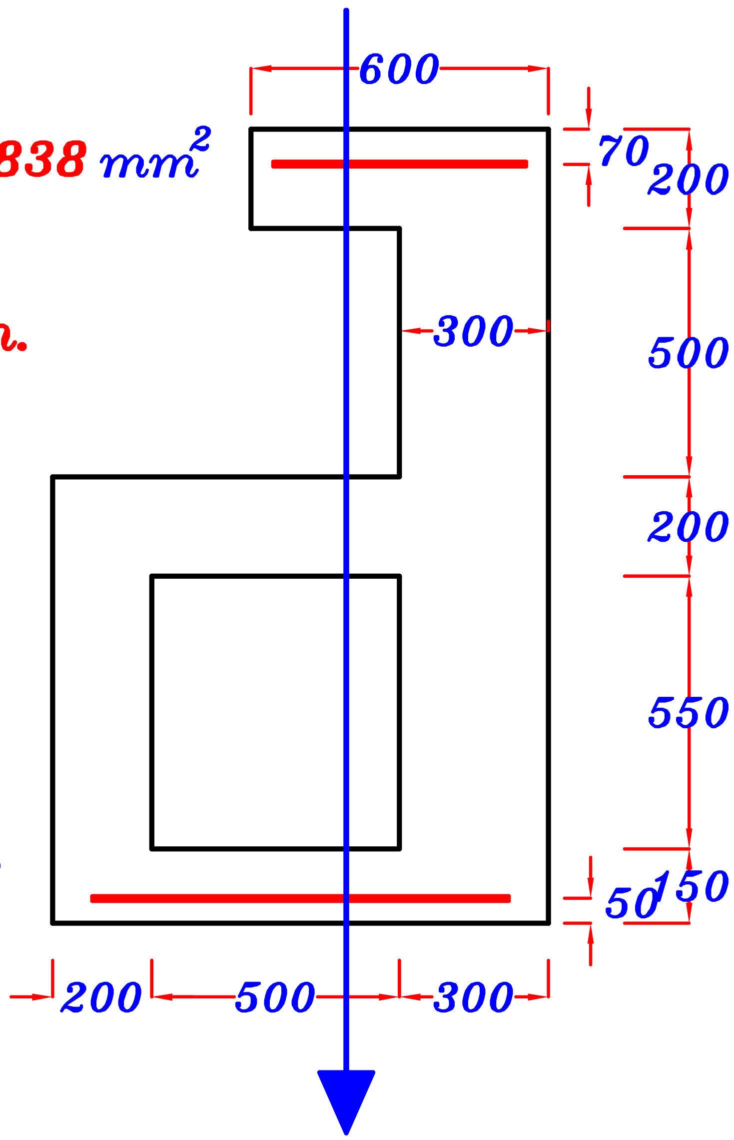
1 – Calculate the cracking moment $(M_{cr.})$, the working moment (M_w) , the ultimate limit moment $(M_{U.L.})$ & the ultimate moment $(M_{ult.})$

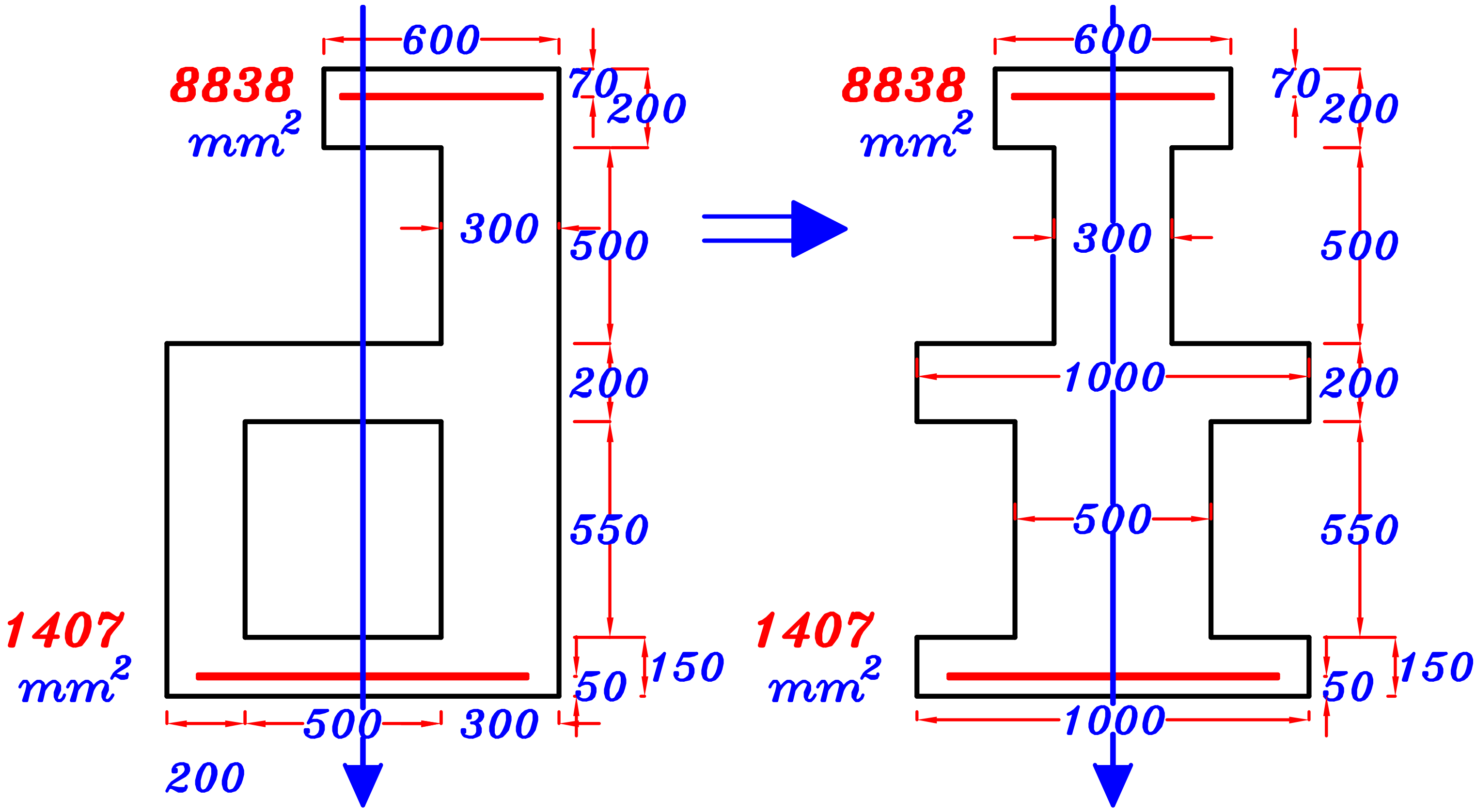
2 – Calculate the Factors of safety For Loads, Materials & Global Factor of safety.

Data : $F_{cu} = 25\ N\backslash mm^2$

, st. 400/600

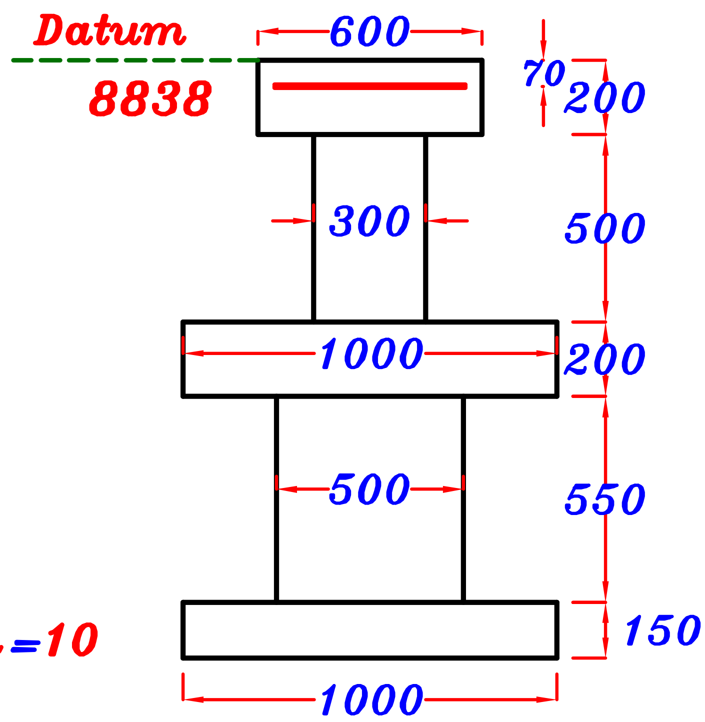
All dimensions in mm.





$$\frac{A_{s'}}{A_s} = \frac{1407}{8838} = 0.159 < 0.2$$

We can neglect $A_{s'}$



a- The Cracking Moment. $(M_{cr.})$

① $n = \dfrac{E_s}{E_c} = \dfrac{2 * 10^5}{4400\sqrt{25}} = 9.1 \rightarrow n = 10$

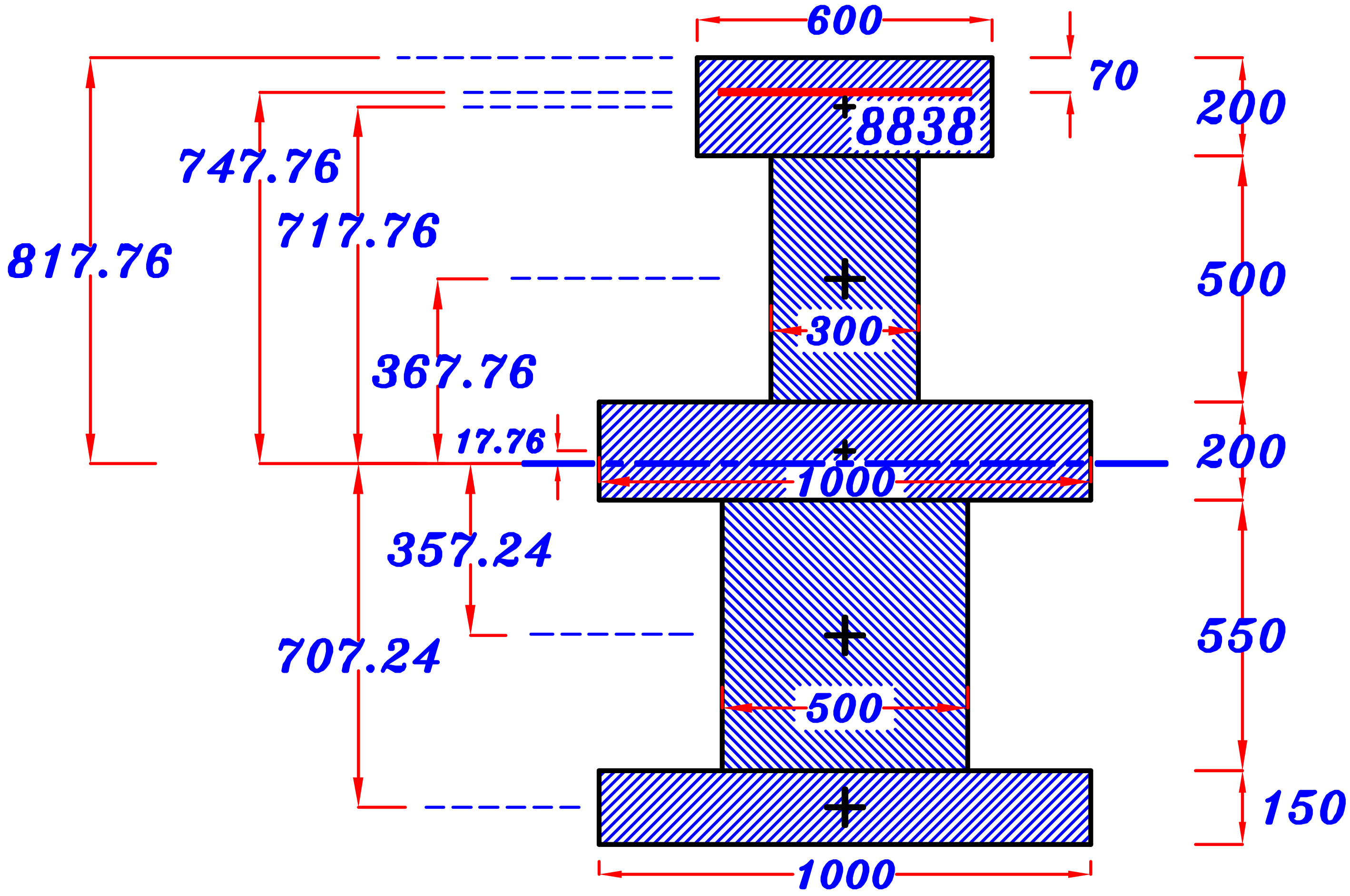
② $A_v = A_c + (n-1)A_s$

$A_v = 600*200 + 300*500 + 1000*200 + 500*550 + 1000*150 + (10-1)(8838)$

$= 974542\ mm^2$

③ $\bar{y}_t = \dfrac{600*200(100) + 300*500(450) + 1000*200(800) + 500*550(1175) + 1000*150(1525) + (10-1)(8838)(70)}{974542}$

$= 817.76\ mm$



④ $$I_g = \frac{600*200^3}{12} + 600*200\,(717.76)^2 + \frac{300*500^3}{12} + 300*500\,(367.76)^2 + \frac{1000*200^3}{12} + 1000*200\,(17.76)^2 + \frac{500*550^3}{12} + 500*550\,(357.24)^2 + \frac{1000*150^3}{12} + 1000*150\,(707.24)^2 + (10-1)(8838)(747.76)^2 = 248176325100 \ mm^4$$

⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0 \ N\backslash mm^2$$

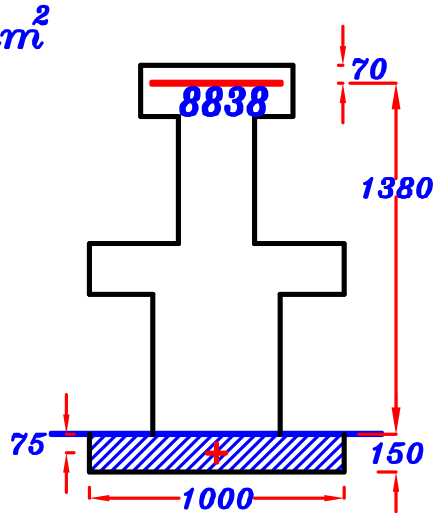
⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\overline{y}_t} = \frac{3.0 * 248176325100}{817.76} = 910449245.8 \ N.mm = 910.45 \ kN.m.$$

$$\boxed{M_{cr} = 910.45 \ kN.m}$$

b – The Working Moment. (M_w)

$F_{cu} = 25 \ N\backslash mm^2 \longrightarrow F_c = 9.50 \ N\backslash mm^2$

$F_y = 400 \ N\backslash mm^2 \longrightarrow F_s = 220 \ N\backslash mm^2$

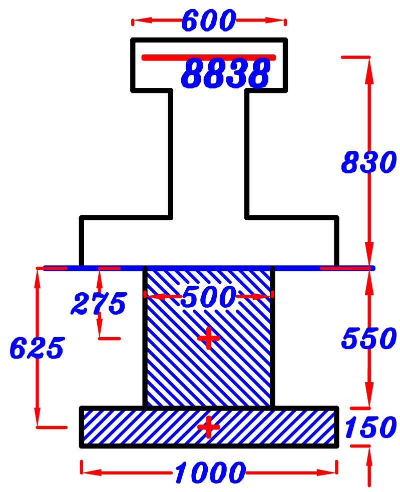


To know if Z bigger or smaller than 150 mm
assume First that $Z = 150$ mm

$S_{nv.}(under) = 1000 * 150 * (75) = 11250000 \ mm^3$

$S_{nv.}(above) = 15 * 8838 * (1380) = 182946600 \ mm^3$

$\because S_{nv.}(above) > S_{nv.}(under) \quad \therefore Z > 150$ mm



To know if Z bigger or smaller than 700 mm
assume First that $Z = 700$ mm

$S_{nv.}(under) = 1000 * 150 * (625) + 500 * 550 * (275)$

$= 169375000 \ mm^3$

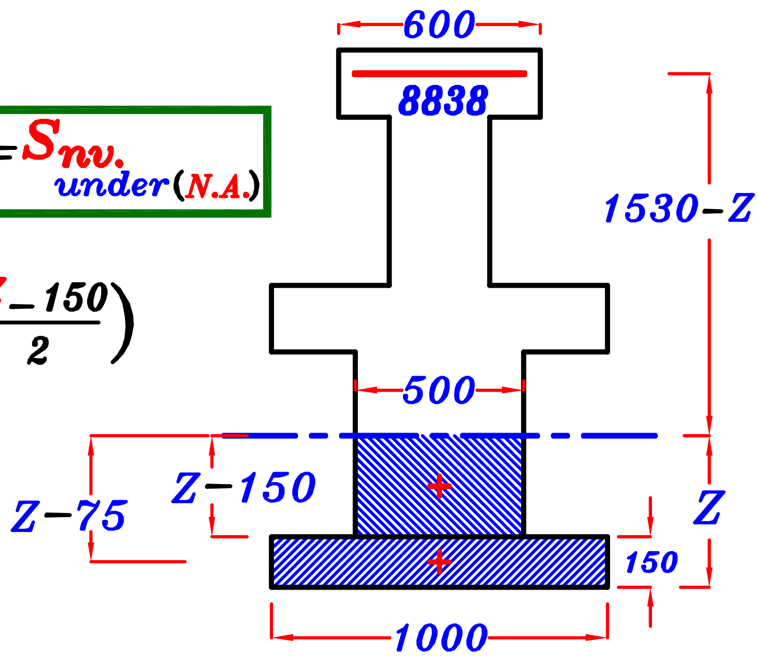
$S_{nv.}(above) = 15 * 8838 * (830) = 110033100 \ mm^3$

$\because S_{nv.}(above) < S_{nv.}(under) \quad \therefore Z < 700$ mm

$\therefore \boxed{150 \text{ mm} < Z < 700 \text{ mm}}$

① Take $n = 15$

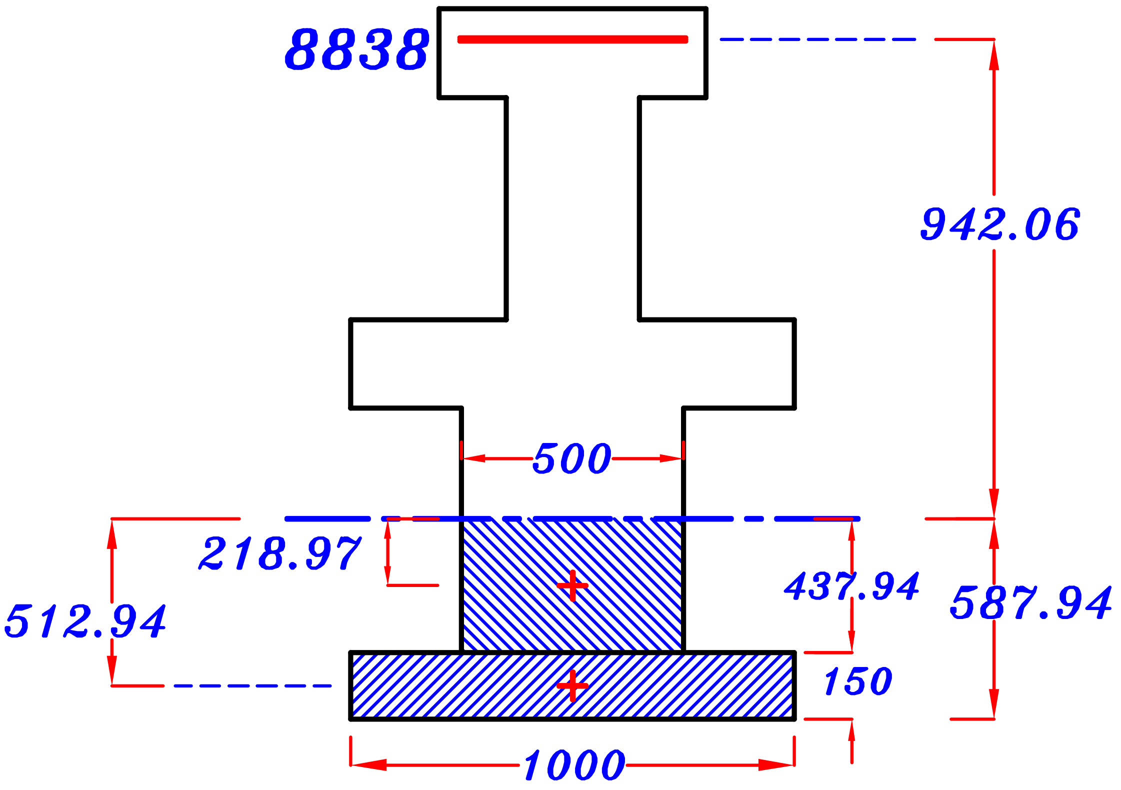
② Get Z by taking $\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$



$$(1000)(150)(Z-75) + (500)(Z-150)\left(\frac{Z-150}{2}\right)$$

$$= (15)(8838)(1530-Z)$$

$\boxed{Z = 587.94 \text{ mm}}$



③ $$I_{nv} = \frac{1000\,(150)^3}{12} + (1000)\,(150)\,(512.94)^2 + \frac{500\,(437.94)^3}{3}$$

$$+ (15)\,(8838)\,(942.06)^2 = 171399055700 \text{ mm}^4$$

④ $$M_{wc} = \frac{F_c * I_{nv}}{Z} \quad \text{------ not as T\_Sec.}$$

$$= \frac{9.5 * 171399055700}{587.94} = 2769485031 \text{ N.mm}$$

$$= 2769.48 \text{ kN.m}$$

⑤ $$M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \frac{\left(\frac{220}{15}\right) * 171399055700}{1530 - 587.94} = 2668463597 \text{ N.mm}$$

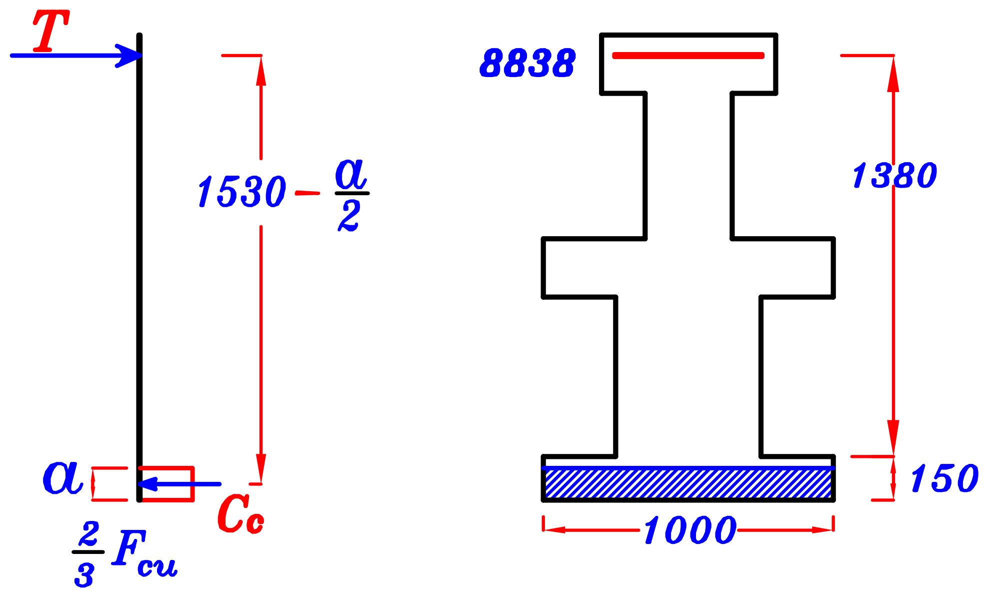
$$= 2668.46 \text{ kN.m}$$

⑥ $$\boxed{M_w = 2668.46 \text{ kN.m}}$$

c – The Failure Moment. (M_{ult})

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 400} * 1530 = 918 \text{ mm}$

② Assume $a < 150$ mm



③ From equilibrium eqn. $C_c = T$

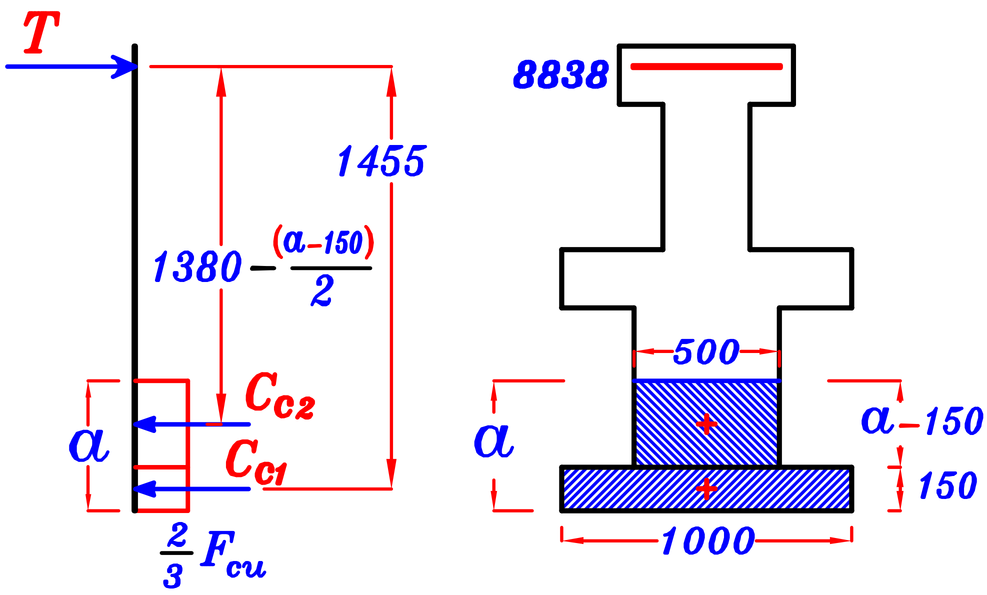
$$\frac{2}{3} F_{cu} * a * B = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3} (25)(a)(1000) = (8838)(400)$$

$\therefore a = 212.1 \text{ mm} > 150 \text{ mm}$ $\therefore$ wrong assumption

$\therefore a > 150$ mm



③ From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * (1000 * 150) + \frac{2}{3} F_{cu} * [500\,(a - 150)] = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

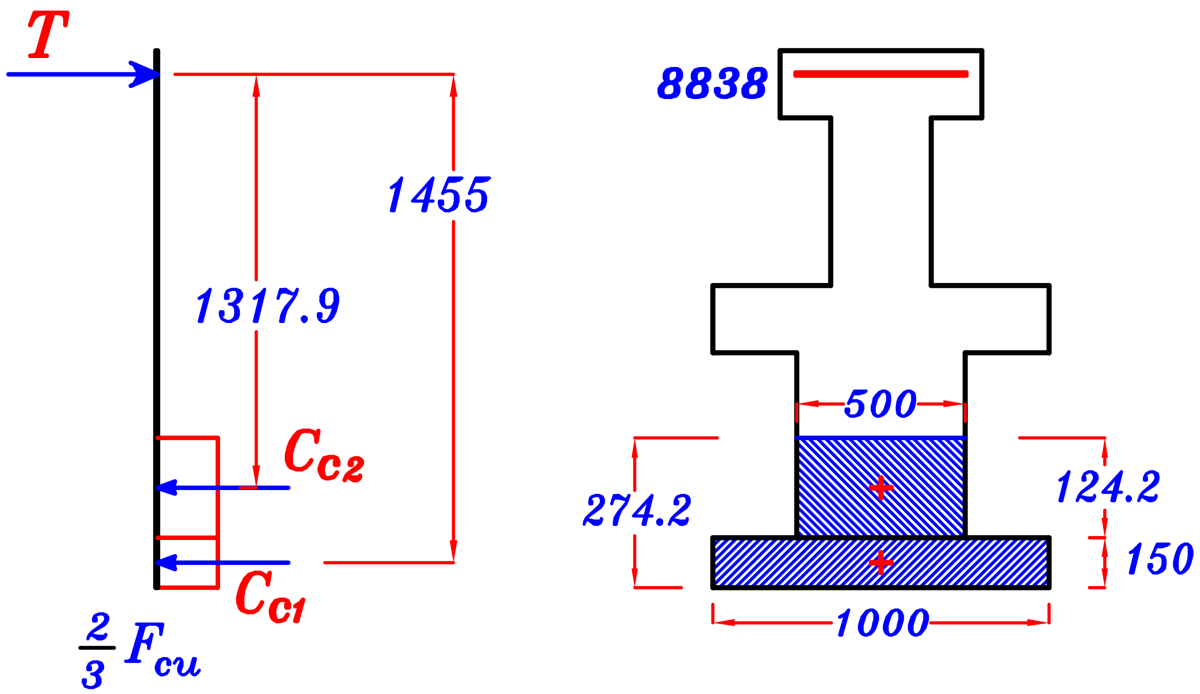
$$\therefore \frac{2}{3}(25)(1000 * 150) + \frac{2}{3}(25) * [500\,(a - 150)] = 8838 * 400$$

$\therefore a = 274.2\ mm > 150\ mm$ $\therefore$ right assumption.

$\therefore C = 1.25\ a = 1.25 * 274.2 = 342.75\ mm < C_b$

$\therefore$ **The Section is Under Reinforced Sec.**

and the assumption is right $F_s = F_y$



$$\therefore M_{ult} = \frac{2}{3}(25)(1000)(150)(1455) + \frac{2}{3}(25)(500)(124.2)(1317.9)$$

$$= 5001526500\ N.mm$$

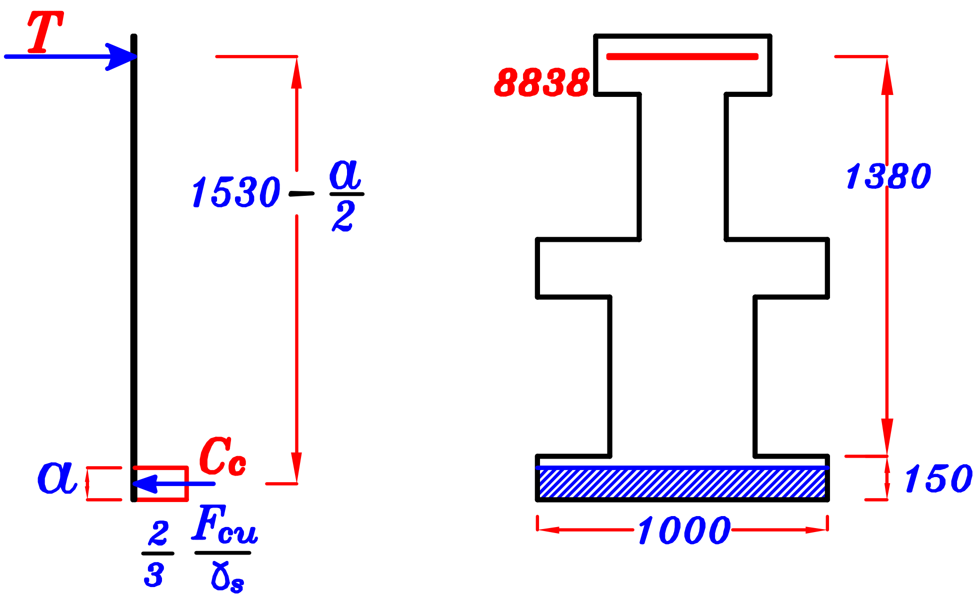
$$\therefore \boxed{M_{ult} = 5001.5\ kN.m}$$

d- The Ultimate Limit Moment. ($M_{U.L.}$)

$a_{min} = 0.1\ d = 0.1 * 1530 = 153.0\ mm$

$a_{max} = 0.8 \left(\frac{2}{3}\right) \left[\frac{600}{600 + (F_y \backslash \delta_s)}\right] * d = 0.337\ d = 0.337 * 1530 = 515.61\ mm$

assume $a < 150\ mm$



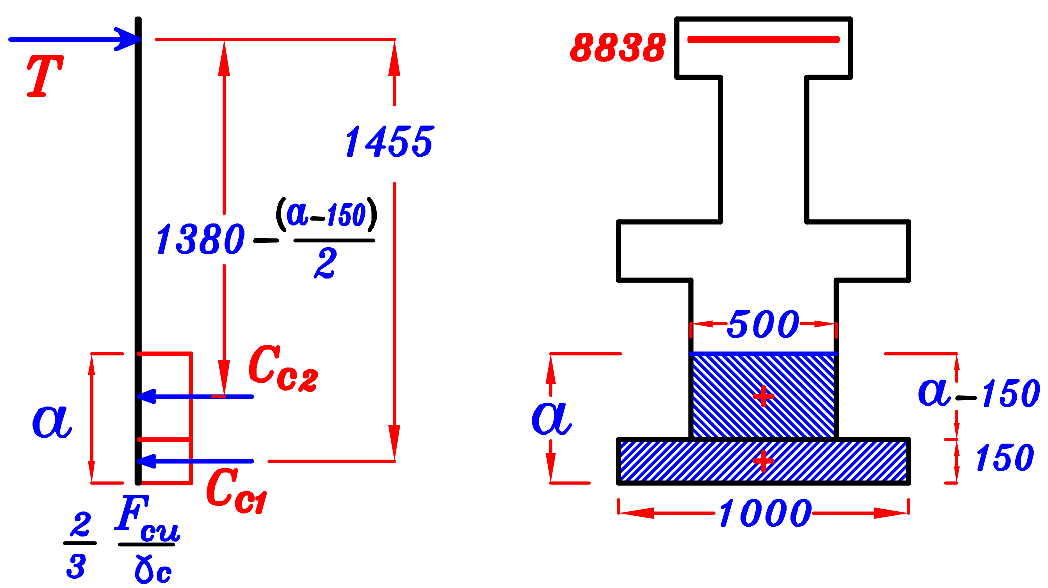
From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\delta_c} * a * B = A_s * F_s$ ---- a , F_s

$F_s = \frac{F_y}{\delta_s}$ (Under reinforced Sec.) $\therefore \frac{2}{3}\frac{F_{cu}}{\delta_c} * a * B = A_s * \frac{F_y}{\delta_s}$

$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)\ (1000) = (8838)\left(\frac{400}{1.15}\right)$

$\longrightarrow a = 276.6\ mm > t_s$ $\therefore$ wrong assumption

$\therefore\ a > 150\ mm$



From equilibrium eqn. $C_c = T$

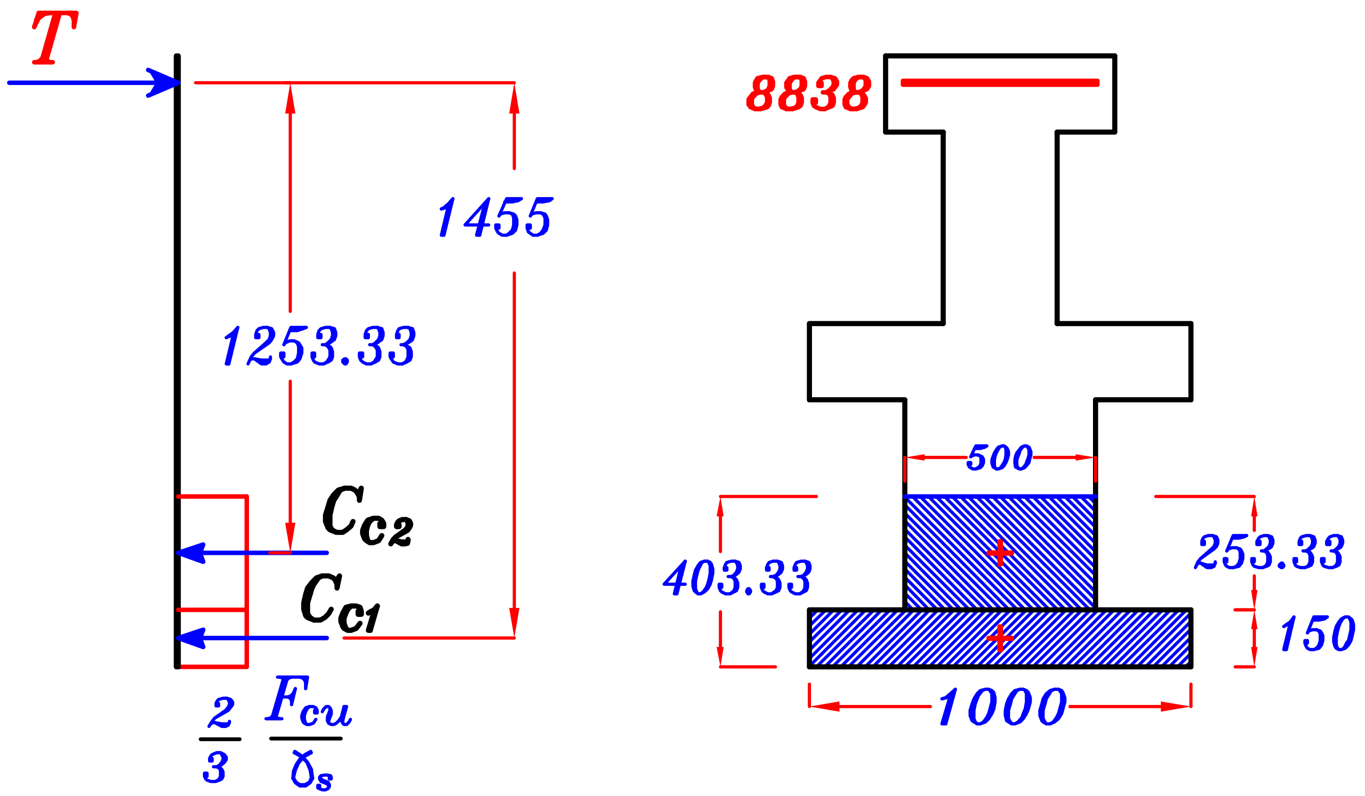
$$\frac{2}{3}\frac{F_{cu}}{\gamma_c} * (1000 * 150) + \frac{2}{3}\frac{F_{cu}}{\gamma_c} * [500\,(a - 150)] = A_s * F_s$$

Assume $F_s = \frac{F_y}{\gamma_s}$ $\longrightarrow$ (under reinforced Sec.)

$$\therefore \frac{2}{3}\left(\frac{25}{1.5}\right)(1000 * 150) + \frac{2}{3}\left(\frac{25}{1.5}\right) * [500\,(a - 150)] = 8838 * \left(\frac{400}{1.15}\right)$$

$\therefore a = 403.33$ mm $>$ 150 mm $\quad\therefore$ right assumption.

$\therefore a_{min} < a < a_{max} \quad \therefore$ right assumption $F_s = \frac{F_y}{\gamma_s}$



$$\therefore M_{U.L.} = \frac{2}{3}\left(\frac{25}{1.5}\right)(1000)(150)(1455) + \frac{2}{3}\left(\frac{25}{1.5}\right)(500)(253.33)(1253.33)$$

$$= 4188922716 \text{ N.mm}$$

$$\therefore \boxed{M_{U.L.} = 4188.92 \text{ kN.m}}$$

- The Factor Of Safety For Loads.

$$= \left(\frac{M_{U.L.}}{M_w}\right) = \frac{4188.92}{2668.46} = 1.57$$

- The Factor Of Safety For Material.

$$= \left(\frac{M_{ult}}{M_{U.L.}}\right) = \frac{5001.5}{4188.92} = 1.193$$

- The Global Factor Of Safety.

$$= \left(\frac{M_{ult}}{M_w}\right) = \frac{5001.5}{2668.46} = 1.87$$

Example.

For the reinforced concrete girder's cross-section shown in the Figure
It is required to:

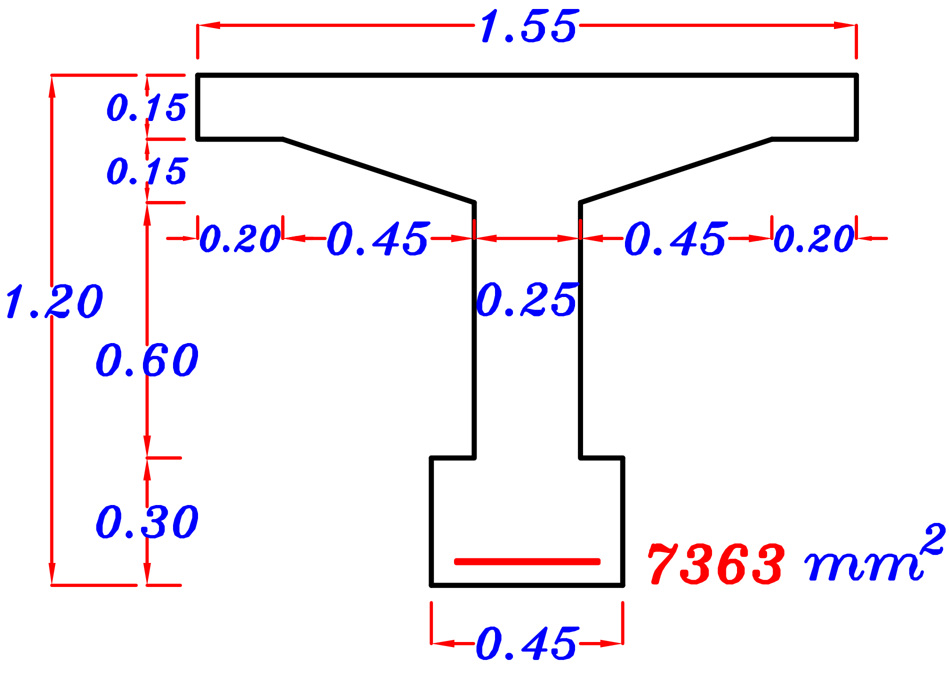
1 – Calculate the cracking moment ($M_{cr.}$), the working moment (M_w), the ultimate limit moment ($M_{U.L.}$) & the ultimate moment ($M_{ult.}$)

2 – Calculate the Factors of safety For Loads, Materials & Global Factor of safety.

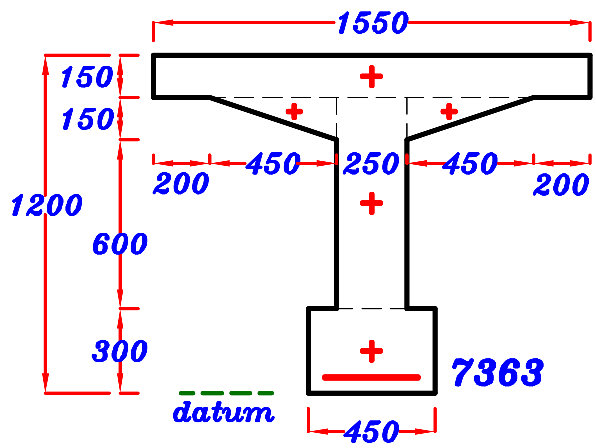
Data :

$F_{cu} = 30\ N\backslash mm^2$

, st. 400/600



a– The Cracking Moment. ($M_{cr.}$)



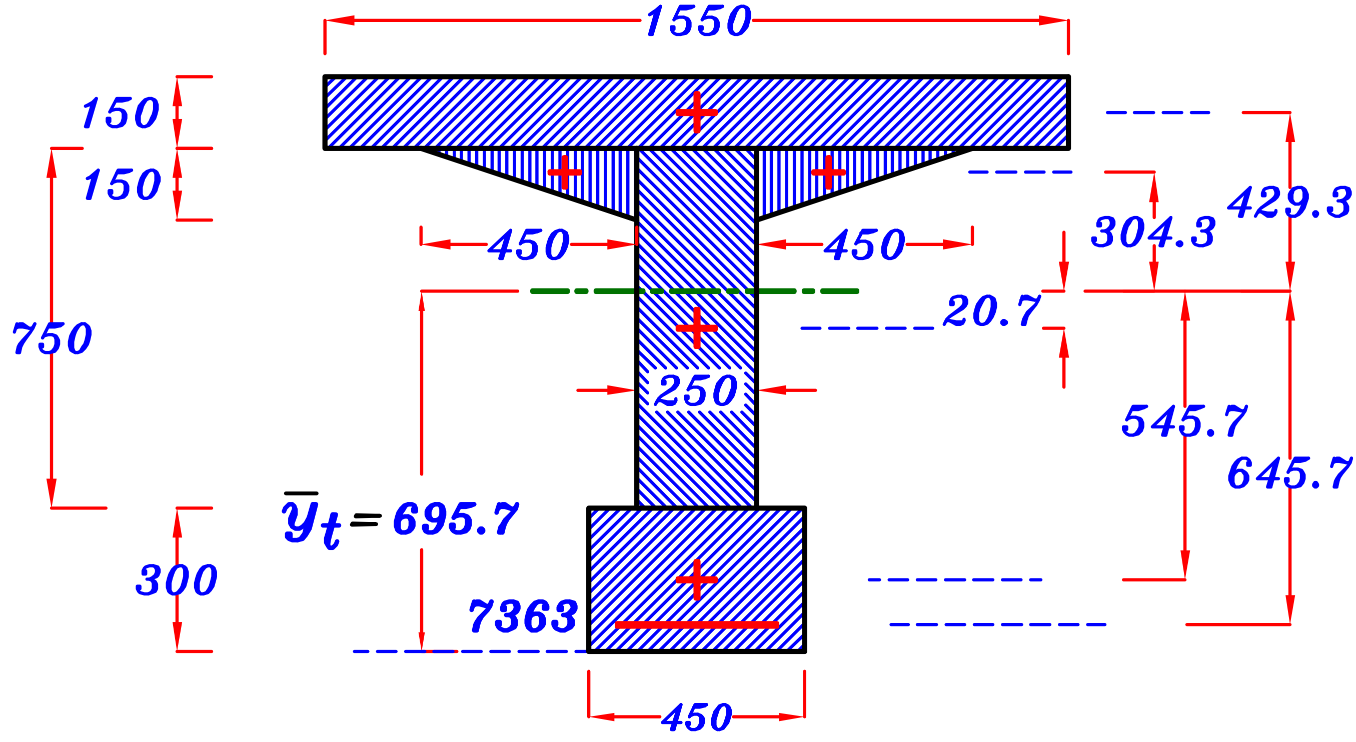
① $n = \frac{E_s}{E_c} = \frac{2*10^5}{4400\sqrt{30}} = 8.30 \longrightarrow n = 10$

② $A_v = A_c + (n-1)A_s$

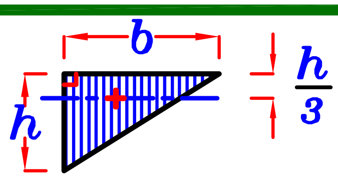
$A_v = 150*1550 + 250*750 + 2(0.5*150*450)$
$+ 300*450 + (10-1)(7363) = 688767\ mm^2$

③ $\overline{y}_t = \frac{1550*150\,(1125) + 250*750\,(675) + 2\,(0.5*150*450)\,(1000) + 300*450\,(150) + (10-1)(7363)(50)}{688767}$

$= 695.7\ mm$



$$I_x = \frac{b\,h^3}{36}$$



④ $$I_g = \frac{1550*150^3}{12} + 1550*150\,(429.3)^2 + 2*\frac{450*150^3}{36} + 2*(0.5*450*150)\,(304.3)^2$$
$$+ \frac{250*750^3}{12} + 250*750\,(20.7)^2 + \frac{450*300^3}{12} + 450*300\,(545.7)^2$$
$$+ (10-1)(7363)(645.7)^2 = 127332060300 \ mm^4$$

⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{30} = 3.28 \ N\backslash mm^2$$

⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.28*\ 127332060300}{695.7} = 600329391.5 \ N.mm$$
$$= 600.32 \ kN.m.$$

$$\boxed{M_{cr} = 600.33 \ kN.m}$$

b – The Working Moment. (M_w)

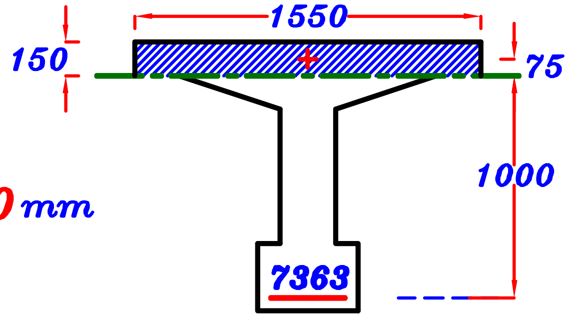
$F_{cu} = 30 \; N\backslash mm^2 \longrightarrow F_c = 10.5 \; N\backslash mm^2$

$F_y = 400 \; N\backslash mm^2 \longrightarrow F_s = 220 \; N\backslash mm^2$

$S_{nv.}(\text{above}) = 150 * 1550 * (75) = 17437500 \; mm^3$

$S_{nv.}(\text{under}) = 15 * 7363 * (1000) = 110445000 \; mm^3$

$\because S_{nv.}(\text{under}) > S_{nv.}(\text{above}) \quad \therefore Z > 150 \; mm$

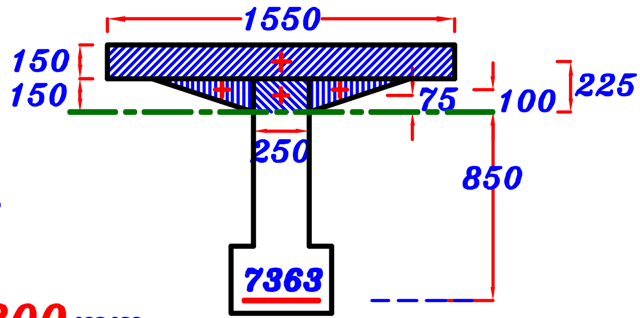


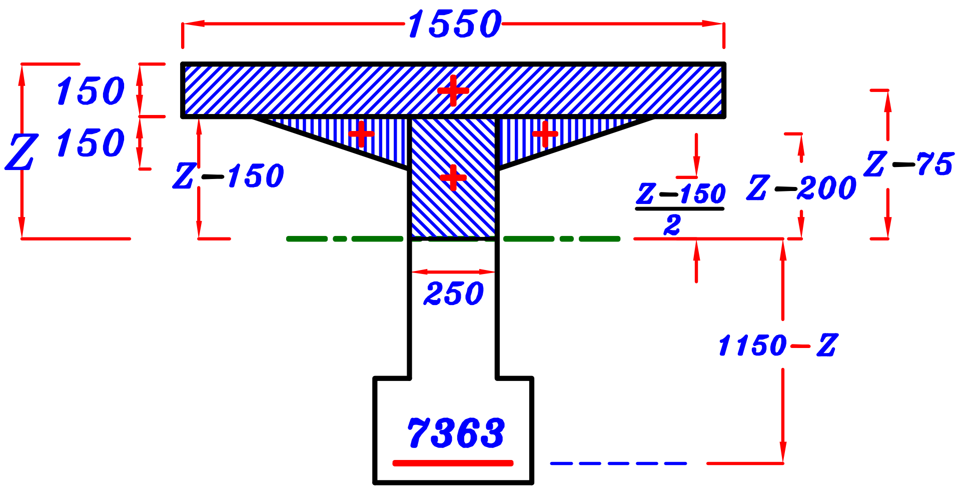
$S_{nv.}(\text{above}) = 150 * 1550 * (225) + 250 * 150 * (75)$

$+ 2 * (0.5 * 450 * 150)(100) = 61875000 \; mm^3$

$S_{nv.}(\text{under}) = 15 * 7363 * (850) = 93878250 \; mm^3$

$\because S_{nv.}(\text{under}) > S_{nv.}(\text{above}) \quad \therefore Z > 300 \; mm$





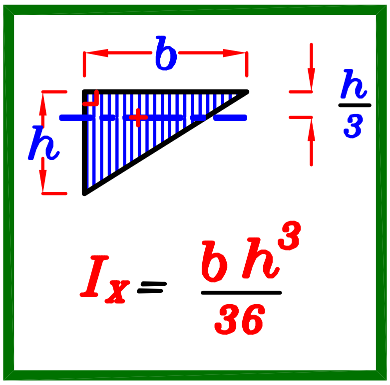
① Take $n = 15$

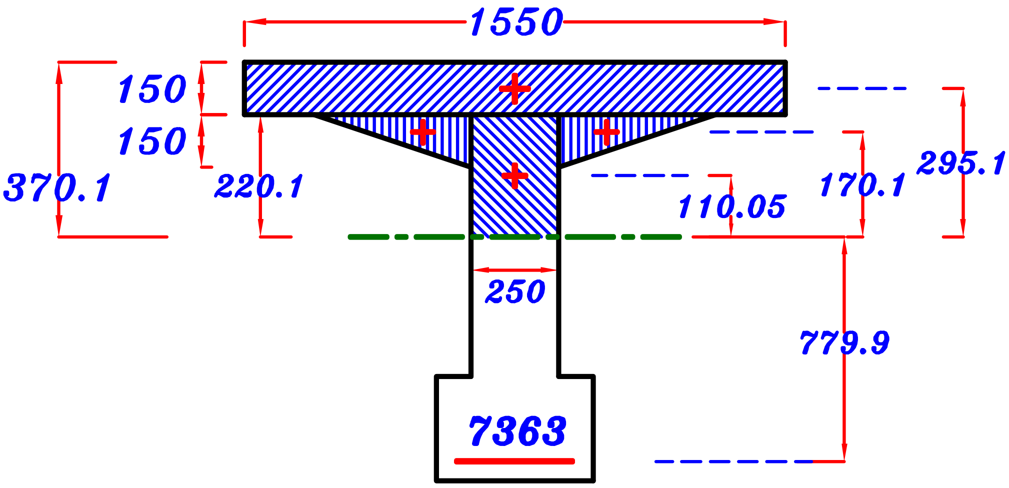
② Get Z by taking $\boxed{S_{nv.\,\text{above}\,(N.A.)} = S_{nv.\,\text{under}\,(N.A.)}}$

$$(1550)(150)(Z-75) + (250)(Z-150)\left(\frac{Z-150}{2}\right) + 2 * (0.5 * 450 * 150)(Z-200)$$

$$= (15)(7363)(1150 - Z)$$

$$\boxed{Z = 370.1 \; mm}$$





③ $$I_{nv} = \frac{1550(150)^3}{12} + (1550)(150)(295.1)^2 + \frac{250\,(220.1)^3}{3} + 2 * \frac{450 * 150^3}{36}$$

$$+\, 2 * (0.5 * 450 * 150)\,(170.1)^2 + (15)(7363)(779.9)^2 = 90786444070 \ mm^4$$

④ $$M_{wc} = \frac{\frac{2}{3} F_c * I_{nv}}{Z} \quad \text{--------} \quad \text{as } T\text{-Sec.}$$

$$= \frac{\left(\frac{2}{3}\right) 10.5 * 90786444070}{370.1} = 1717117289 \ N.mm$$

$$= 1717.1 \ kN.m$$

⑤ $$M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \frac{\left(\frac{220}{15}\right) * 90786444070}{1150 - 370.1} = 1707314416 \ N.mm$$

$$= 1707.3 \ kN.m$$

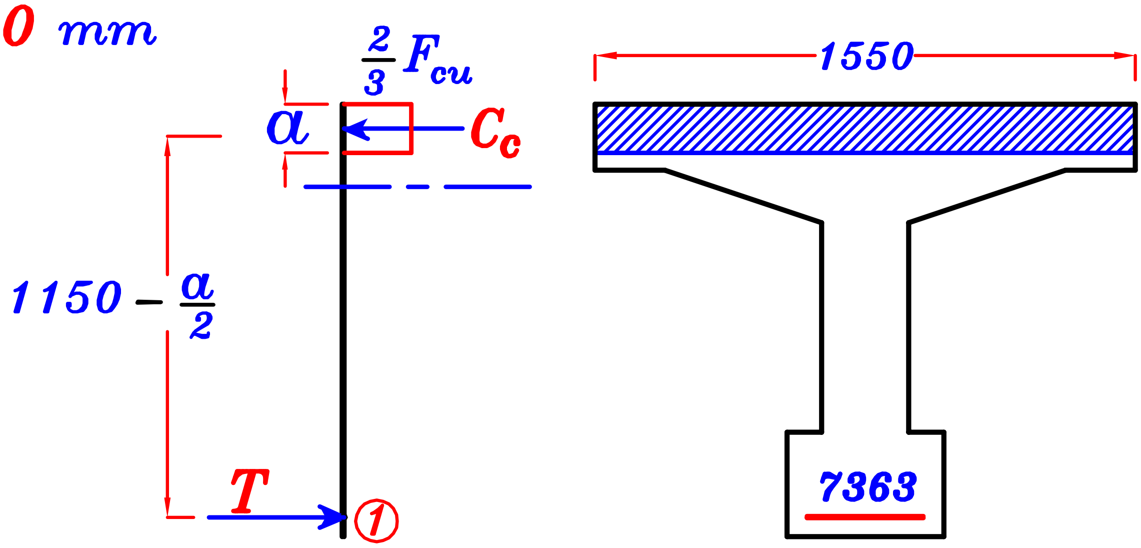
⑥ $$\boxed{M_w = 1707.3 \ kN.m}$$

C – The Failure Moment. (M_{ult})

① $C_b = \dfrac{600}{600+F_y} * d = \dfrac{600}{600+400} * 1150 = 690\ mm$

② Assume $a \leqslant t_s$

$a < 150\ mm$



③ From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * B = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$\frac{2}{3}(30)(a)(1550) = (7363)(400) \longrightarrow a = 95.0\ mm < t_s \therefore$ O.K.

$\therefore C = 1.25\ a = 1.25 * 95.0 = 118.75\ mm < C_b$

$\therefore$ **The Section is Under Reinforced Sec.**

and the assumption is right $F_s = F_y$

$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, B\, (d - \frac{a}{2})$

$= \frac{2}{3}(30)(95.0)(1550)\left(1150 - \frac{95.0}{2}\right) = 3246862500\ N.mm$

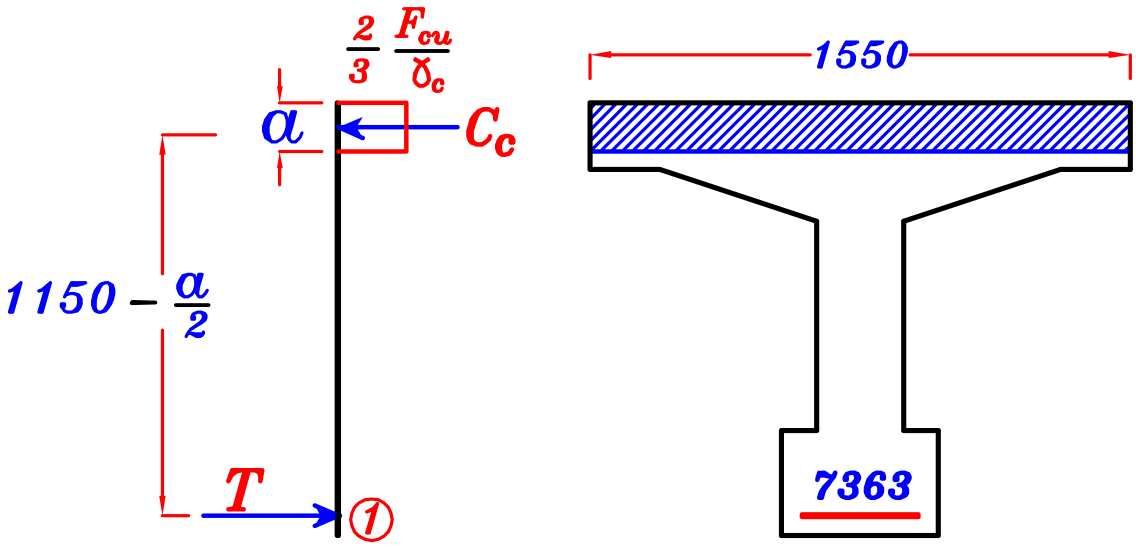
$$\therefore \boxed{M_{ult} = 3246.86\ kN.m}$$

d- The Ultimate Limit Moment. ($M_{U.L.}$)

$a_{min} = 0.1\ d = 0.1 * 1150 = 115$ mm

$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)}\right] * d = 0.337\ d = 0.337 * 1150 = 387.55$ mm

assume $a \leqslant t_s$ $\quad a < 150$ mm



From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * B = A_s * F_s$ ---- a, F_s

assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$\therefore \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * B = A_s * \frac{F_y}{\gamma_s}$

$\frac{2}{3}\left(\frac{30}{1.5}\right)(a)(1550) = (7363)\left(\frac{400}{1.15}\right)$

$\longrightarrow a = 123.92$ mm $< t_s$ $\quad \therefore$ o.k.

$\therefore a_{min} < a < a_{max}$ $\quad \therefore$ right assumption $F_s = \frac{F_y}{\gamma_s}$

$\therefore M_{U.L.} = \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * B\left(d - \frac{a}{2}\right)$

$= \frac{2}{3}\left(\frac{30}{1.5}\right)(123.92)(1550)\left(1150 - \frac{123.92}{2}\right) = 2786484947$ N.mm

$= 2786.48$ kN.m

$$\boxed{M_{U.L.} = 2786.48\ kN.m}$$

- The Factor Of Safety For Loads.

$$= \left(\frac{M_{U.L.}}{M_w}\right) = \frac{2786.48}{1707.3} = 1.63$$

- The Factor Of Safety For Material.

$$= \left(\frac{M_{ult}}{M_{U.L.}}\right) = \frac{3246.86}{2786.48} = 1.165$$

- The Global Factor Of Safety.

$$= \left(\frac{M_{ult}}{M_w}\right) = \frac{3246.86}{1707.3} = 1.90$$

Example.

Data. $F_{cu} = 25\ N/mm^2$

$st.\ 360/520$

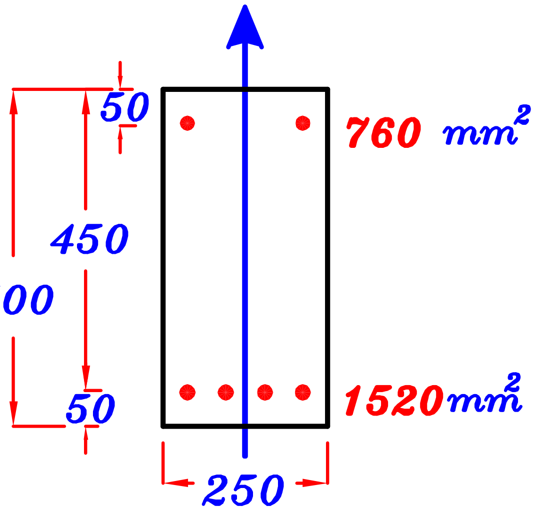
Req. For the shown Cross-Section

1- Calculate $M_{cr.}$

2- Calculate M_w

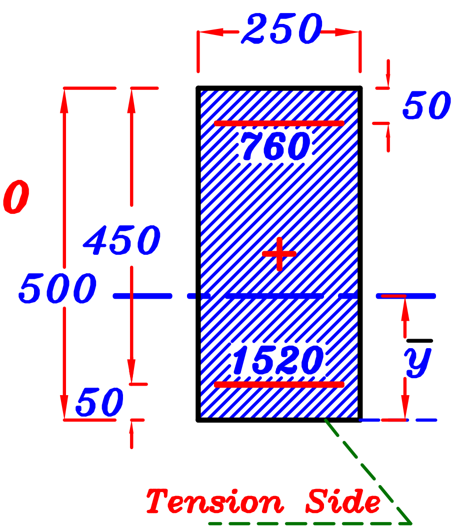
3- Calculate M_{ult}

4- Calculate $M_{U.L.}$



$\boxed{\dfrac{A_s`}{A_s} > 0.2 \longrightarrow \text{don't neglect } A_s`}$

Solution. 1- $M_{cr.}$



① $n = \dfrac{E_s}{E_c} = \dfrac{2*10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$

② $A_v = b*t + (n-1)A_s + (n-1)A_s`$

$A_v = 250*500 + (10-1)(1520) + (10-1)(760)$

$= 145520\ mm^2$

③ $\bar{y}_t = \dfrac{250*500*250 + (10-1)(1520)(50) + (10-1)(760)(450)}{145520} = 240.6\ mm$

④ $I_{gross} = \dfrac{250*500^3}{12} + 250*500(250-240.6)^2 + (10-1)(1520)(240.6-50)^2$

$+ (10-1)(760)(450-240.6)^2 = 3412106414\ mm^4$

⑤ $F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0\ N\backslash mm^2$

⑥ $M_{cr} = \dfrac{F_{ctr} * I_g}{\bar{y}_t} = \dfrac{3.0 * 3412106414}{240.6} = 42544967.7\ N.mm$

$= \dfrac{42544967.7\ N.mm}{10^6} = 42.54\ kN.m$

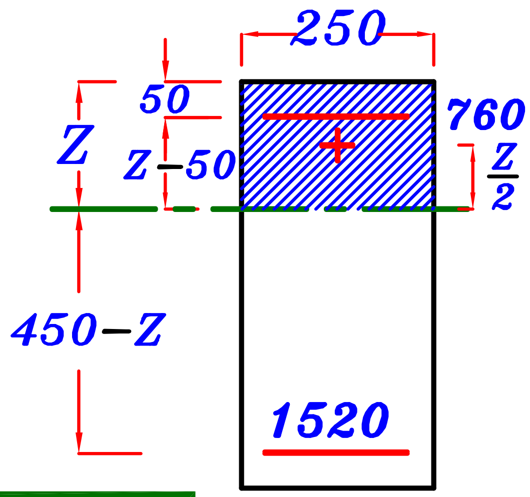
$\boxed{M_{cr} = 42.54\ kN.m}$

2- M_w

Allowable stresses

$F_{cu} = 25 \ N\backslash mm^2 \longrightarrow F_c = 9.5 \ N\backslash mm^2$

$F_y = 360 \ N\backslash mm^2 \longrightarrow F_s = 200 \ N\backslash mm^2$



① Take $n = 15$

② Get Z by taking $\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$

$$b\,(Z)\left(\frac{Z}{2}\right) + (n-1)\,A_s`\,(Z - d`) = n\,A_s\,(d - Z)$$

$$250\,(Z)\left(\frac{Z}{2}\right) + (14)(760)(Z-50) = (15)(1520)(450 - Z)$$

$$\boxed{Z = 189.1 \ mm}$$

③ Get $I_{nv} = \dfrac{bZ^3}{3} + (n-1)\,A_s`\,(Z-d`)^2 + n\,A_s\,(d-Z)^2$

$$I_{nv} = \frac{250\,(189.1)^3}{3} + (14)(760)(189.1-50)^2 + (15)(1520)(450-189.1)^2$$

$$= 2321339454 \ mm^4$$

④ $M_{wc} = \dfrac{F_c * I_{nv}}{Z} = \dfrac{9.5 * 2321339454}{189.1} = 116619380 \ N.mm$

$= 116.62 \ kN.mm$

⑤ $M_{ws} = \dfrac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \dfrac{\left(\frac{200}{15}\right) * 2321339454}{450 - 189.1}$

$= 118632398 \ N.mm = 118.6 \ kN.m$

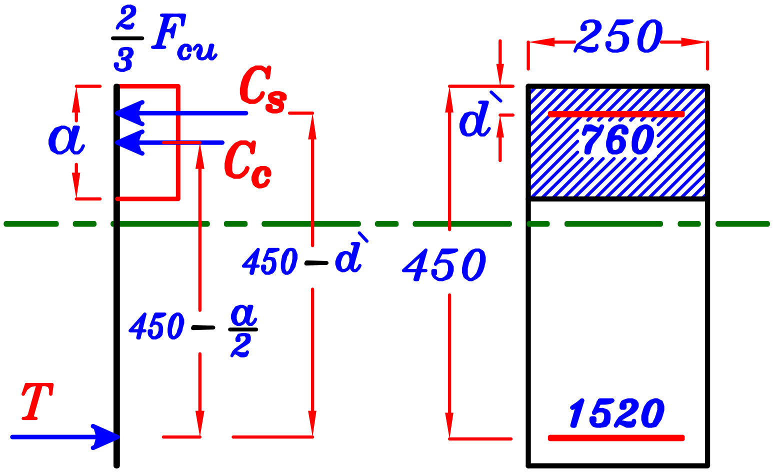
⑥ $\boxed{M_w = 116.62 \ kN.m}$

3- M_{ult}

Take للتسهيل

$F_s^{`}$ (For compression steel) $= F_y$

$C_s = A_s^{`} * F_y$



① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 450 = 281.2$ mm

② From equilibrium eqn. $C_c + C_s = T$

$$\frac{2}{3} F_{cu} * a * b + A_s^{`} * F_y = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(25)(a)(250) + (760)(360) = (1520)(360) \longrightarrow a = 65.6 \text{ mm}$$

$\therefore$ $C = 1.25\,a = 1.25 * 65.6 = 82.0$ mm $< C_b$

$\therefore$ **The Section is Under Reinforced Sec.**

and the assumption is right $F_s = F_y$

M_{ult} = The moment about the steel.

$$M_{ult} = C_c * (d - \frac{a}{2}) + C_s * (d - d^{`})$$

$$= \frac{2}{3} F_{cu} * a * b \,(d - \frac{a}{2}) + A_s^{`} * F_y \,(d - d^{`})$$

$$= \frac{2}{3}(25)(65.6)(250)\,(450 - \frac{65.6}{2}) + (760)(360)(450 - 50)$$

$= 223474666$ N.mm $= 223.47$ kN.m

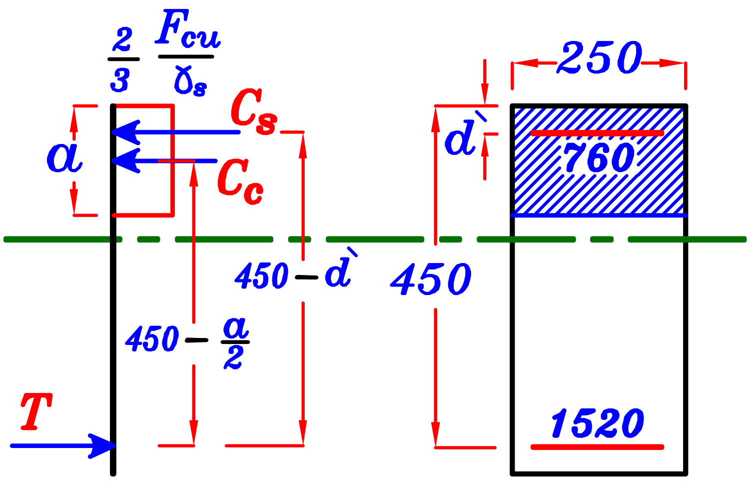
$$\boxed{M_{ult} = 223.47 \text{ kN.m}}$$

4- $M_{U.L.}$

Take للتسهيل

$F_{s}`$ (For compression steel) $= \frac{F_y}{\gamma_s}$

$C_s = A_{s}` * \frac{F_y}{\gamma_s}$



$$a_{min} = 0.1\ d = 0.1 * 450 = 45\ mm$$

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)}\right] * d = 0.35\ d = 0.35 * 450 = 157.5\ mm$$

From equilibrium eqn. $C_c + C_s = T$

$$\frac{2}{3}\frac{F_{cu}}{\gamma_s} * (a * b) + A_{s}` * \frac{F_y}{\gamma_s} = A_s * F_s \quad ---- \quad a\ ,\ F_s$$

assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$$\frac{2}{3}\frac{F_{cu}}{\gamma_s} * (a * b) + A_{s}` * \frac{F_y}{\gamma_s} = A_s * \frac{F_y}{\gamma_s}$$

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)\ (250) + (760)\left(\frac{360}{1.15}\right) = (1520)\left(\frac{360}{1.15}\right)$$

$\longrightarrow a = 85.64\ mm$ $\qquad \therefore\ a_{min} < a < a_{max}$

$\therefore$ right assumption

$M_{U.L.}$ = The moment about the steel.

$$M_{U.L.} = C_c * (d - \frac{a}{2}) + C_s * (d - d`)$$

$$= \frac{2}{3}\frac{F_{cu}}{\gamma_s} * a * b\ (d - \frac{a}{2}) + A_{s}` * \frac{F_y}{\gamma_s}(d - d`)$$

$$= \frac{2}{3}\left(\frac{25}{1.5}\right)(85.64)(250)\left(450 - \frac{85.64}{2}\right) + (760)\left(\frac{360}{1.15}\right)(450 - 50)$$

$= 192028815\ N.mm \quad = 192.0\ kN.m$

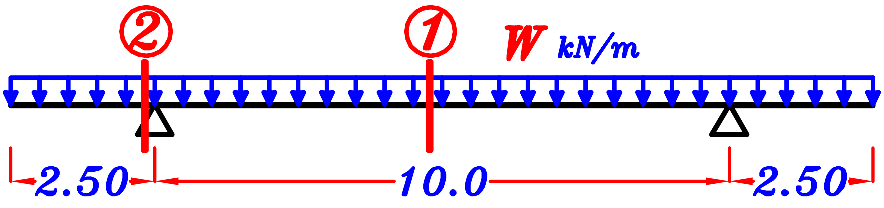
$$\boxed{M_{U.L.} = 192.0\ kN.m}$$

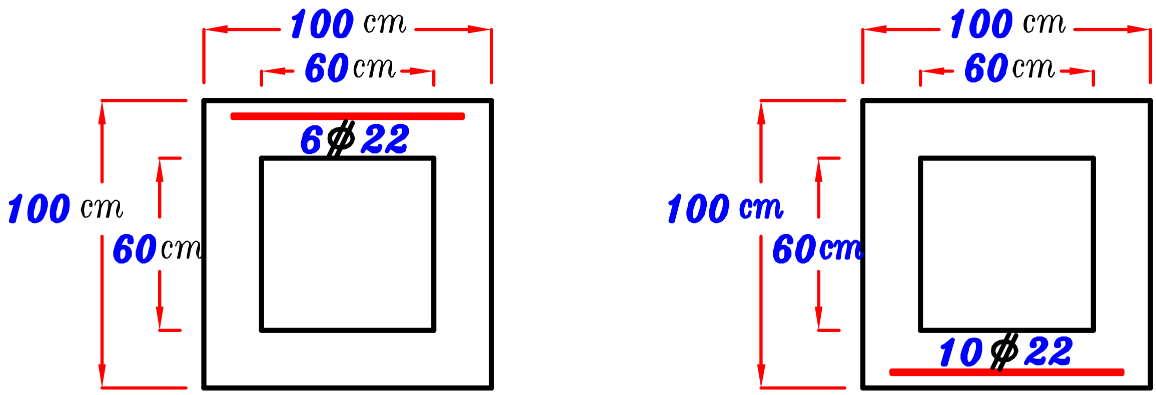
Example.

The Figure shows a statical system of an overhanging beam, subjected to uniform distributed load (W) with the shown sections. It is required to calculate the critical value of the load (W) in each of the Following cases:

1- The cracking load of the girder (Steel reinforcement can be ignored)

2- The ultimate load of the girder.

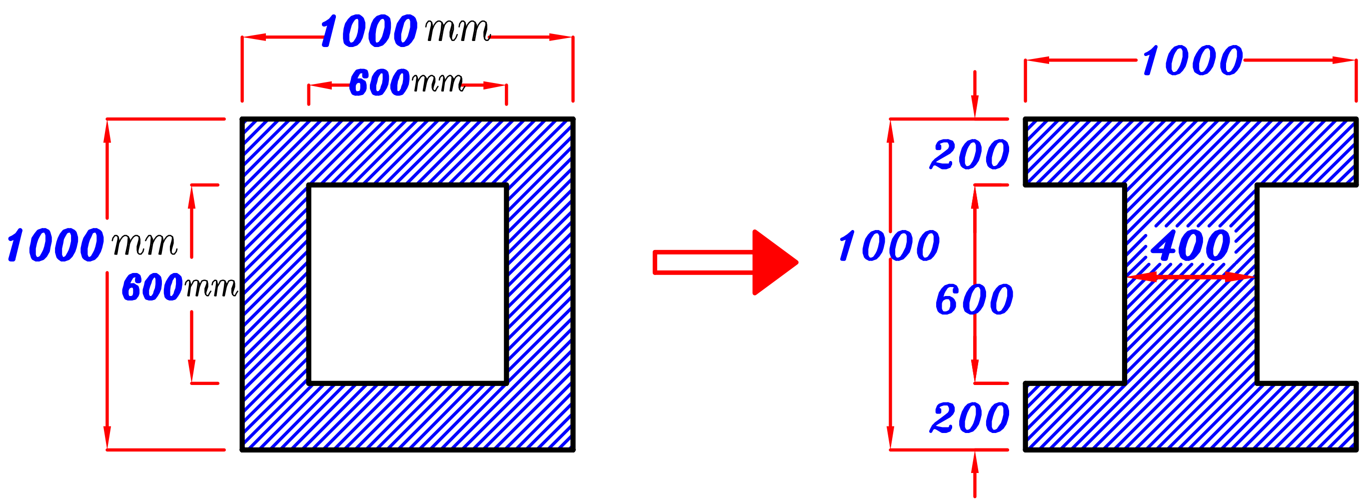




Section 2   Section 1

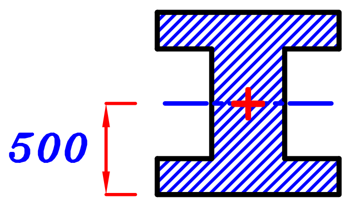
For Cracking Moment. M_{cr}

IF we neglect the steel. $M_{cr}(Sec.1) = M_{cr}(Sec.2)$



① $A_v = A_c = 2(200 * 1000) + 400 * 600 = 640000\ mm^2$

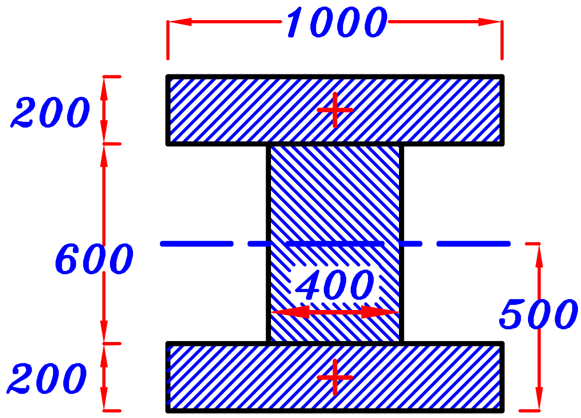
② $\bar{y}_t = 500\ mm$



③ $I_g = \frac{400 * 600^3}{12}$

$+ 2\left[\frac{1000 * 200^3}{12} + 1000 * 200(500 - 100)^2\right]$

$= 72533333333 \ mm^4$



④ $F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{30} = 3.28 \ N\backslash mm^2$

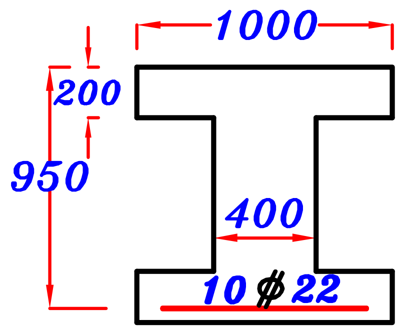
⑤ $M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.28 * 72533333333}{500} = 475818666 \ N.mm$

$= 475.8 \ kN.m$

$$\boxed{M_{cr1} = M_{cr2} = 475.8 \ kN.m}$$

For ***Ultimate Moment.*** M_{ult}

Section 1

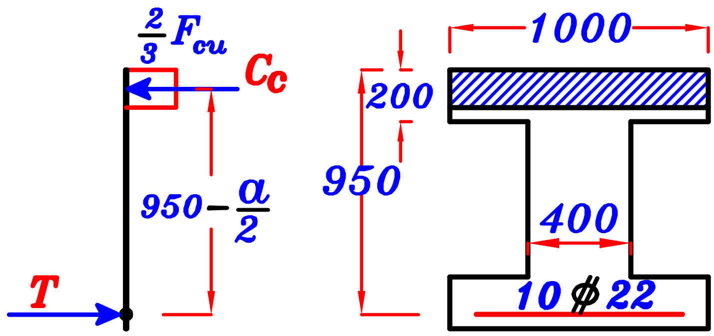


$A_s = 10 \phi 22 = 10\left[\frac{\pi * 22^2}{4}\right] = 3801 \ mm^2$

① $C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 950 = 593.7 \ mm$

② **Assume** $a \leqslant t_s$

$a < 200 \ mm$



③ *From equilibrium eqn.*

$\frac{2}{3} F_{cu} * a * B = A_s * F_s$

Assume $F_s = F_y \longrightarrow$ (*under reinforced or Balanced Sec.*)

$\frac{2}{3}(30)(a)(1000) = (3801)(360) \longrightarrow a = 68.4 \ mm < t_s \quad \therefore$ **O.K.**

$\therefore \ C = 1.25 \ a = 1.25 * 68.4 = 85.52 \ mm < C_b$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

$\therefore$ **The Section is Under Reinforced Sec.**

and the assumption is right $F_s = F_y$

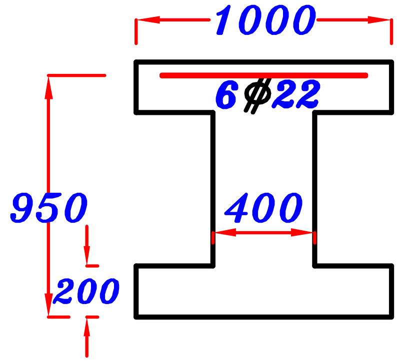
$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, B \left(d - \frac{a}{2}\right)$$

$$M_{ult} = \frac{2}{3}(30)(68.4)(1000)\left(950 - \frac{68.4}{2}\right) = 1252814400 \text{ N.mm} = 1252.8 \text{ kN.m}$$

$$\therefore \boxed{M_{ult} = 1252.8 \text{ kN.m}}$$

Section 2

$$A_s = 6\phi 22 = 6\left[\frac{\pi * 22^2}{4}\right] = 2280 \text{ mm}^2$$

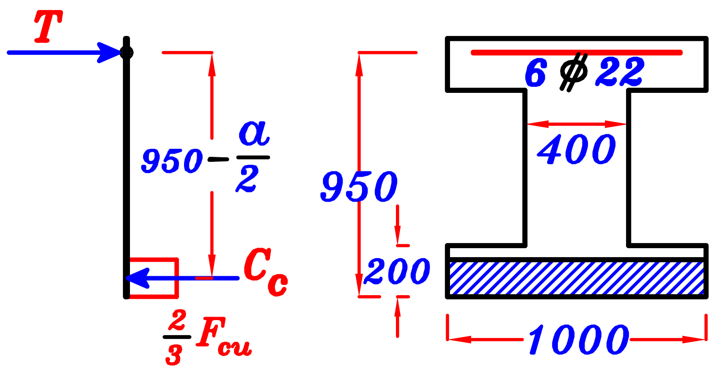


① $$C_b = \frac{600}{600 + F_y} * d$$

$$= \frac{600}{600 + 360} * 950 = 593.7 \text{ mm}$$

② Assume $a \leqslant t_s$

$a < 200$ mm



③ From equilibrium eqn.

$$\frac{2}{3} F_{cu} * a * B = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(30)(a)(1000) = (2280)(360) \longrightarrow a = 41.04 \text{ mm} < t_s \quad \therefore \text{O.K.}$$

$$\therefore C = 1.25\, a = 1.25 * 41.04 = 51.3 \text{ mm} < C_b$$

$\therefore$ **The Section is Under Reinforced Sec.**

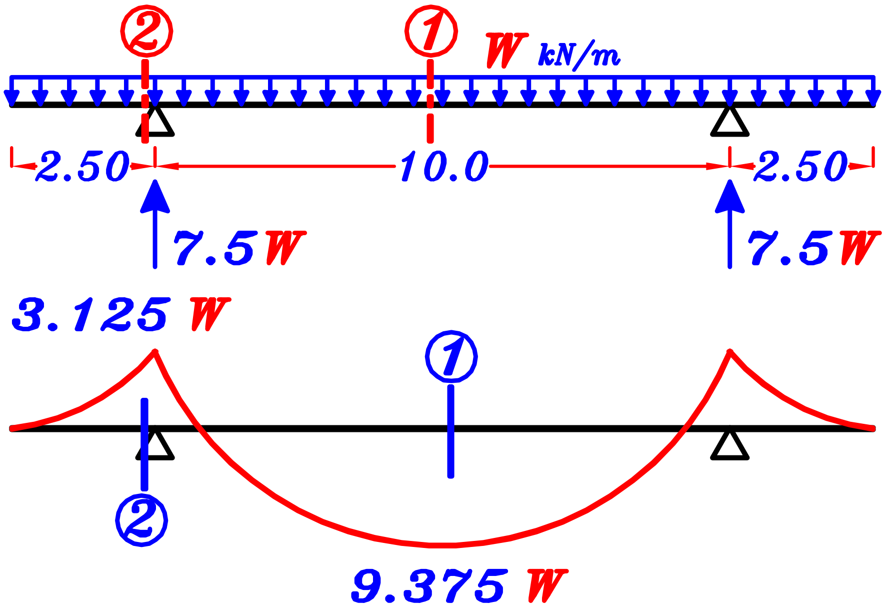
and the assumption is right $F_s = F_y$

$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, B \left(d - \frac{a}{2}\right)$$

$$\therefore M_{ult} = \frac{2}{3}(30)(41.04)(1000)\left(950 - \frac{41.04}{2}\right) = 762917184 \text{ N.mm} = 762.9 \text{ kN.m}$$

$$\therefore \boxed{M_{ult} = 762.9 \text{ kN.m}}$$

Actual Moment.



1 – The cracking load of the girder. (W_{cr})

Sec. ① $M_{act.} = 9.375\ W$ $\qquad M_{cr1} = 475.8\ kN.m$

$\therefore\ 9.375\ W_{cr.} = 475.8\ kN.m \longrightarrow W_{cr.1} = 50.75\ kN/m$

Sec. ② $M_{act.} = 3.125\ W$ $\qquad M_{cr2} = 475.8\ kN.m$

$\therefore\ 3.125\ W_{cr.} = 475.8\ kN.m \longrightarrow W_{cr.2} = 152.2\ kN/m$

$$\boxed{W_{cr.} = 50.75\ kN/m}$$

1 – The ultimate load of the girder. (W_{ult})

Sec. ① $M_{act.} = 9.375\ W$ $\qquad M_{ult1} = 1252.8\ kN.m$

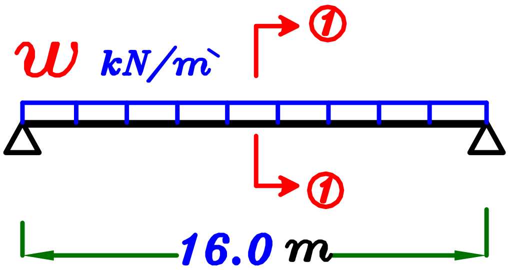
$\therefore\ 9.375\ W_{ult} = 1252.8\ kN.m \longrightarrow W_{ult1} = 133.6\ kN/m$

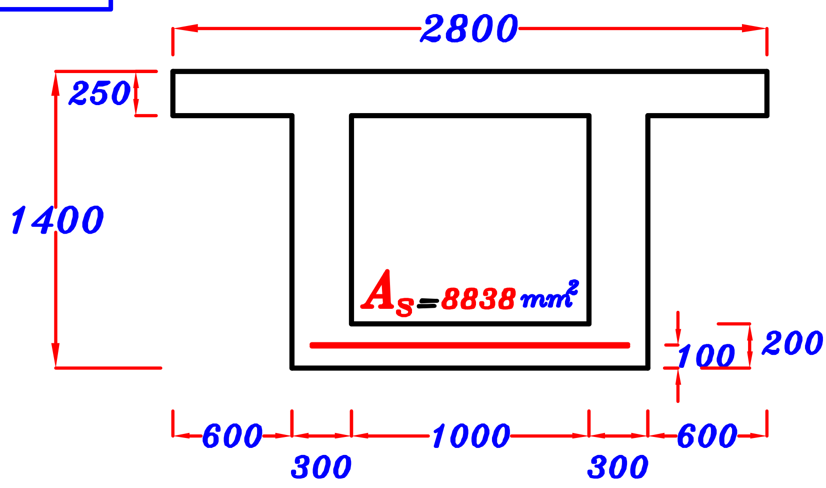
Sec. ② $M_{act.} = 3.125\ W$ $\qquad M_{ult2} = 762.9\ kN.m$

$\therefore\ 3.125\ W_{ult} = 762.9\ kN.m \longrightarrow W_{ult2} = 244.12\ kN/m$

$$\boxed{W_{ult} = 133.6\ kN/m}$$

Example.





Sec. (1−1)

Data.

$F_{cu} = 30 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2$

Floor Cover $= 3.50 \quad kN\backslash m^2$

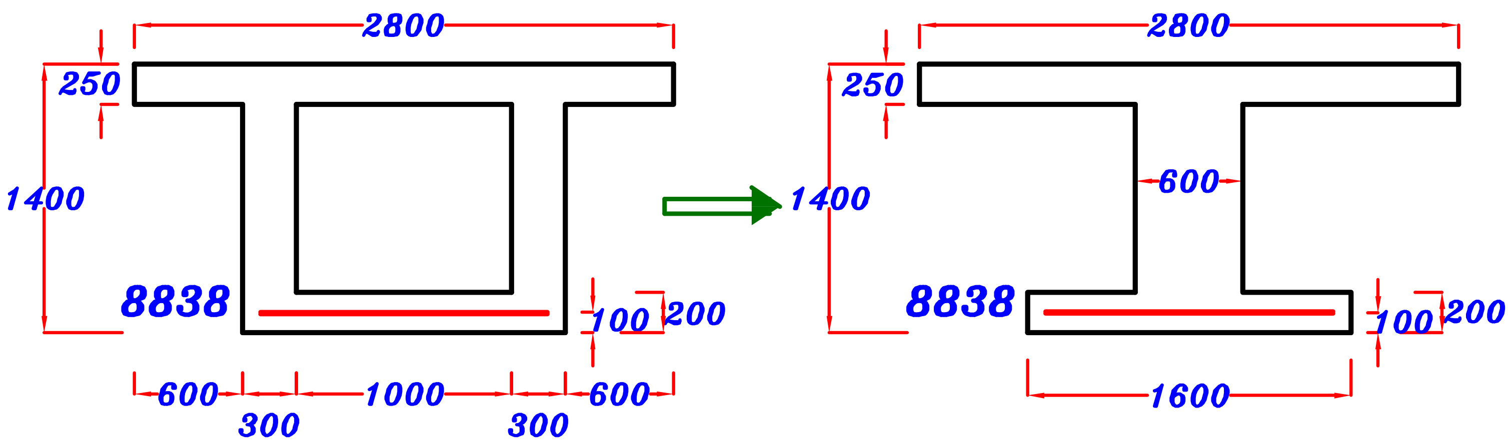
Req.

Find the allowable working live load $(kN\backslash m^2)$

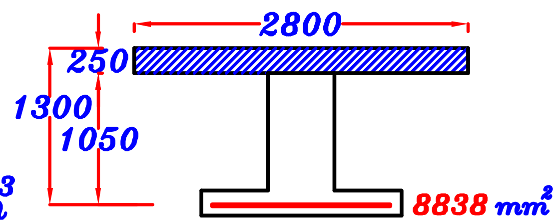
Allowable stresses

$F_{cu} = 30 \quad N\backslash mm^2 \longrightarrow F_c = 10.5 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2 \longrightarrow F_s = 200 \quad N\backslash mm^2$



To know if Z is bigger or smaller than the Flange thickness $= 250$ mm

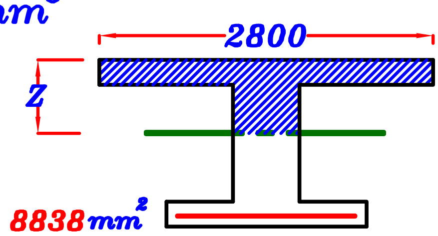


$S_{nv.}(above) = 250 * 2800 * (125) = 87500000\ mm^3$

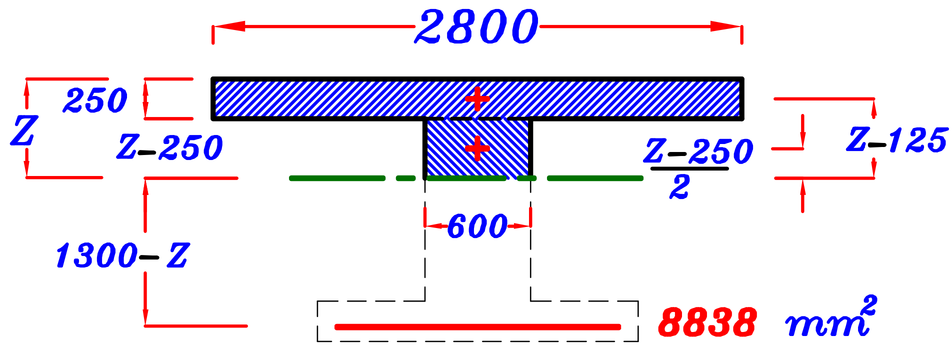
$S_{nv.}(under) = 15 * 8838 * (1050) = 139198.5\ mm^3$

$\because\ S_{nv.}(under) > S_{nv.}(above)$

$\therefore\ Z > 250 \quad mm$



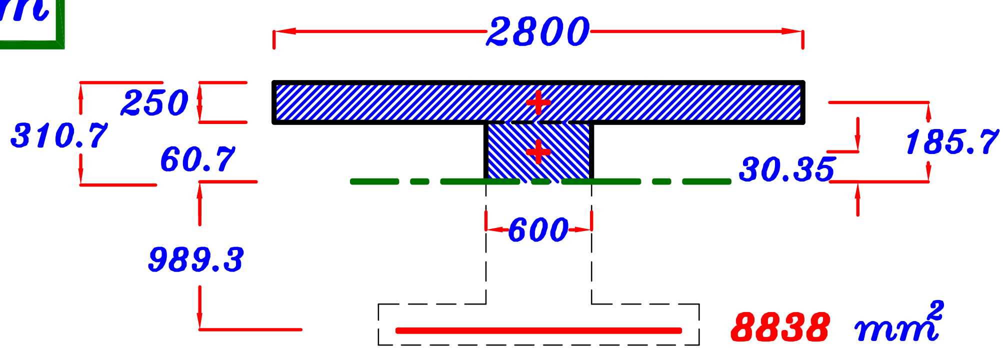
① Take $n = 15$



② Get Z by taking $\boxed{S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}}$

$$(2800)(250)(Z-125) + (600)(Z-250)\left(\frac{Z-250}{2}\right) = (15)(8838)(1300-Z)$$

$$\boxed{Z = 310.7 \ mm}$$



③ $$I_{nv} = \frac{2800(250)^3}{12} + (2800)(250)(185.7)^2 + \frac{600(60.7)^3}{3} + (15)(8838)(989.3)^2 = 157577886000 \ mm^4$$

④ $$M_{wc} = \frac{F_c * I_{nv}}{Z}$$ ------ not as T_Sec.

$$= \frac{10.5 * 157577886000}{310.7} = 5325290643 \ N.mm = 5325.3 \ kN.m$$

⑤ $$M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z}$$

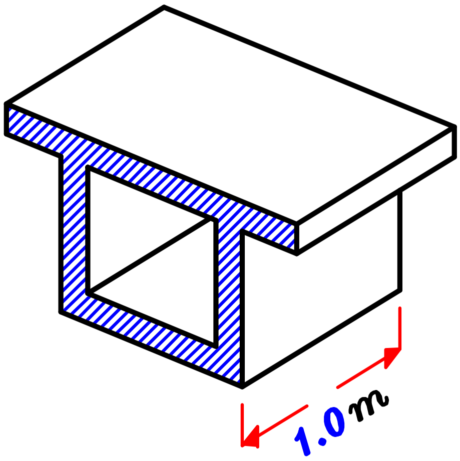
$$= \frac{\left(\frac{200}{15}\right) * 157577886000}{1300 - 310.7} = 2123762741 \ N.mm = 2123.7 \ kN.m$$

⑥ $$\boxed{M_w = 2123.7 \ kN.m}$$

للتحويل من $kN\backslash m^2$ إلى $kN\backslash m`$ نضرب فى العرض بالمتر

للتحويل من $kN\backslash m`$ إلى $kN\backslash m^2$ نقسم على العرض بالمتر

$w = O.W. + F.C. + L.L. = \checkmark\ kN\backslash m`$

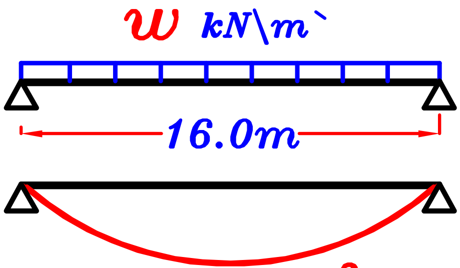


O.W. of the beam For 1.0 m.

$= \text{Volume} * \gamma_c$

$= [0.25(2.8) + 0.95(0.6) + 0.20(1.6)]\ (25)$

$= 39.75\ kN\backslash m`$



$M_{act.} = \frac{wL^2}{8} = \frac{w(16)^2}{8} = 32.0\,w$

$M_{act.} = \frac{wL^2}{8}$

To get the allowable L.L.

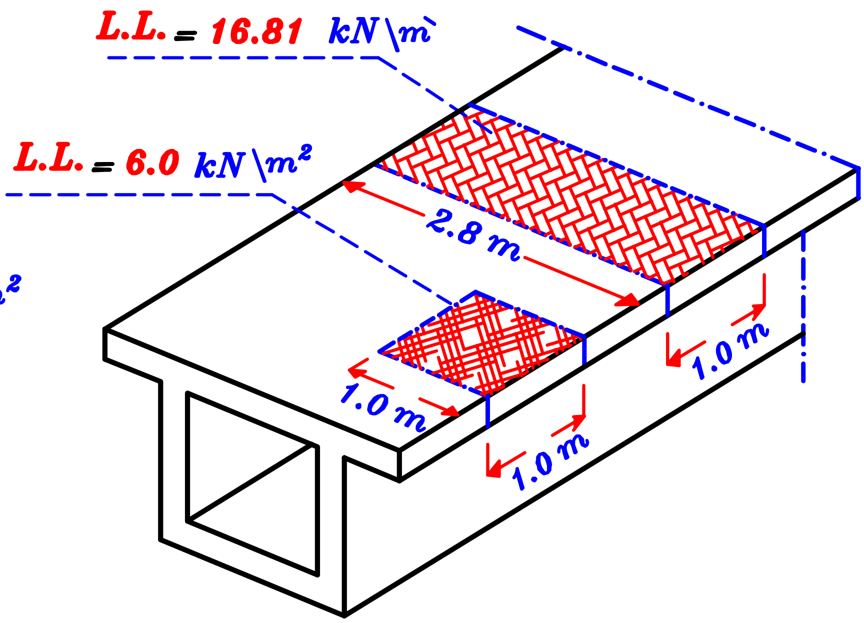
$M_{act.} = M_w$

$32\,w = 2123.7\ kN.m \longrightarrow \boxed{w = 66.36\ kN\backslash m`}$

$\because\ w = O.W. + F.C. + L.L.$ العرض بالمتر

$\therefore\ 66.36 = 39.75 + (3.5 * 2.8) + L.L. \longrightarrow \boxed{L.L. = 16.81\ kN\backslash m`}$

$L.L.\,(kN\backslash m^2) = \frac{L.L.\,(kN\backslash m`)}{\text{العرض بالمتر}}$



$\therefore\ L.L. = \frac{16.81}{2.80} = 6.0\ kN\backslash m^2$

$\boxed{L.L. = 6.0\ kN\backslash m^2}$

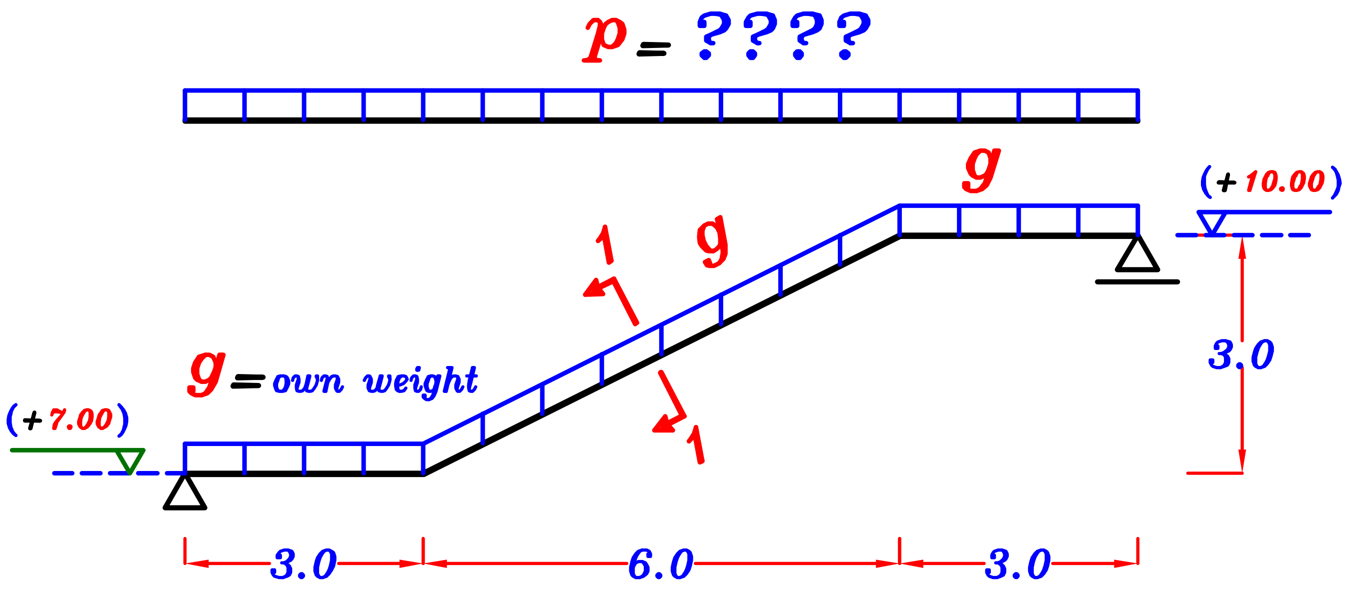
Example.

Figure 1 shows an elevation and cross section For a ramp path structure connecting the two levels (+7.00) and (+10.00). **It is required to:**

1- Calculate the maximum working uniform live load acting on horizontal projection which could be carried by the ramp structure (taking into consideration its own weight).

2- Calculate the Failure uniform live load of the ramp structure (taking into consideration its own weight) and state the type of Failure.

$F_{cu} = C = 30$ MPa , steel 36/52



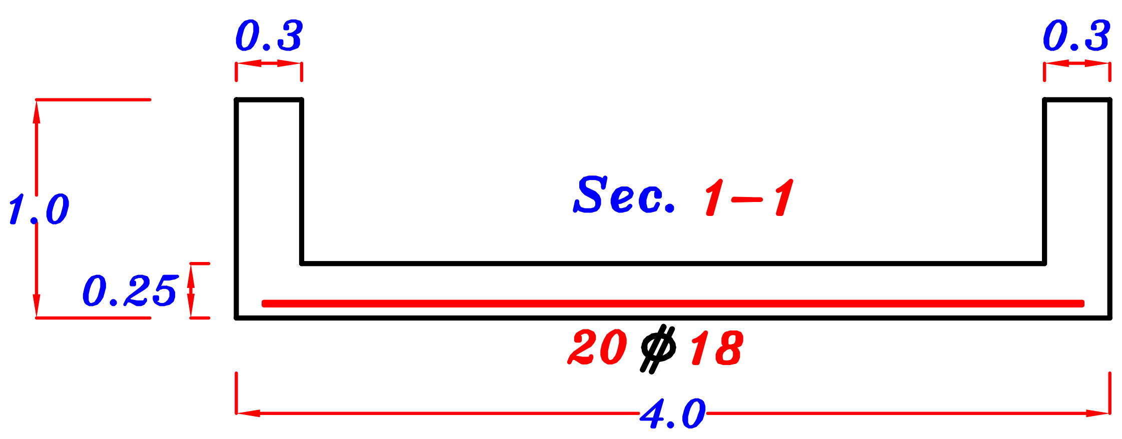
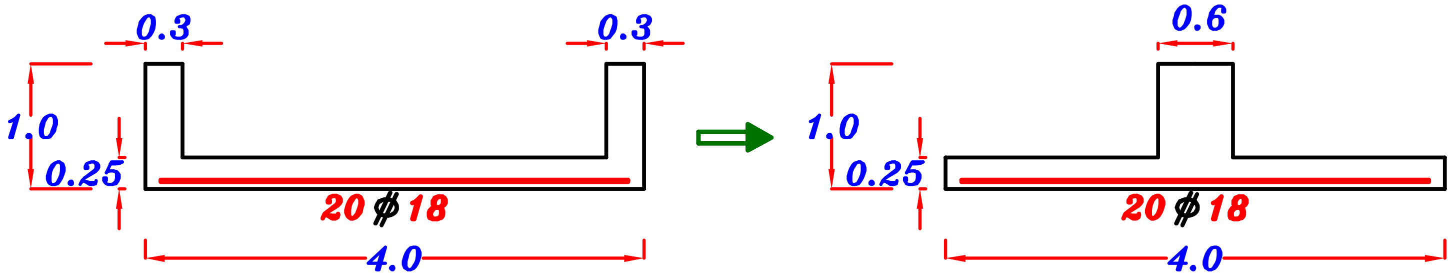
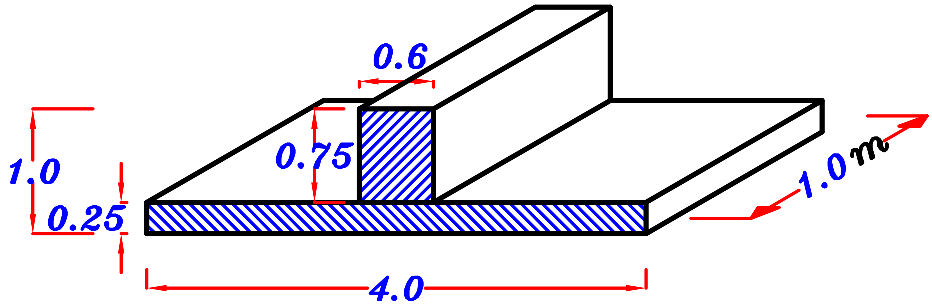


Figure 1

1 – Calculate the maximum working uniform live load acting on horizontal projection which could be carried by the ramp structure (taking into consideration its own weight).





$$o.w. = \left[(0.25 * 4.0) + (0.75 * 0.6)\right] * 1.0 * 25 = 36.25 \text{ kN/m}$$

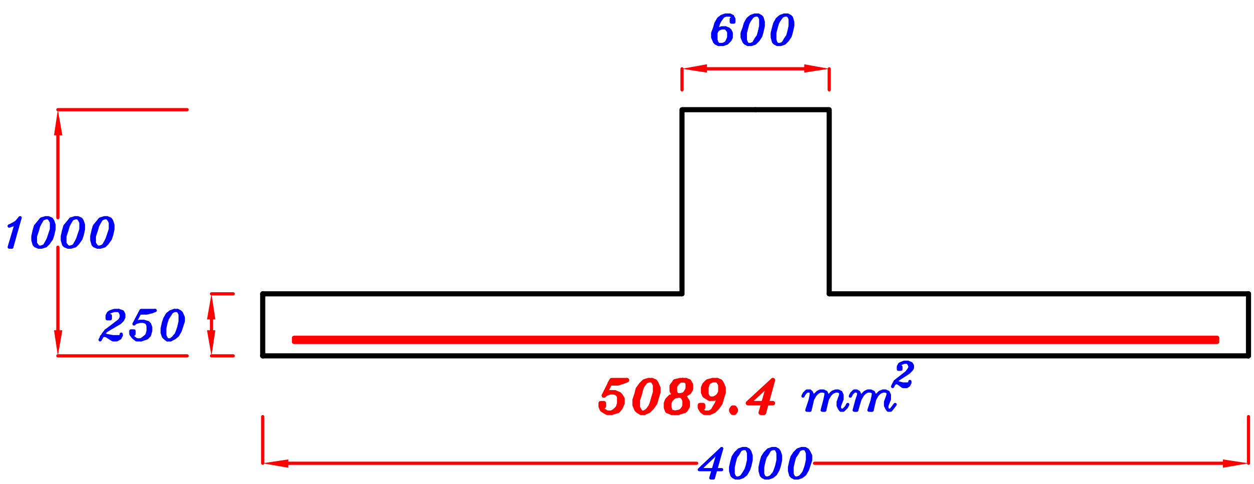
Allowable working moment. M_w

Allowable stresses

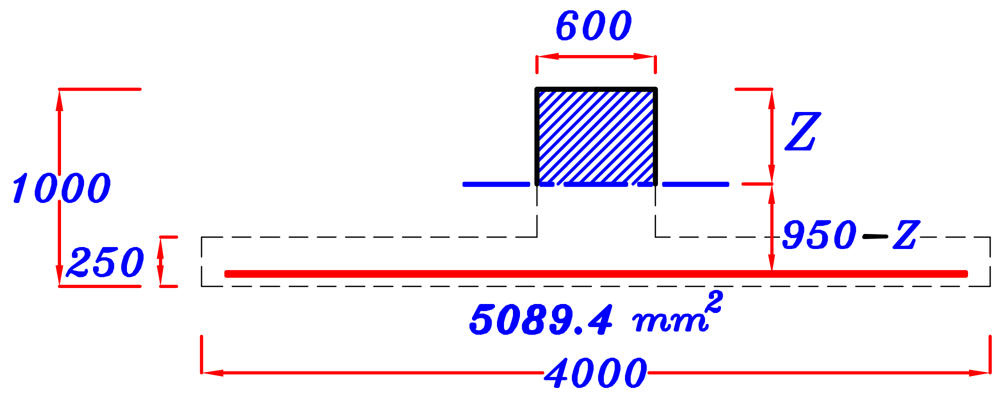
$F_{cu} = 30 \; N \backslash mm^2 \longrightarrow F_c = 10.5 \; N \backslash mm^2$

$F_y = 360 \; N \backslash mm^2 \longrightarrow F_s = 200 \; N \backslash mm^2$

$$A_s = 20 \phi 18 = 20 \left[\frac{\pi * 18^2}{4}\right] = 5089.4 \; mm^2$$



① Take $n = 15$



② Get Z by taking $S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}$

$$b\,(Z)\left(\frac{Z}{2}\right) = n\,A_s\,(d - Z)$$

$$600\,(Z)\left(\frac{Z}{2}\right) = (15)\,(5089.4)\,(950 - Z)$$

$$\boxed{Z = 380.6 \text{ mm}}$$

③ Get $I_{nv} = \dfrac{bZ^3}{3} + n\,A_s\,(d - Z)^2$

$$I_{nv} = \frac{600\,(380.6)^3}{3} + (15)\,(5089.4)\,(950 - 380.6)^2 = 35777467260 \text{ mm}^4$$

④ $M_{wc} = \dfrac{F_c * I_{nv}}{Z} = \dfrac{10.5 * 35777467260}{380.6} = 987029443$ N.mm

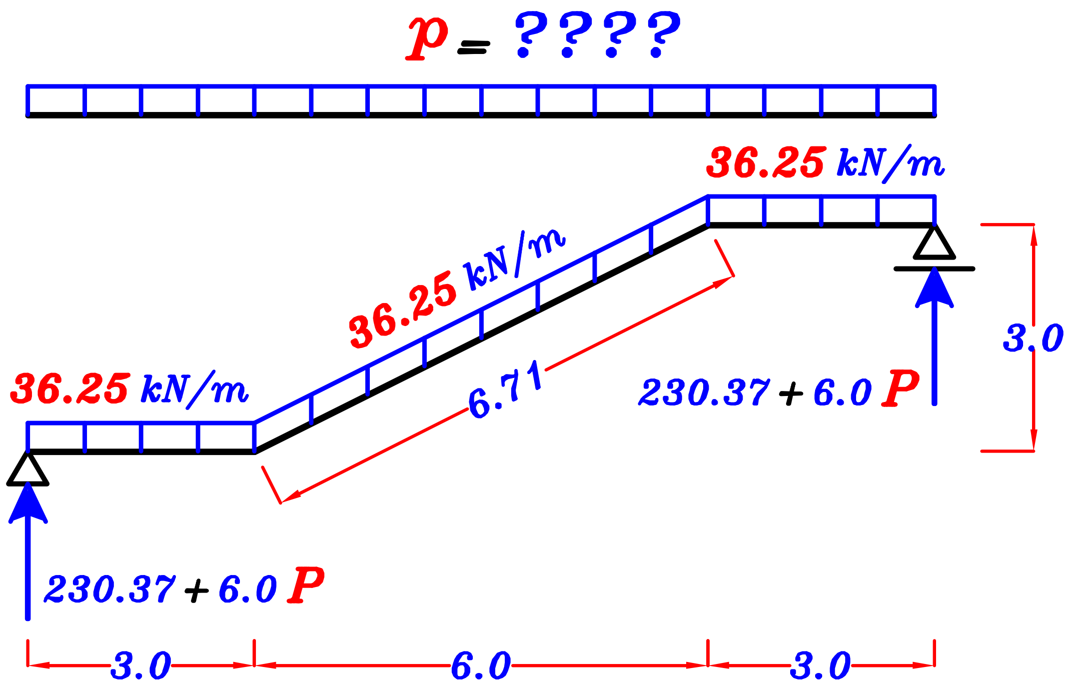
$= 987.03$ kN.m

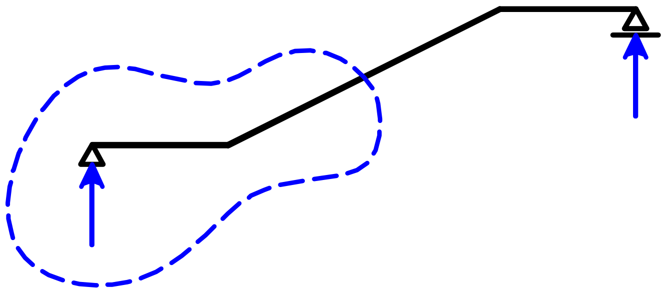
⑤ $M_{ws} = \dfrac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \dfrac{\left(\frac{200}{15}\right) * 35777467260}{950 - 380.6} = 837781694$ N.mm

$= 837.78$ kN.m

⑥ $$\boxed{Mw_{all} = 837.78 \text{ kN.m}}$$

Actual working moment.





moment at mid span.

$$(230.37 + 6.0\,P)(6.0) - (36.25 * 3.0)(4.5) - \left(36.25 * \frac{6.71}{2}\right)(1.5)$$

$$-\,(P * 6.0)(3.0) = 18.0\,P + 710.41$$

$$\boxed{M_{act} = 18.0\,P + 710.41}$$

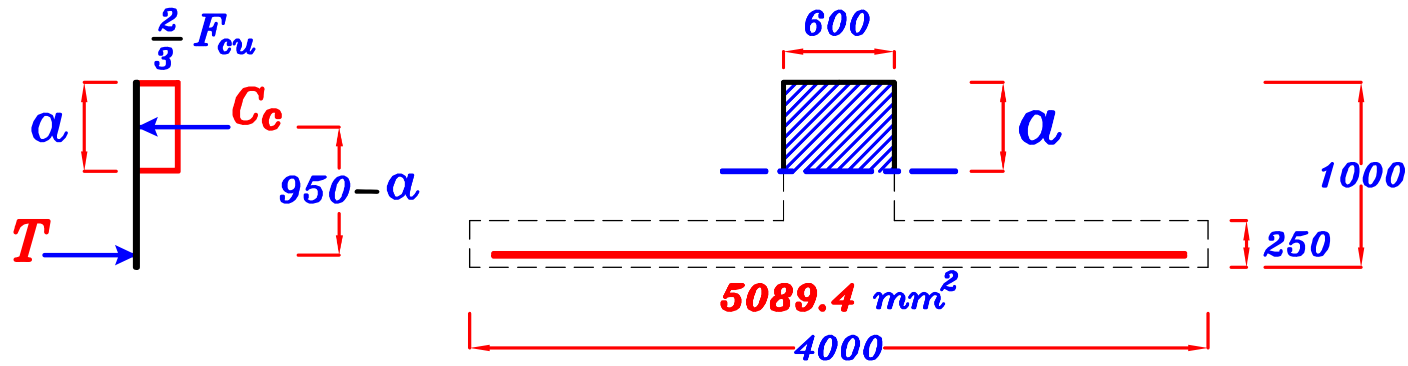
To calculate the maximum working uniform live load acting on horizontal projection.

$$M_{w_{all}} = M_{act}$$

$$837.78 = 18.0\,P_w + 710.41 \longrightarrow \boxed{P_w = 7.076\ kN/m}$$

2- Calculate the Failure uniform live load of the ramp structure (taking into consideration its own weight) and state the type of Failure.

Ultimate moment. M_{ult}



$$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 950 = 593.75 \text{ mm}$$

② From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * b = F_s * A_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(30)(a)(600) = (360)(5089.4) \longrightarrow a = 152.68 \text{ mm}$$

③ $\therefore C = 1.25\, a = 1.25 * 152.68 = 190.85 \text{ mm} < C_b$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

and the assumption is right $F_s = F_y$

④ By taking the moment about the steel.

$$\therefore M_{ult} = \frac{2}{3}(30)(152.68)(600)\left(950 - \frac{152.68}{2}\right)$$

$$= 1600684906 \text{ N.mm} = 1600.7 \text{ kN.m}$$

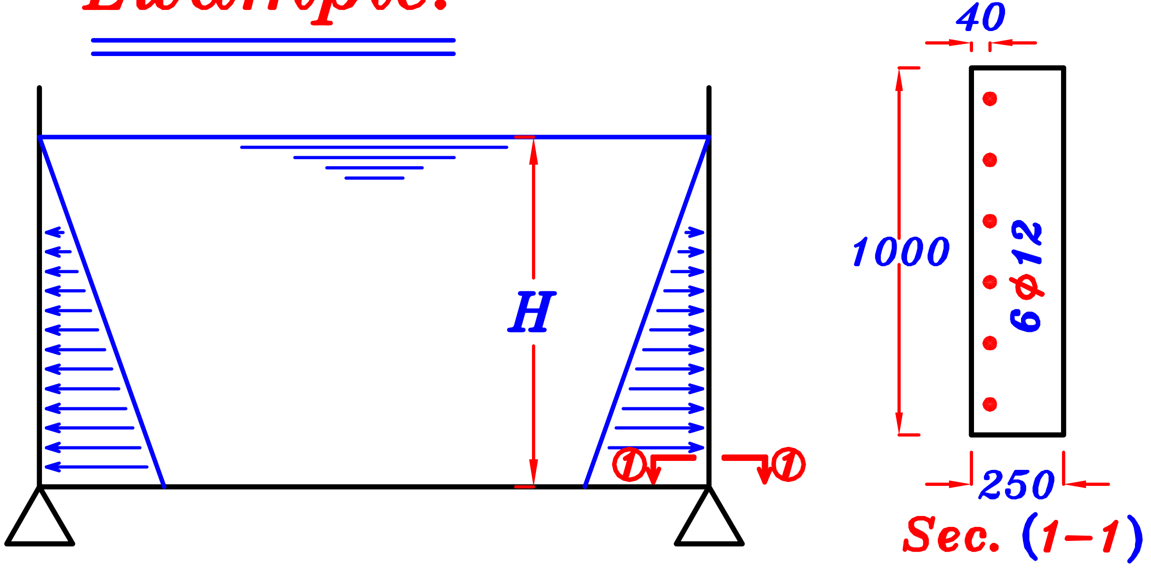
$$\boxed{M_{ult} = 1600.7 \text{ kN.m}}$$

To calculate the Failure uniform live load.

$$M_{ult} = M_{act}$$

$$1600.7 = 18.0\, P_{ult} + 710.41 \longrightarrow \boxed{P_{ult} = 49.46 \text{ kN/m}}$$

Example.



For the given statical system & cross section of a water tank with 0.25 m thick cantilever walls, It is required to Find the max safe height of water (H) in the tank.

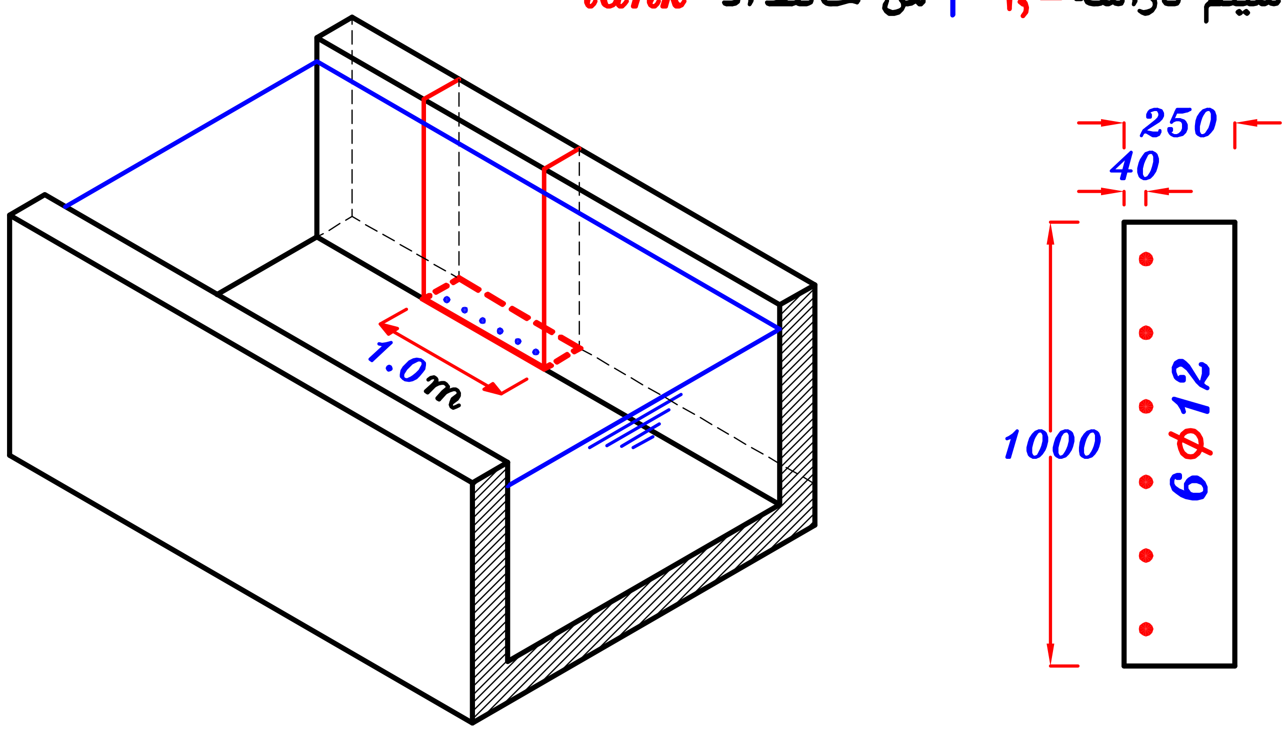
$F_{cu} = 30\ N\backslash mm^2$ st. 240/350

في المنشآت المائيه يجب منع حدوث أى شروخ فى الخرسانه حتى لا تصل المياه الى حديد التسليح فيصدأ .

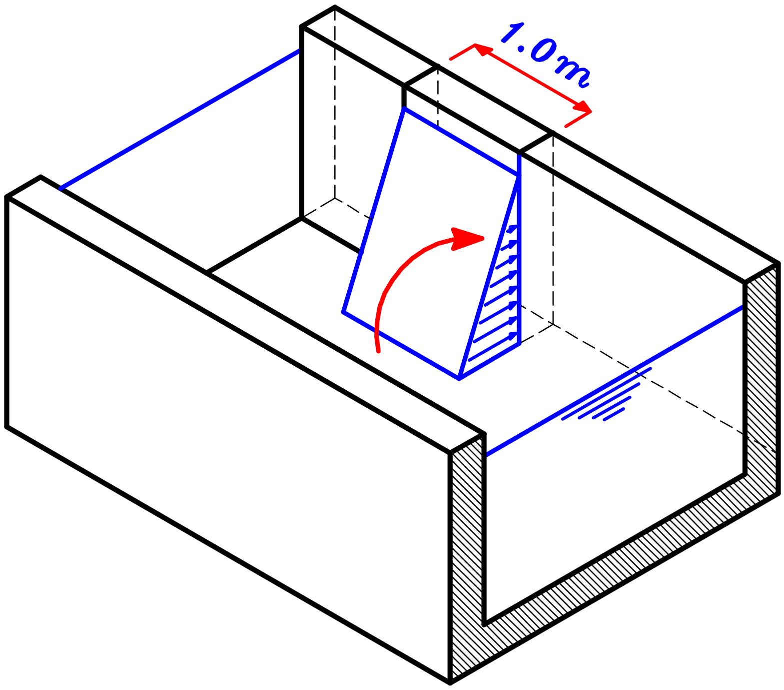
لذا أى قطاع موجود فى ال tank يجب أن لا يتعدى العزم عليه عن $M_{cr.}$

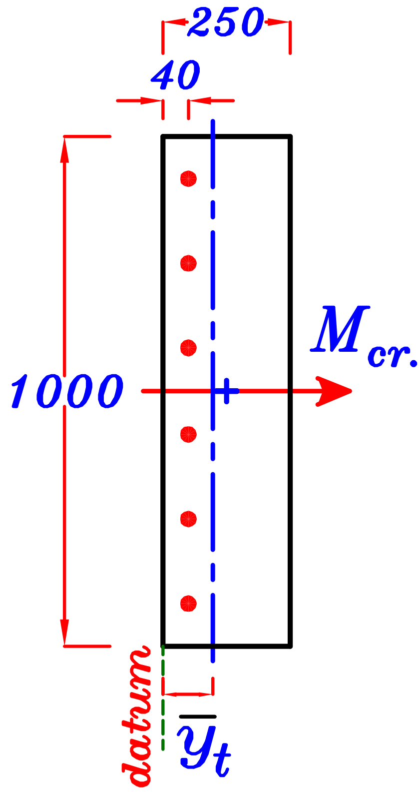
∴ لحساب أكبر ارتفاع للماء ممكن أن تتحمله حوائط ال tank هو الارتفاع الذى يجعل العزم على القطاع السفلى للحائط مساوى تماماً ل $M_{cr.}$

سيتم دراسه ١,٠ م من حائط ال tank



$M_{cr.}$ For the section.





$$A_s = 6\phi 12 = 678.5 \ mm^2$$

① $$n = \frac{E_s}{E_c} = \frac{2*10^5}{4400\sqrt{30}} = 8.29 \longrightarrow n = 10$$

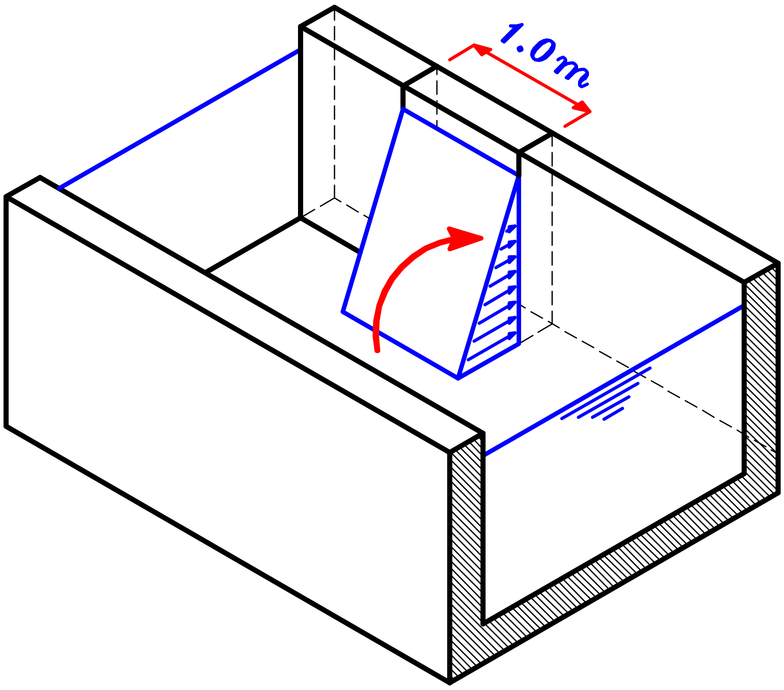
② $$A_v = 250*1000 + (10-1)(678.5) = 256106 \ mm^2$$

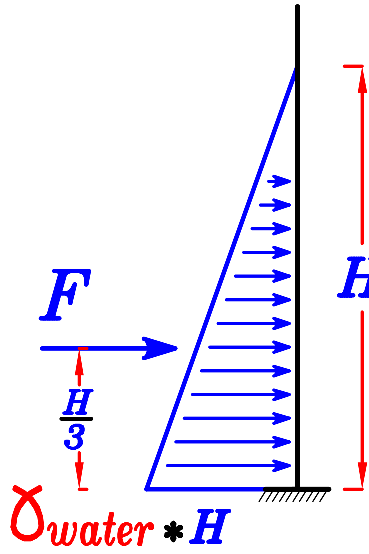
③ $$\bar{y}_t = \frac{250*1000*125 + (10-1)(678.5)(40)}{256106} = 123 \ mm$$

④ $$I_g = \frac{1000*250^3}{12} + 1000*250(125-123)^2 + (10-1)(678.5)(123-40)^2$$
$$= 1345151012 \ mm^4$$

⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{30} = 3.28 \ N\backslash mm^2$$

⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.28 * 1345151012}{123} = 35870693.6 \ N.mm$$
$$= 35.87 \ kN.m$$





$$\gamma_{water} = 1.0 \ t\backslash m^3 = 10.0 \ kN\backslash m^3$$

$$\text{water pressure (at base)} = \gamma_{water} * H = 10\,H \ kN\backslash m^2$$

$$\text{water Force} \quad F = \tfrac{1}{2}(\gamma_{water} * H) * H = \tfrac{1}{2}(10\,H) * H = 5.0\,H^2 \ kN$$

$$\text{Actual moment at Base} = F * \frac{H}{3} = 5.0\,H^2 * \frac{H}{3} = \frac{5}{3}\,H^3 \ kN.m$$

$$\boxed{\text{Actual moment at Base} = \frac{5}{3}\,H^3 \ kN.m}$$

To get the max. safe height (H)

$$\therefore \ \text{Actual moment at Base} = M_{cr.}$$

$$\therefore \ \frac{5}{3}\,H^3 = 35.87 \ kN.m \qquad \therefore \ \boxed{H = 2.781 \ m}$$

∴ اذا زاد إرتفاع الماء عن $2.781\ m$ سوف يكون العزم المؤثر على القطاع السفلى للحائط أكبر من ال $M_{cr.}$ فتتشرخ الخرسانه فيصل الماء إلى الحديد فيصدأ الحديد .

Example.

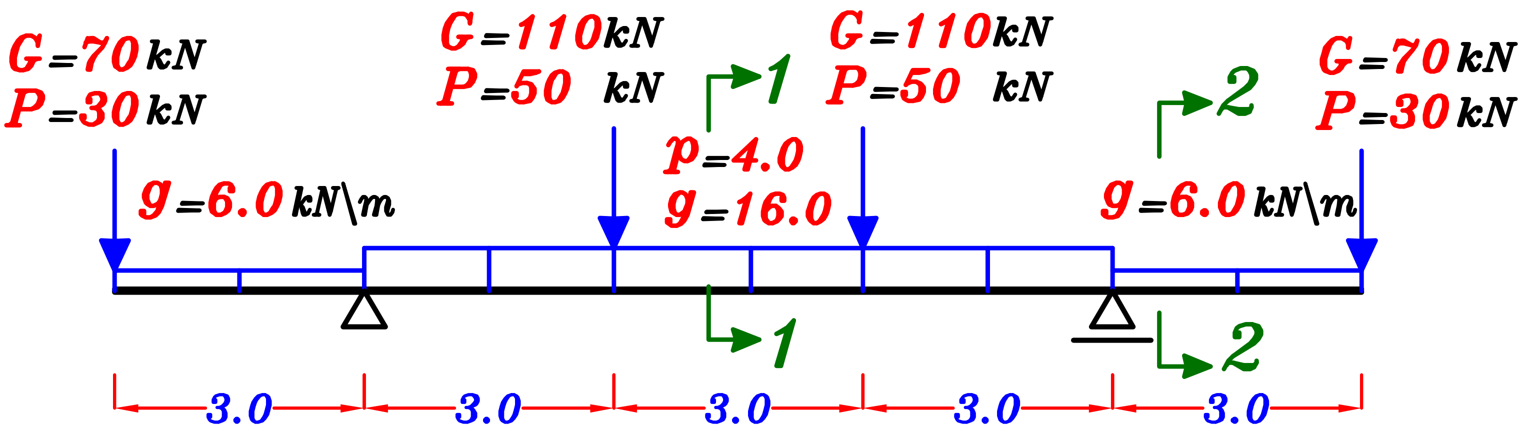
For the given statical system, it is required to:

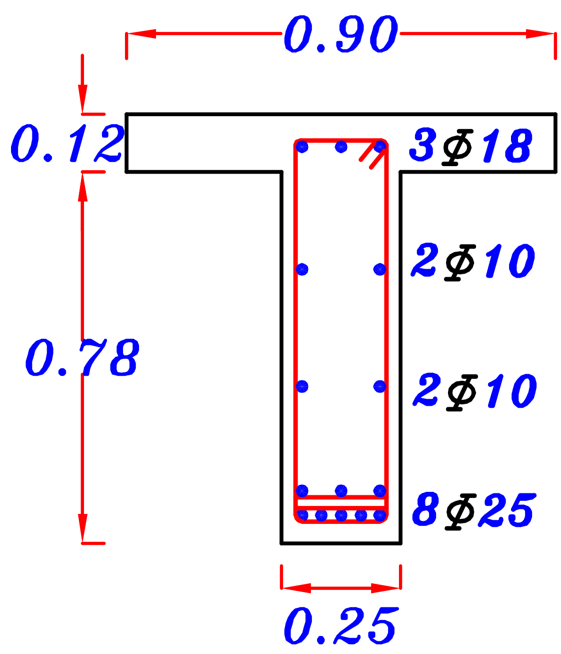
1- Draw the max.-max. B.M.D.

2- Calculate the compressive and tensile stresses on both concrete and steel bars respectivly at sections 1 and 2 (using the working loads).

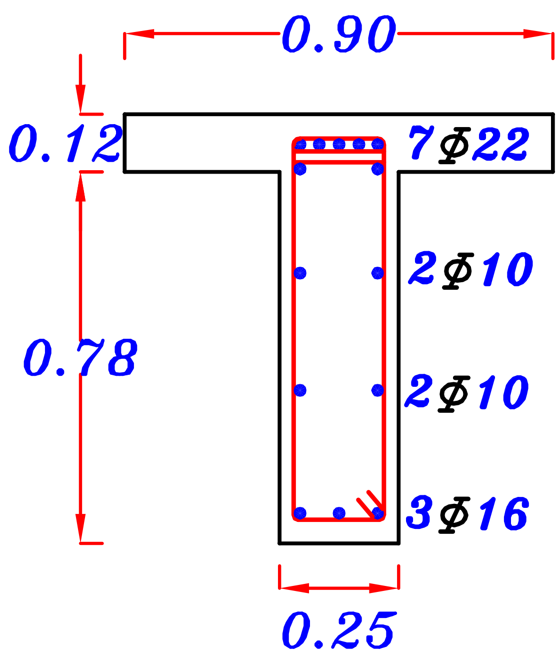
3- Comment on the results From No. 2 in the light of Egyptian code.

$F_{cu} = 30$ N\mm^2 , st. 400/600

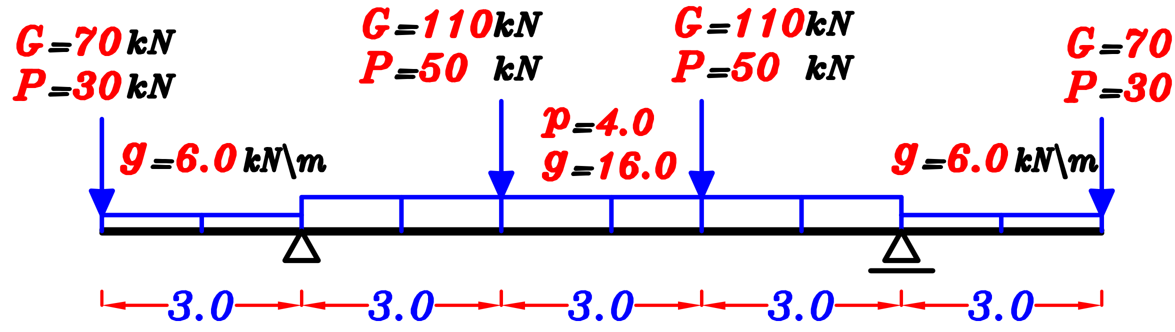




Sec. (1-1)

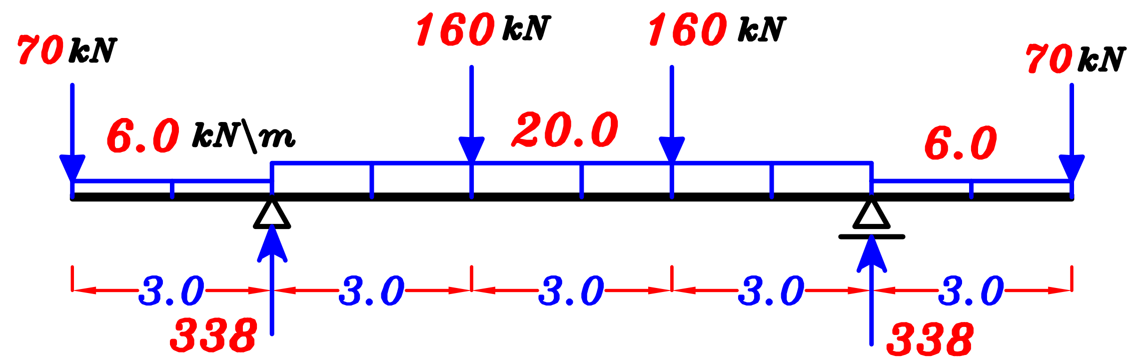


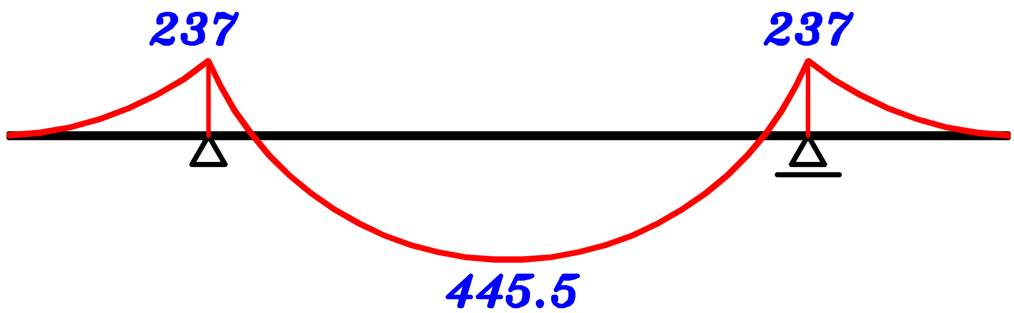
Sec. (2-2)



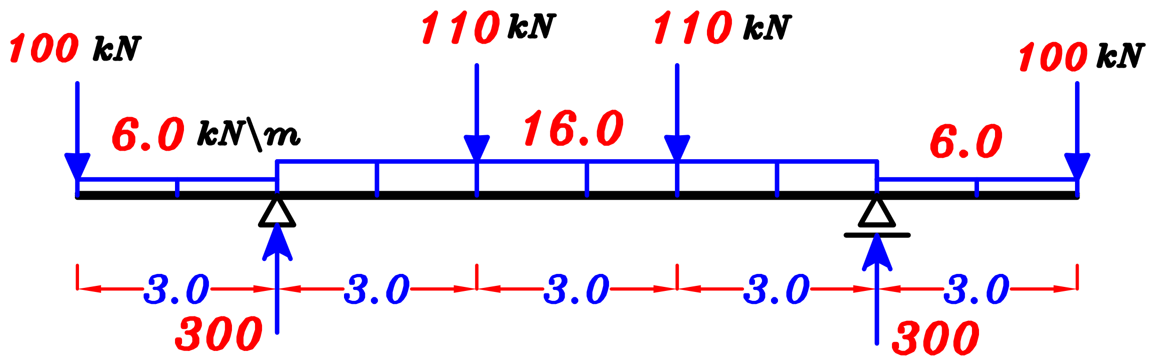
1 - Draw the max.-max. B.M.D.

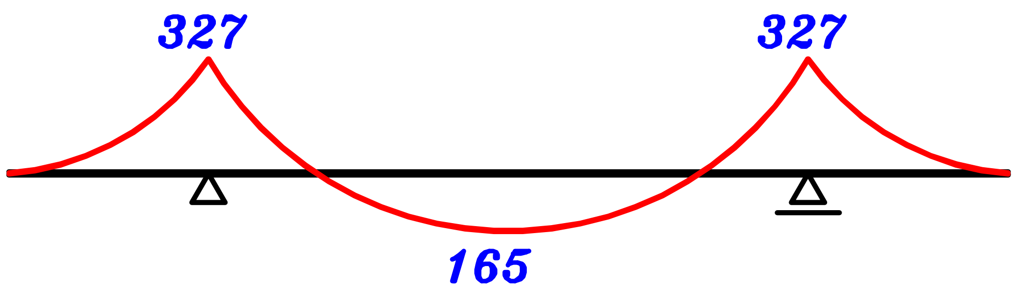
1- max. +Ve B.M.D.



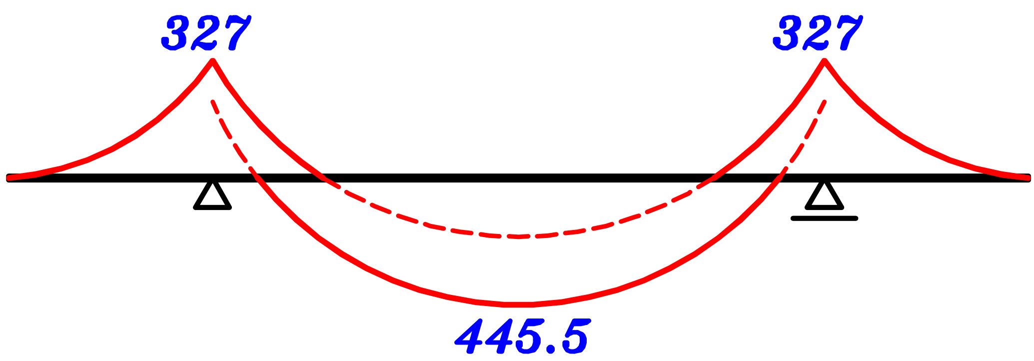


2- max. -Ve B.M.D.



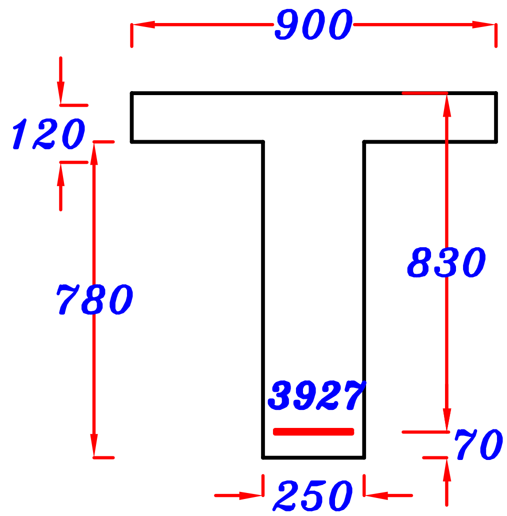


max-max B.M.D.



2- Calculate the compressive and tensile stresses on both concrete and steel bars respectivly at sections 1 and 2 (using the working loads).

Sec. (1-1)

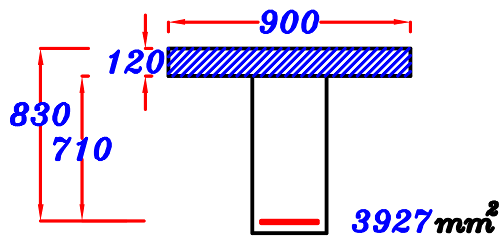


$F_{cu} = 30$ N\mm² $\longrightarrow$ $F_c = 10.5$ N\mm²

$F_y = 400$ N\mm² $\longrightarrow$ $F_s = 220$ N\mm²

T-Sec. No A_s'

To know IF Z is bigger or smaller than the Flange thickness = 120 mm

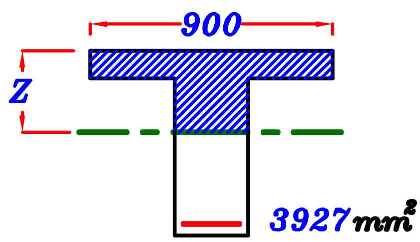


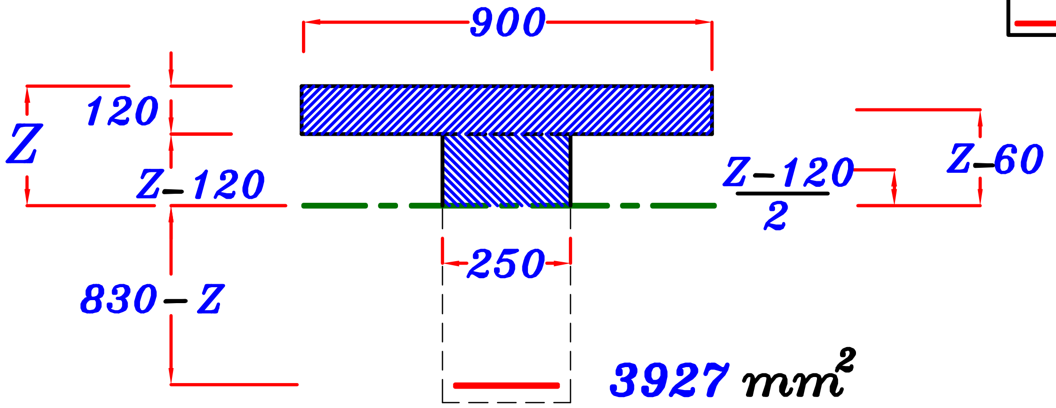
$S_{nv.}(\text{above}) = 120 * 900 * (60) = 6480000 \ mm^3$

$S_{nv.}(\text{under}) = 15 * 3927 * (710) = 41822550 \ mm^3$

$\because \ S_{nv.}(\text{under}) > S_{nv.}(\text{above})$

$\therefore \ Z > 120$ mm

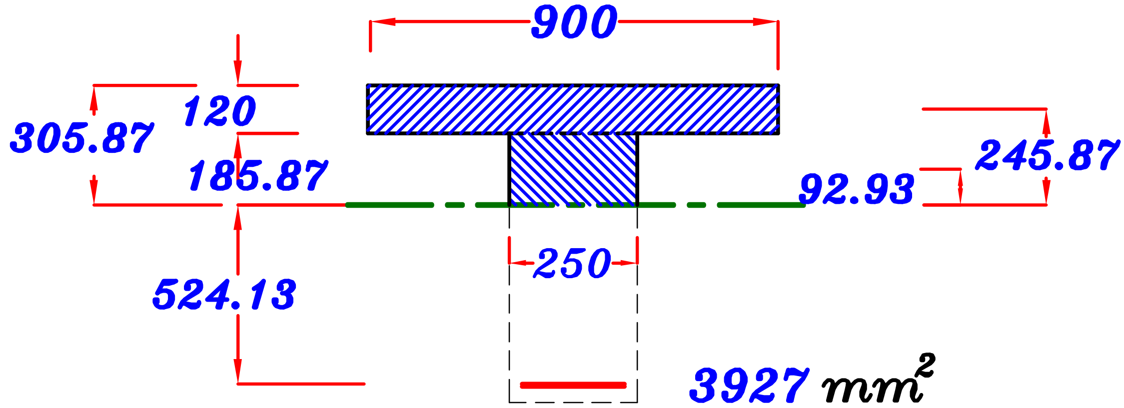




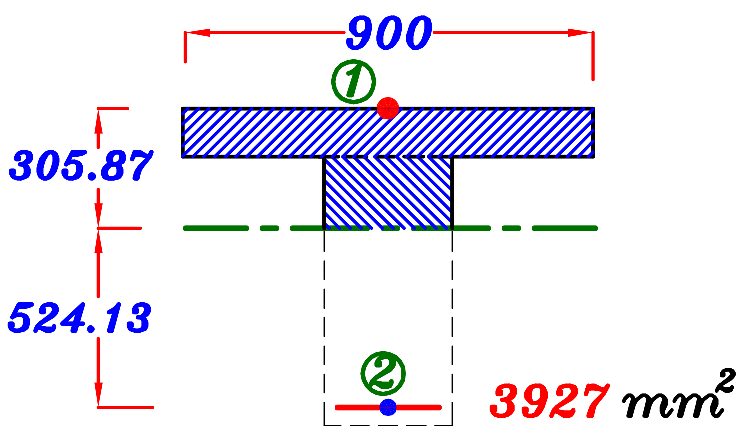
② Get Z by taking $S_{nv.} = 0.0$

$$(900)(120)(Z-60)+(250)(Z-120)\left(\frac{Z-120}{2}\right)-(15)(3927)(830-Z)=0.0$$

$$\boxed{Z = 305.87\,mm}$$



③ $$I_{nv} = \frac{900\ (120)^3}{12} + (120)(900)(245.87)^2 + \frac{250\,(185.87)^3}{3} + (15)(3927)(524.13)^2 = 23375462050\ mm^4$$



Actual Stresses On Concrete.

$$F_1 = \frac{M\,Z}{I_{nv}} = \frac{445.5*10^6*305.87}{23375462050} = 5.82\ N\backslash mm^2 < \frac{2}{3}F_c$$

T-sec.

Actual Stresses On Steel.

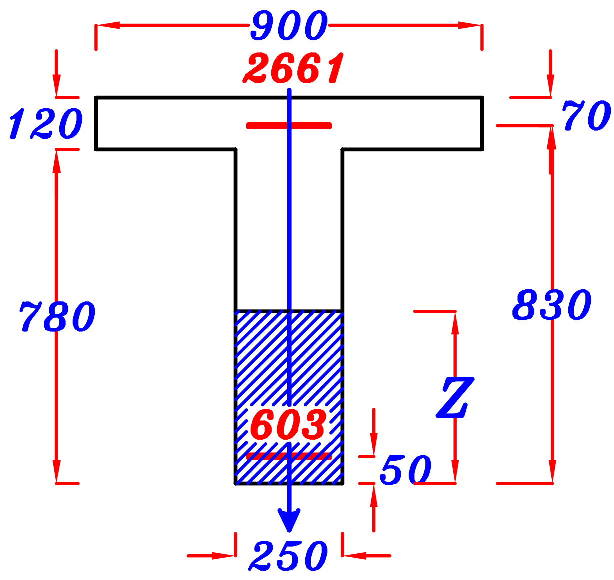
$$F_2 = n*\frac{M\,(d-Z)}{I_{nv}} = 15\left(\frac{445.5*10^6*(524.13)}{23375462050}\right) = 149.83\ N\backslash mm^2 < F_s$$

Comment Sec. (1-1) is Safe.

Sec. (2−2)

$\because \dfrac{A_{s`}}{A_s} = \dfrac{603}{2661} = 0.22 > 0.2$

$\therefore$ $\boxed{\text{Don't neglect } A_{s`}}$



Take $n = 15$

Get Z by taking $S_{nv.} = 0.0$

$$b(Z)\left(\frac{Z}{2}\right) + (n-1)A_{s`}(Z-d`) - n\,A_s(d-Z) = 0.0$$

$$250(Z)\left(\frac{Z}{2}\right) + (14)(603)(Z-50) - (15)(2661)(830-Z) = 0.0$$

$$\boxed{Z = 359.6 \text{ mm}}$$

Get $I_{nv} = \dfrac{bZ^3}{3} + (n-1)A_{s`}(Z-d`)^2 + n\,A_s(d-Z)^2$

$$I_{nv} = \frac{250(359.6)^3}{3} + (14)(603)(359.6-50)^2 + (15)(2661)(830-359.6)^2 = 13516476260 \text{ mm}^4$$

Actual Stresses On Concrete.

$$F_1 = \frac{M\,Z}{I_{nv}} = \frac{327 * 10^6 * 359.6}{13516476260} = 8.70 \text{ N}\backslash\text{mm}^2 < F_c$$

R−sec.

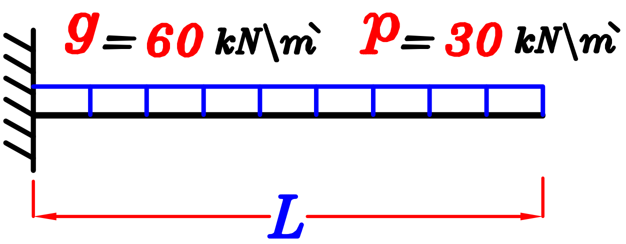
Actual Stresses On Steel.

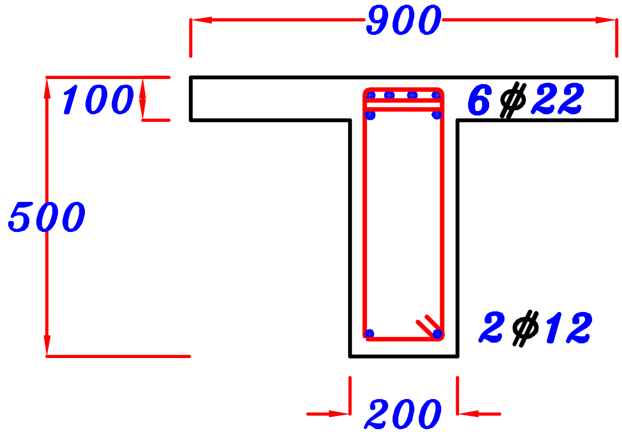
$$F_2 = n * \frac{M(d-Z)}{I_{nv}} = 15\left(\frac{327 * 10^6 * (470.4)}{13516476260}\right) = 170.7 \text{ N}\backslash\text{mm}^2 < F_s$$

Comment Sec. (2−2) is Safe.

The Beam is Safe according to Egyptian code.

Example.





$F_{cu} = 25$ N\mm^2 st. 360/250

Req.

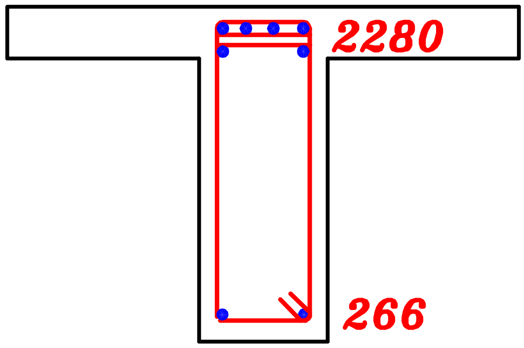
① Fined the max. design length of such cantilever to safely carry these loads according to Ultimate Limit Method.

② Check stresses in both concrete & steel at working level and comment your result according the New Egyptian Code.

Solution.

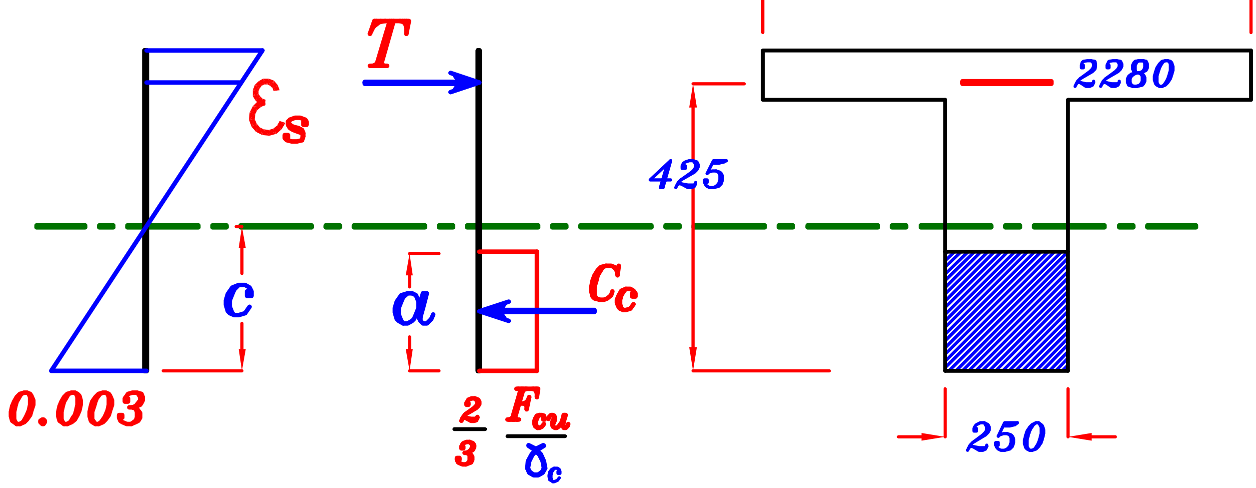
$A_s = 6\phi 22 = 2280\ mm^2$

$A_{s`} = 2\phi 12 = 226\ mm^2$



Neglect $A_{s`}$

① $M_{U.L.}$



$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600+(F_y\backslash\gamma_s)}\right] * d = 0.35\, d = 0.35 * 425 = 148.75\ mm$$

From equilibrium eqn.

$$\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b = A_s * F_s \qquad \text{assume } \varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = \frac{F_y}{\gamma_s}$$

$$\therefore \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b = A_s * \frac{F_y}{\gamma_s}$$

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)(200) = (2280)\left(\frac{360}{1.15}\right)$$

$$\longrightarrow a = 321.18\ mm > a_{max.} \xrightarrow{\text{Take}} a = a_{max.} = 148.75\ mm$$

$$\therefore M_{U.L.} = \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a_{max.}\, b\left(d - \frac{a_{max.}}{2}\right)$$

$$\therefore M_{U.L.} = \frac{2}{3}\left(\frac{25}{1.5}\right)(148.75)(200)\left(425 - \frac{148.75}{2}\right) = 115901041.7\ N.mm$$

$$= 115.90\ kN.m$$

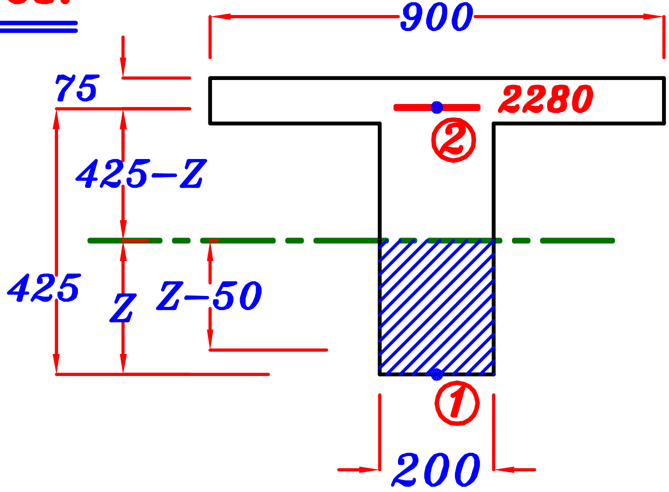
$$(w)_{U.L.} = 1.4\,(60) + 1.6\,(30) = 132.0\ kN\backslash m'$$

$$(M_{U.L.})_{act.} = \frac{w\,L^2}{2} = \frac{132.0 * L^2}{2} = 66.0\ L^2$$

To get the max. design length $\xrightarrow{\text{Take}}$ $(M_{U.L.})_{act.} = (M_{U.L.})_{all.}$

$$\therefore 66.0\ L^2 = 115.90 \longrightarrow \boxed{L = 1.325\ m}$$

② Check working stresses.



$$(w)_w = (60) + (30) = 90 \ kN\backslash m'$$

$$(M_w)_{act.} = \frac{w L^2}{2}$$

$$= \frac{90 * 1.325^2}{2} = 79.0 \ kN.m$$

① Take $n = 15$

② Get Z by taking $S_{nv.} = 0.0$

$$(200)(Z)\left(\frac{Z}{2}\right) - (15)(2280)(425 - Z) = 0.0$$

$$Z = 246.84 \ mm$$

③ Get $I_{nv} = \frac{bZ^3}{3} + n A_s (d-Z)^2$

$$I_{nv} = \frac{200\,(246.84)^3}{3} + (15)(2280)(425 - 246.84)^2$$

$$= 2088205551 \ mm^4$$

Actual Stresses.

On Concrete $F_1 = \frac{M Z}{I_{nv}} = \frac{79.0 * 10^6 * 246.84}{2088205551} = 9.338 \ N\backslash mm^2$

On Steel $F_2 = n * \frac{M (d-Z)}{I_{nv}} = 15 \left(\frac{79.0 * 10^6 * 178.16}{2088205551}\right) = 101.1 \ N\backslash mm^2$

Allowable Stresses (Due to N.E.C. 2007)

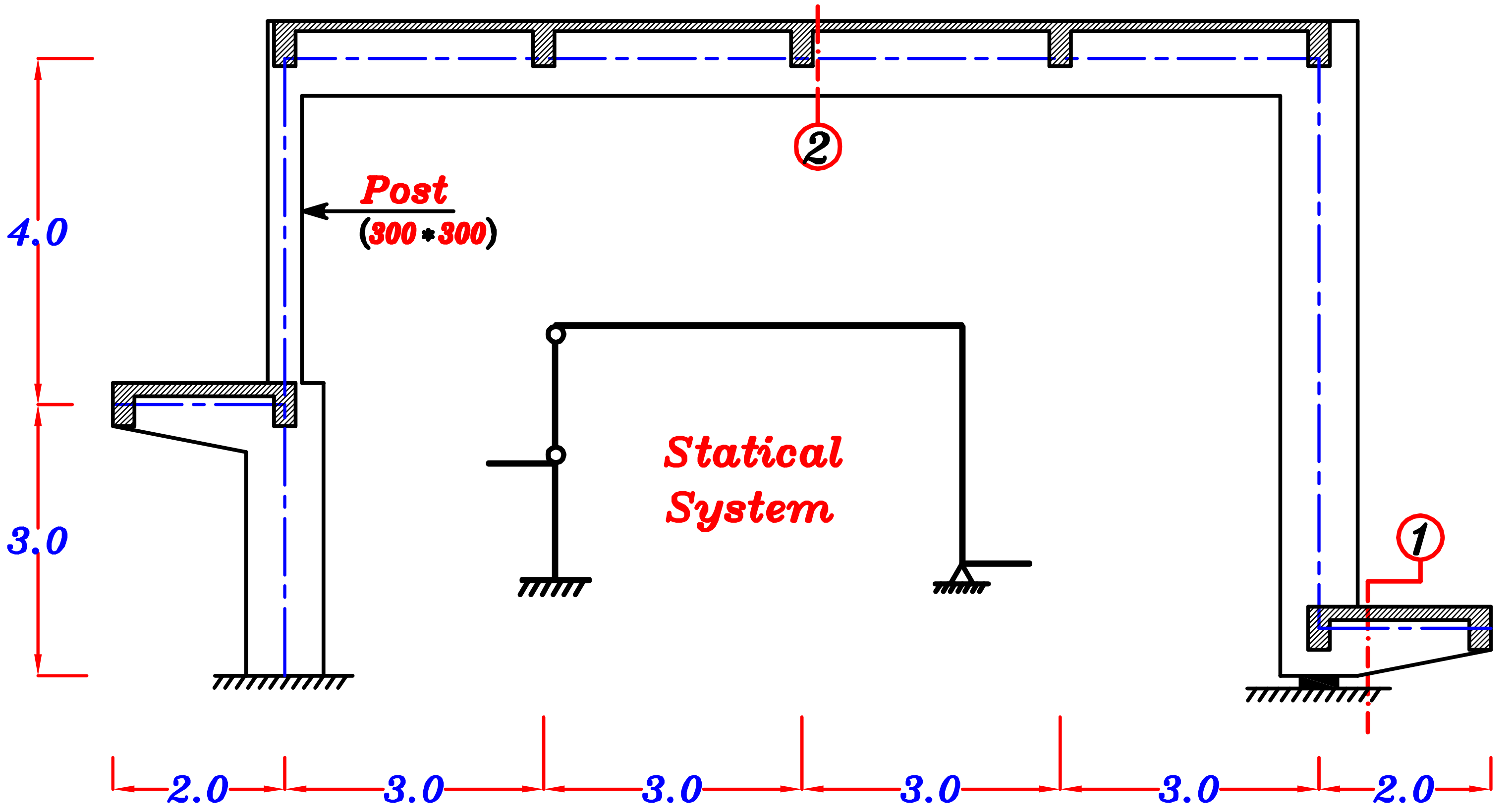
On Concrete $F_{cu} = 25 \ N\backslash mm^2 \longrightarrow F_c = 9.5 \ N\backslash mm^2$

On Steel $F_y = 360 \ N\backslash mm^2 \longrightarrow F_s = 200 \ N\backslash mm^2$

$\because$ Actual Stresses. $<$ Allowable Stresses

$\therefore$ The Section is Safe in working method too.

Example.



Data:

t_s = 120 mm

F_{cu} = 25 N\mm²

st. 360/250

b (Beam) = 250 mm

b (Frame) = 300 mm

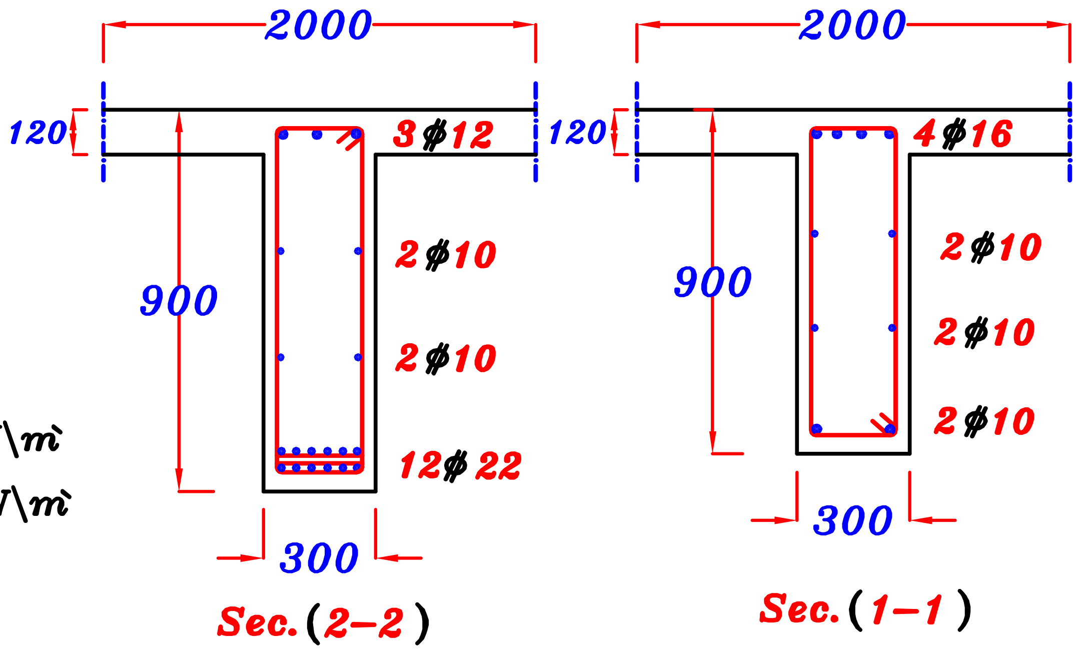
L.L. = 1.0 kN\m²

F.C. = 1.50 kN\m²

O.W. (beam) = 3.0 kN\m`

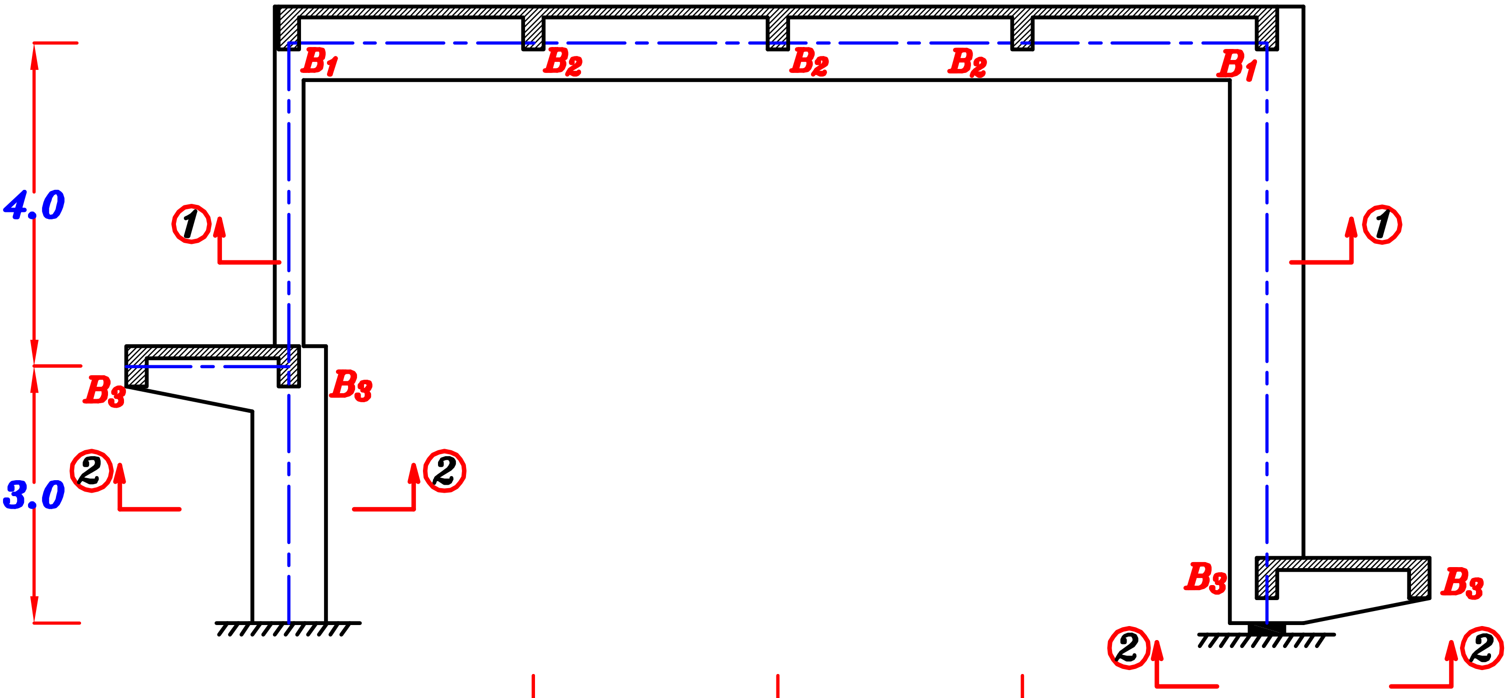
O.W. (Frame) = 6.0 kN\m`

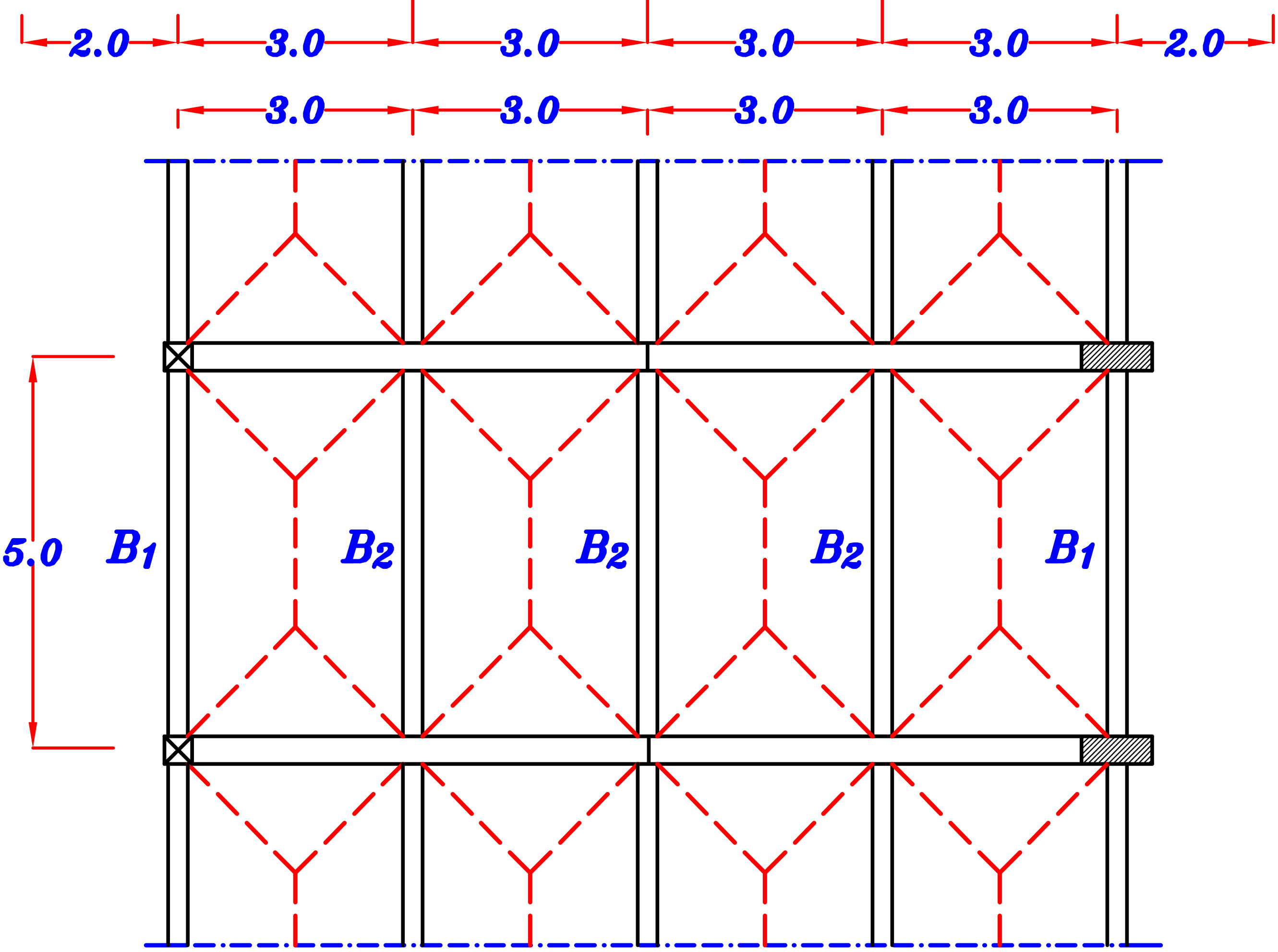
Spacing = 5.0 m



Req.

1. Draw I.F.D. For the Frame (Case of Total Loads only)
2. Check the safety For Sec.(1-1) , Sec.(2-2) (Using Working Method)
3. Calculate F.O.S. For Sec.(1-1) , Sec.(2-2)
4. Draw a sketch illustrate the position of main RFT.





Plan (1-1)

$$w_s = t_s * \gamma_c + F.C. + L.L.$$

$$= 0.12 * 25 + 1.50 + 1.0 = 5.50 \ kN \backslash m^2$$

$$\boxed{w_s = 5.50 \ kN \backslash m^2}$$

B_1

For Trapezoid $C_a = 1 - \frac{1}{2}\left(\frac{L_s}{L}\right) = 1 - \frac{1}{2}\left(\frac{3.0}{5.0}\right) = 0.70$

$w_a = O.W. + C_a\, w_s\, \frac{L_s}{2} = 3.0 + 0.70\,(5.50)\left(\frac{3.0}{2}\right) = 8.77$ kN\m

$R_1 = w_a * \text{Spacing}$

$R_1 = 8.77 * 5.0 = 43.85$ kN

$$\boxed{R_1 = 43.85\ kN}$$

B_2

For Trapezoid $C_a = 0.70$, $C_e = 1 - \frac{1}{3}\left(\frac{L_s}{L}\right)^2 = 1 - \frac{1}{3}\left(\frac{3.0}{5.0}\right)^2 = 0.88$

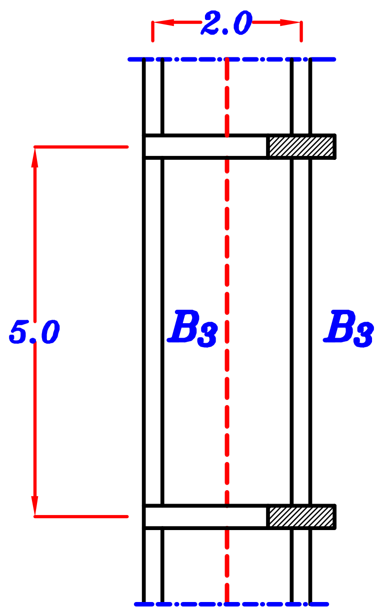
$w_a = O.W. + 2\, C_a\, w_s\, \frac{L_s}{2} = 3.0 + 2\,(0.70)(5.50)\left(\frac{3.0}{2}\right) = 14.55$ kN\m

$R_2 = 14.55 * 5.0 = 72.75$ kN

$$\boxed{R_2 = 72.75\ kN}$$

$w_e = O.W. + 2\, C_e\, w_s\, \frac{L_s}{2} = 3.0 + 2\,(0.88)(5.50)\left(\frac{3.0}{2}\right) = 17.52$ kN\m

B_3



Plan (2-2)

$w_a = O.W. + w_s\, \frac{L_s}{2}$

$= 3.0 + (5.50)\left(\frac{2}{2}\right) = 8.50$ kN\m

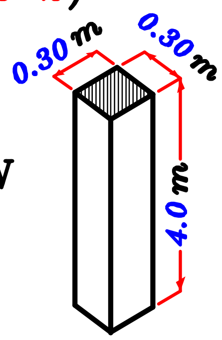
$R_3 = 8.50 * 5.0 = 42.5$ kN

$$\boxed{R_3 = 42.5\ kN}$$

Post (Can be neglected)

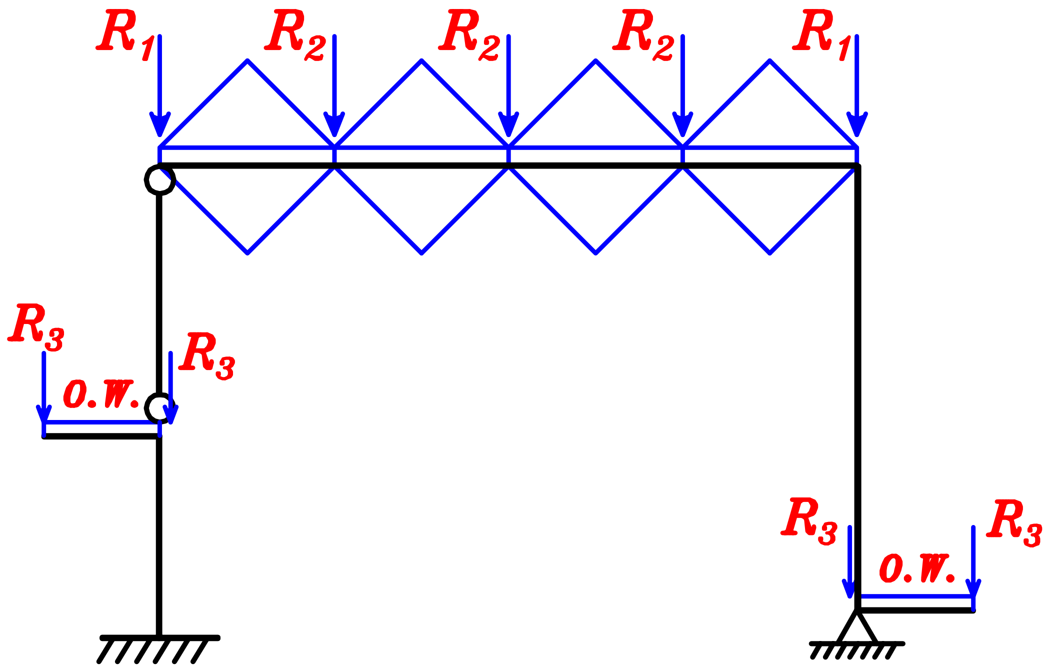
Weight of the Post = Volume * Density

$= (0.30 * 0.30 * 4.0)\,(25) = 9.0$ kN



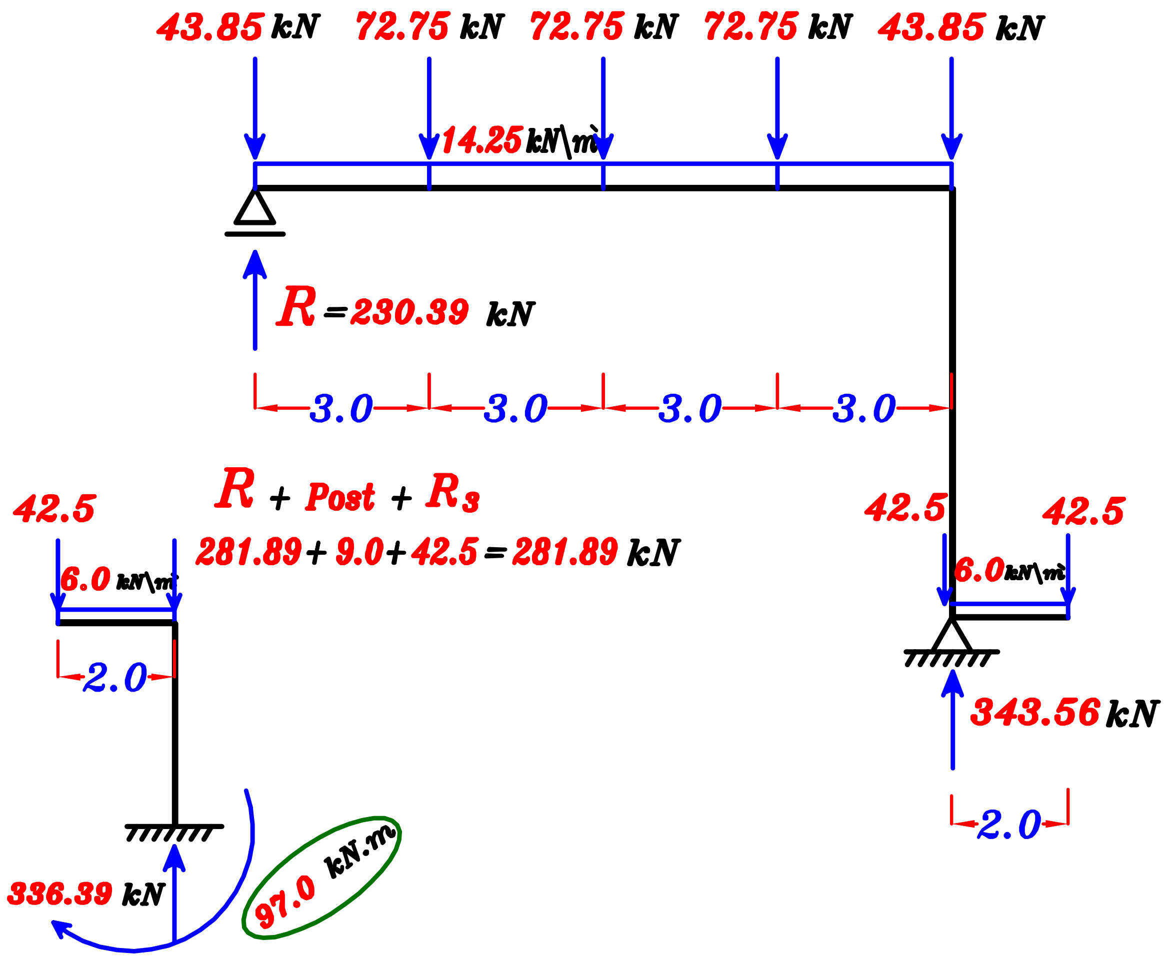
$$\boxed{\text{Weight of the Post} = 9.0\ kN}$$

Loads on the Frame.



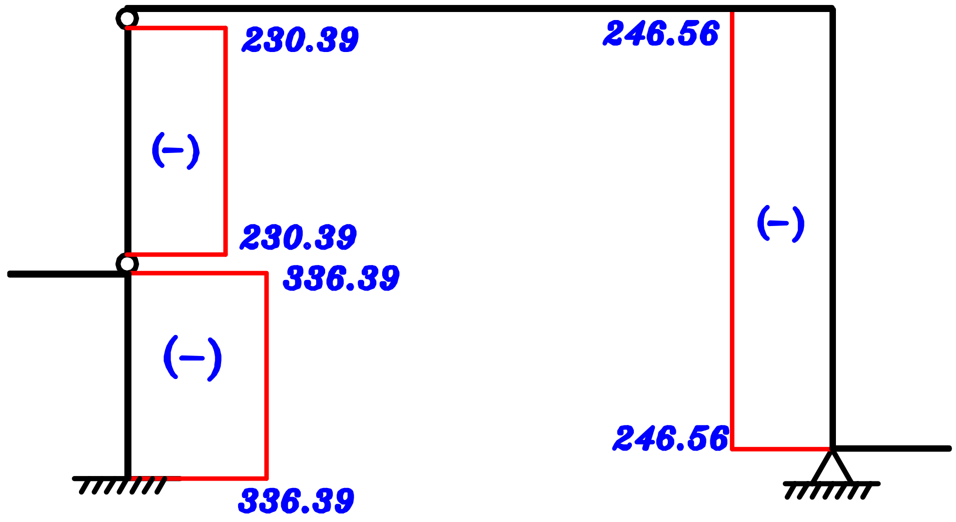
$$\frac{\sum area}{span} = \frac{8\left(\frac{1}{2}(3.0)(1.5)\right)}{12.0} = 1.50$$

$$w_1 = O.W. + \frac{\sum area}{span} * w_s = 6.0 + (1.50)(5.50) = 14.25 \text{ kN}\backslash m`$$

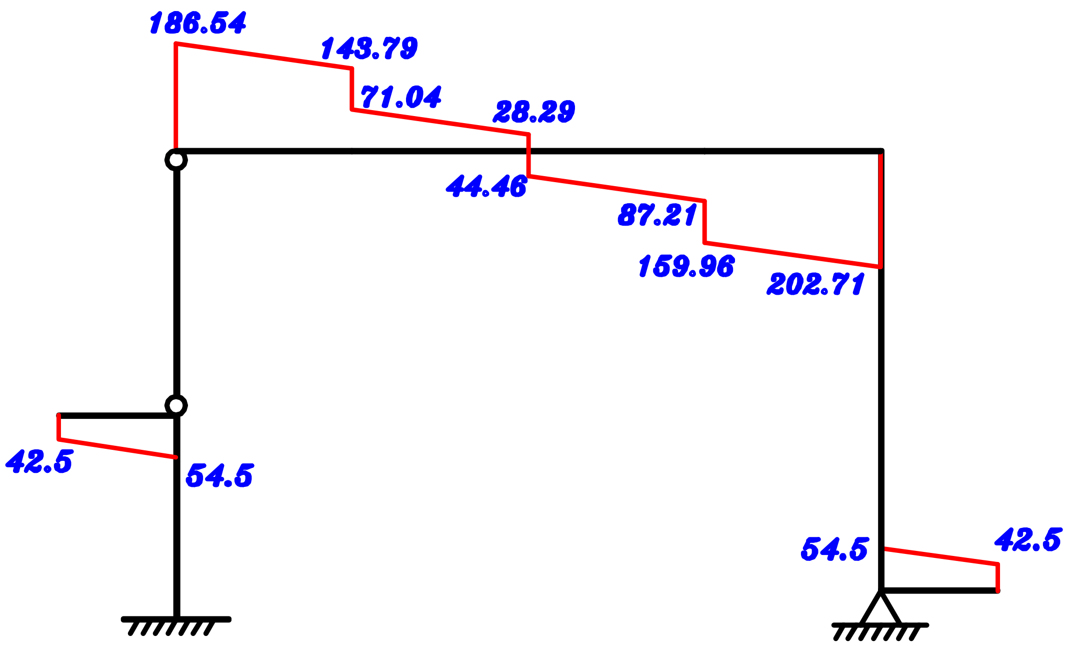


1 I.F.D. For the Frame.

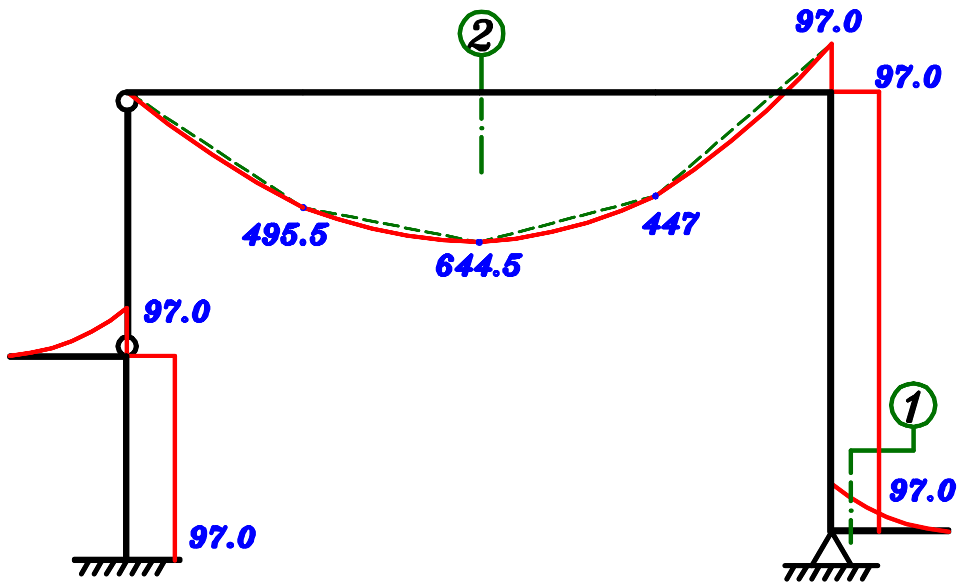
N.F.D.



S.F.D.

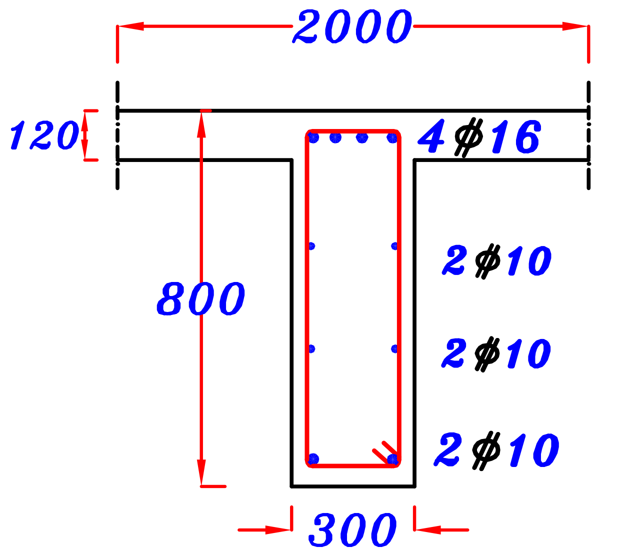


B.M.D.



3 Check the safety For Sec.(1-1) , Sec.(2-2).

Sec.(1-1)



$A_s = 4\phi16 = 804 \quad mm^2$

$A_{s'} = 2\phi10 = 157 \quad mm^2$

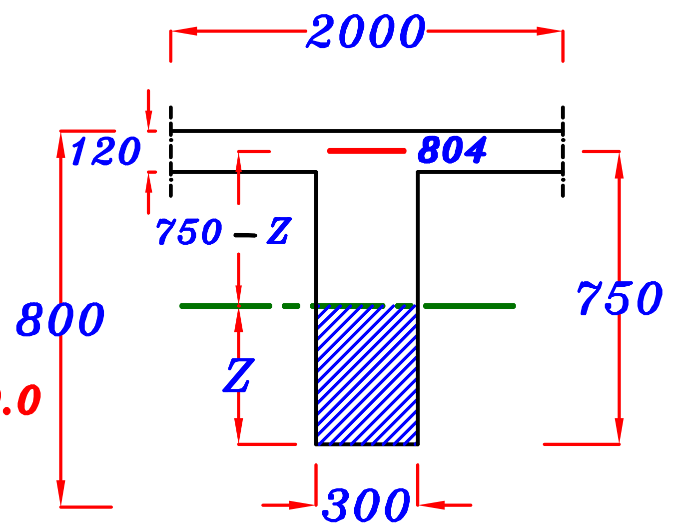
$\therefore$ Neglect $A_{s'}$

① M_w

Allowable stresses

$F_{cu} = 25 \quad N\backslash mm^2 \longrightarrow F_c = 9.5 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2 \longrightarrow F_s = 200 \quad N\backslash mm^2$



① Take $n = 15$

② Get Z by taking $S_{nv.} = 0.0$

$$(300)(Z)\left(\frac{Z}{2}\right) - (15)(804)(750 - Z) = 0.0$$

$$\boxed{Z = 208.63 \ mm}$$

③ Get $I_{nv} = \frac{bZ^3}{3} + n\,A_s(d - Z)^2$

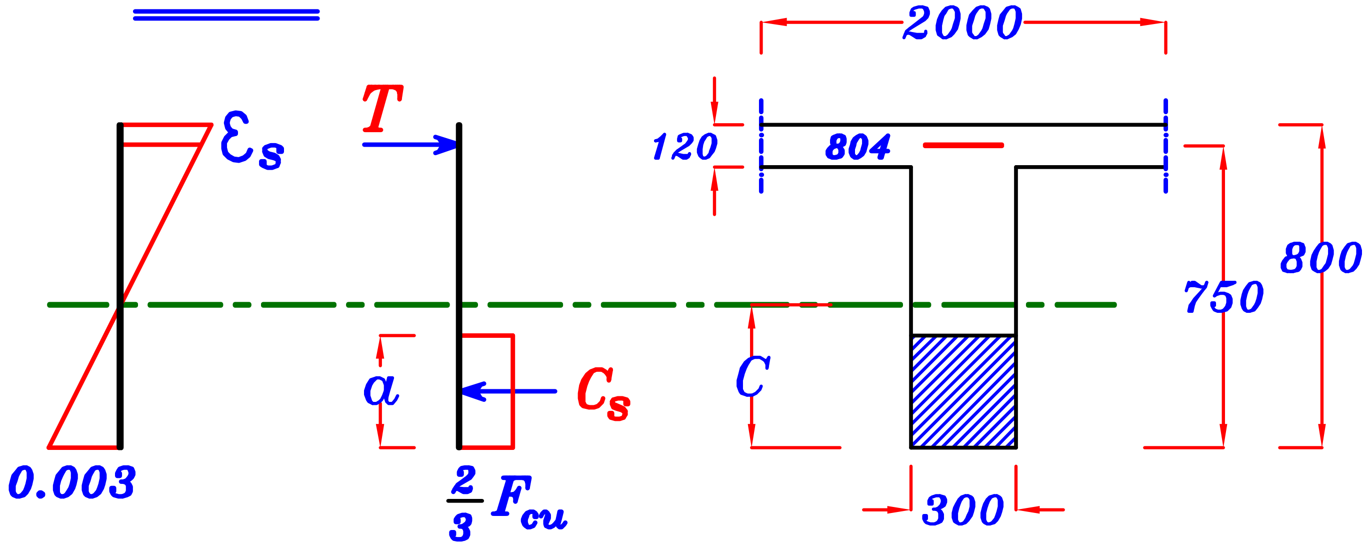
$$I_{nv} = \frac{300\,(208.63)^3}{3} + (15)(804)(750 - 208.63)^2 = 4442655499 \ mm^4$$

④ $$M_{wc} = \frac{F_c * I_{nv}}{Z} = \frac{9.5 * 4442655499}{208.63} = 202297019.8 \ \ N.mm = 202.3 \ \ kN.m$$

⑤ $$M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \frac{\left(\frac{200}{15}\right) * 4442655499}{750 - 208.63} = 109417601 \, N.mm = 109.4 \ \ kN.m$$

⑥ $$\boxed{M_w = 109.4 \quad kN.m}$$

② M_{ult}



Sec. (1-1)

① $$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 750 = 468.75 \text{ mm}$$

② From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * b = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(25)(a)(300) = (804)(360) \longrightarrow a = 57.88 \text{ mm}$$

$$\therefore C = 1.25\, a = 72.36 \text{ mm} < C_b$$

$\therefore$ **The Section is Under Reinforced Sec.**

and the assumption is right $F_s = F_y$

$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right)$$

$$= \frac{2}{3}(25)(57.88)(300)\left(750 - \frac{57.88}{2}\right) = 208674764 \text{ N.mm}$$

$$208.67 \text{ kN.m}$$

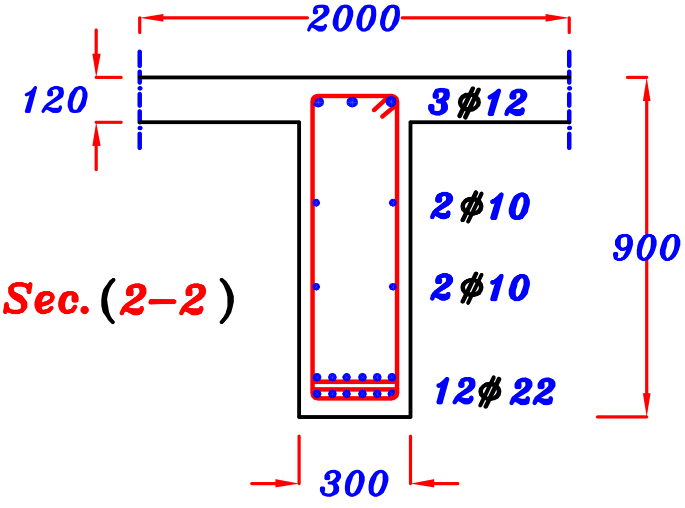
$$\therefore \boxed{M_{ult} = 208.67 \text{ kN.m}}$$

Sec. (2-2)

$A_s = 12\phi22 = 4560\ mm^2$

$A_s` = 3\ \phi12 = 339\ mm^2$

Neglect $A_s`$



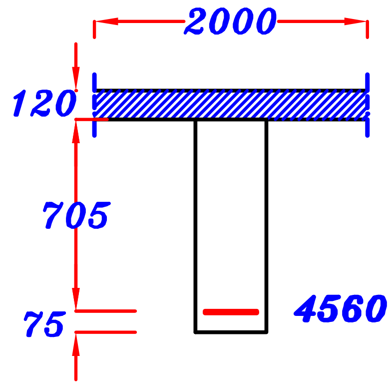
Sec. (2-2)

① $\underline{M_w}$

Allowable stresses

$F_{cu} = 25\ \ N\backslash mm^2 \longrightarrow F_c = 9.5\ \ N\backslash mm^2$

$F_y = 360\ \ N\backslash mm^2 \longrightarrow F_s = 200\ \ N\backslash mm^2$

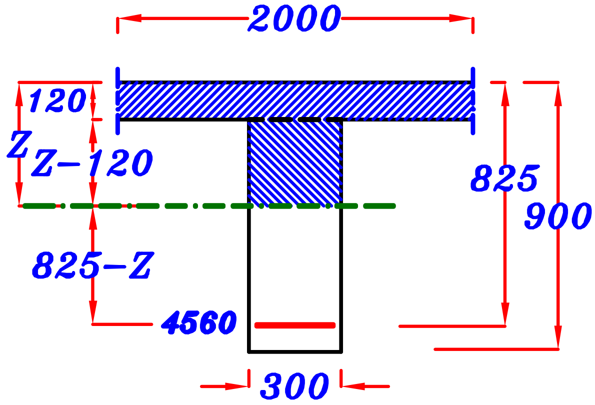


$S_{nv.}(above) = 120 * 2000 * (60) = 14400000\ mm^3$

$S_{nv.}(under) = 15 * 4560 * (705) = 48222000\ mm^3$

$\because\ S_{nv.}(under) > S_{nv.}(above)$

$\therefore\ Z > 120\ mm$



① Take $n = 15$

② Get Z by taking $S_{nv.} = 0.0$

$$(2000)(120)(Z-60) + (300)(Z-120)\left(\frac{Z-120}{2}\right) - (15)(4560)(825-Z) = 0.0$$

$$\boxed{Z = 224.37\ mm}$$

③ $$I_{nv} = \frac{2000(120)^3}{12} + (120)(2000)(224.37-60)^2 + \frac{300(224.37-120)^3}{3} + (15)(4560)(825-224.37)^2$$

$$= 31561628060\ mm^4$$

④ $$M_{wc} = \frac{\left(\frac{2}{3}\right) F_c * I_{nv}}{Z} \quad \text{------} \quad T\_Sec.$$

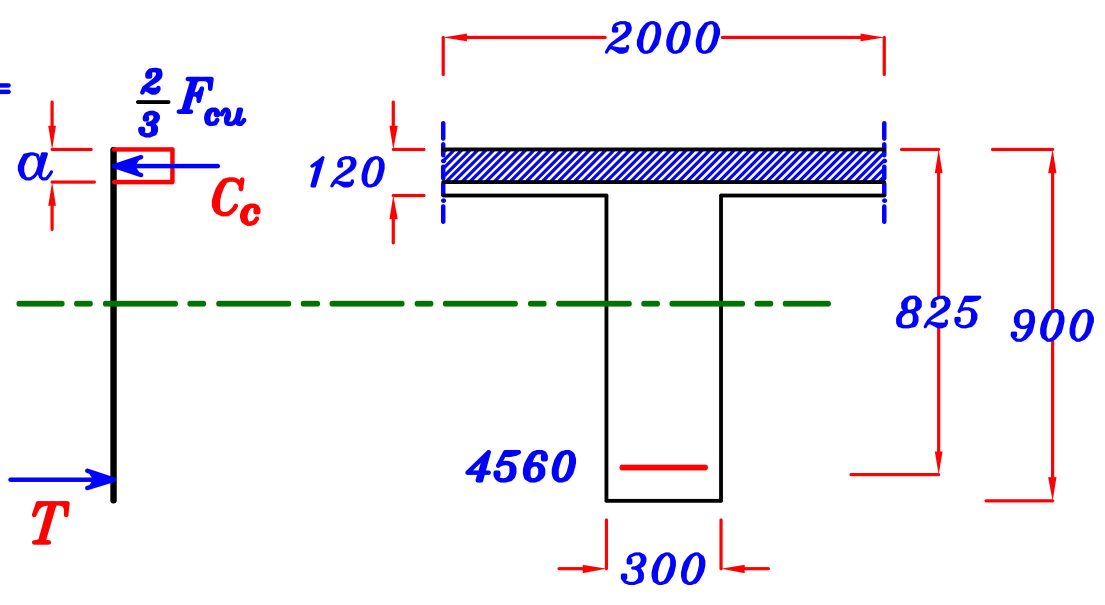
$$= \frac{\left(\frac{2}{3}\right) 9.5 * 31561628060}{224.37} = 890895890\ N.mm = 890.89\ kN.m$$

⑤ $$M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z} = \frac{\left(\frac{200}{15}\right) * 31561628060}{825 - 224.37} = 700633846\ N.mm$$

$$= 700.6\ kN.m$$

⑥ $$\boxed{M_w = 700.6\ kN.m}$$

② M_{ult}



Sec. (2–2)

① $C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 825 = 515.625$ mm

② Assume $a \leqslant t_F$

$a < 120$ mm

③ From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * B = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$\frac{2}{3}(25)(a)(2000) = (4560)(360) \longrightarrow a = 49.25$ mm $< t_s \quad \therefore$ O.K.

$\therefore \; C = 1.25\, a = 61.56$ mm $< C_b$

$\therefore$ **The Section is Under Reinforced Sec.**

and the assumption is right $F_s = F_y$

$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, B \left(d - \frac{a}{2}\right)$$

$$= \frac{2}{3}(25)(49.25)(2000)\left(825 - \frac{49.25}{2}\right) = 1313948958 \text{ N.mm} = 1313.94 \text{ kN.m}$$

$$\therefore \boxed{M_{ult} = 1313.94 \text{ kN.m}}$$

3 Check the safety For Sec.(1−1) , Sec.(2−2).

Sec.(1−1)

Using Working Stress Method

$(M_w)_{act.} = 97.0\ kN.m$ $\quad (M_w)_{all.} = 109.4\ kN.m$

$\because\ (M_w)_{act.} < (M_w)_{all.} \longrightarrow \therefore$ The Sec. is Safe.

Sec.(2−2)

Using Working Stress Method

$(M_w)_{act.} = 644.5\ kN.m$ $\quad (M_w)_{all.} = 700.6\ kN.m$

$\because\ (M_w)_{act.} < (M_w)_{all.} \longrightarrow \therefore$ The Sec. is Safe.

4 F.O.S. For Sec.(1−1) , Sec.(2−2)

Sec.(1−1)

$$F.O.S. = \frac{(M_{ult})}{(M_w)_{act.}} = \frac{208.67}{97.0} = 2.15$$

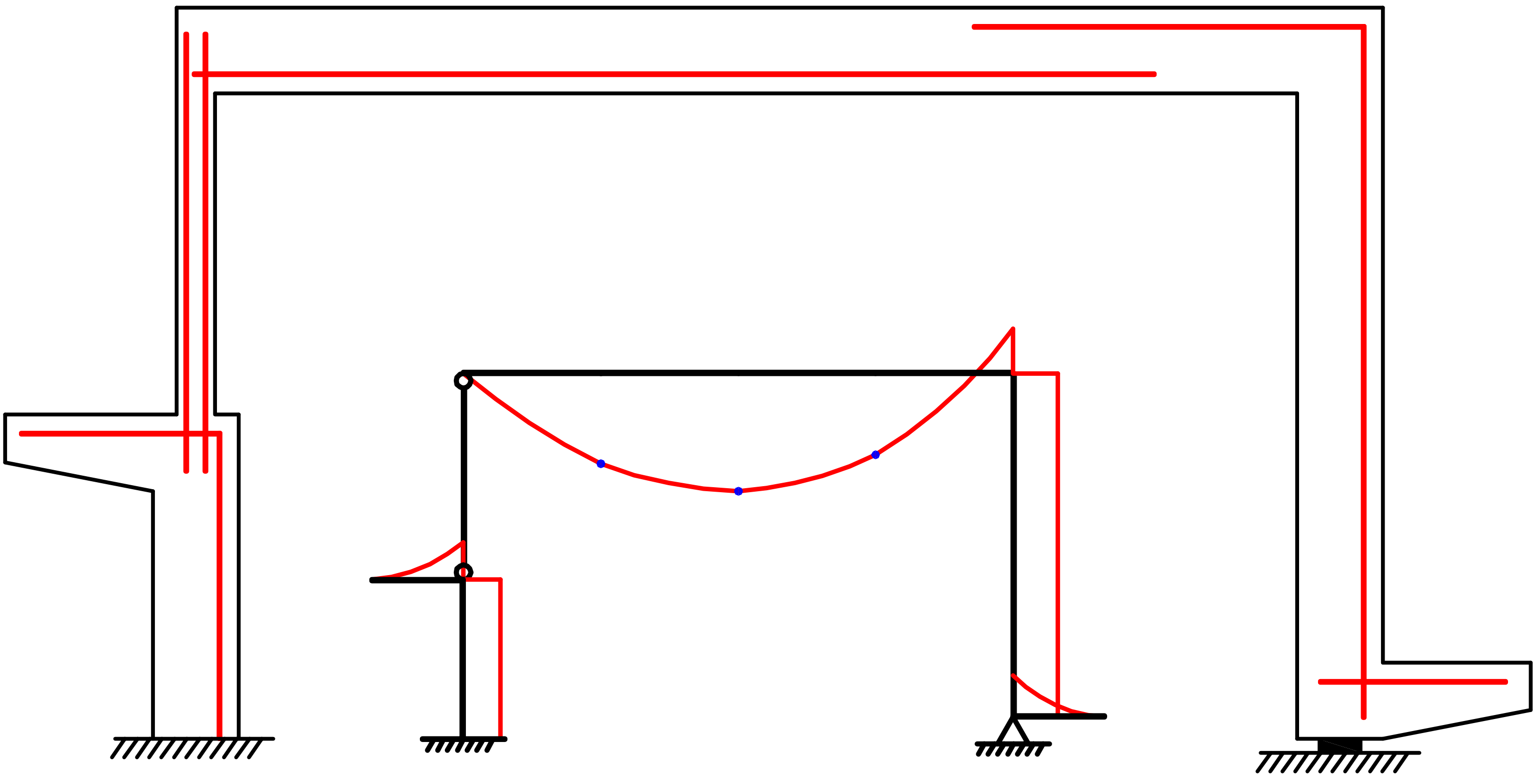
$$\boxed{F.O.S. = 2.15}$$

Sec.(2−2)

$$F.O.S. = \frac{(M_{ult})}{(M_w)_{act.}} = \frac{1313.94}{644.5} = 2.04$$

$$\boxed{F.O.S. = 2.04}$$

5 Sketch illustrate the Main RFT. For the Frame.



moment مكان التسليح الرئيسى يكون دائما جهه ال

Example.

Figure 1 shows different **RC** sections. It is required to arrange them in an ascending order according to the value of **Cracking Moment** and once again according to the value of the **Ultimate Moment.**

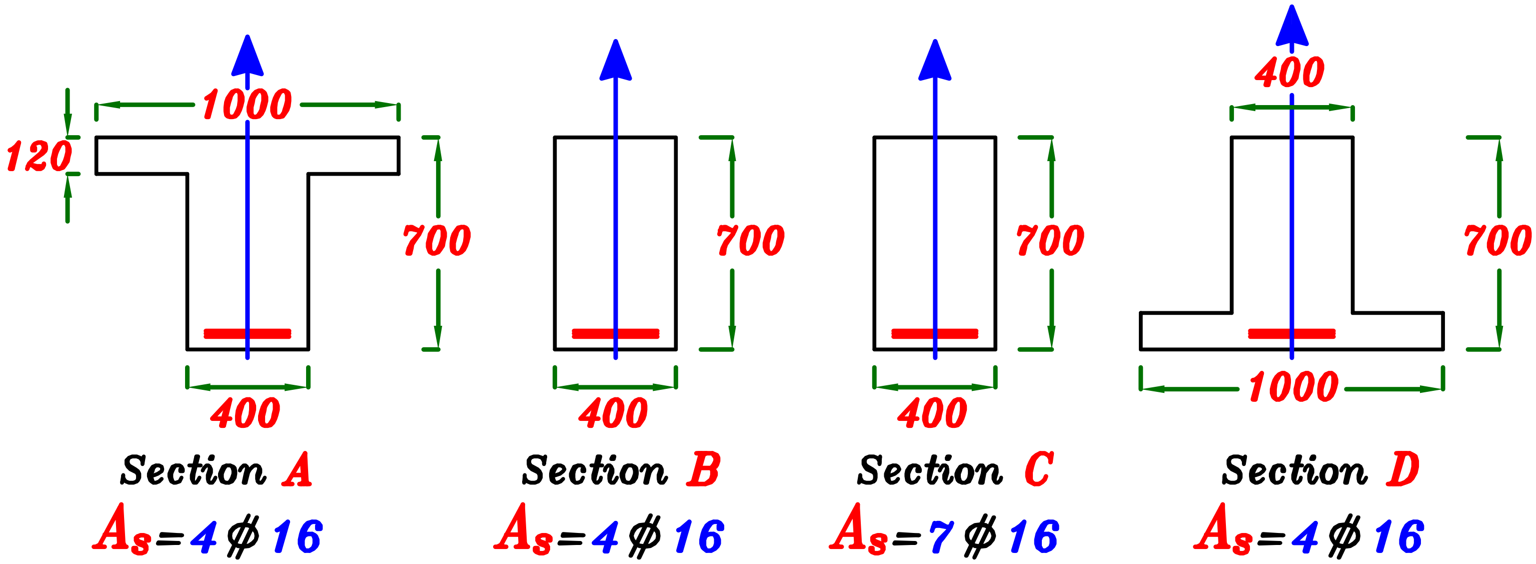


Figure 1

ملحوظه هامه

لم يطلب حساب قيم $M_{cr.}$ و M_{ult} بل طلب فقط ترتيبهم من الاصغر للاكبر

لذا لن نحتاج لعمل اى حسابات.

For Cracking Moment.

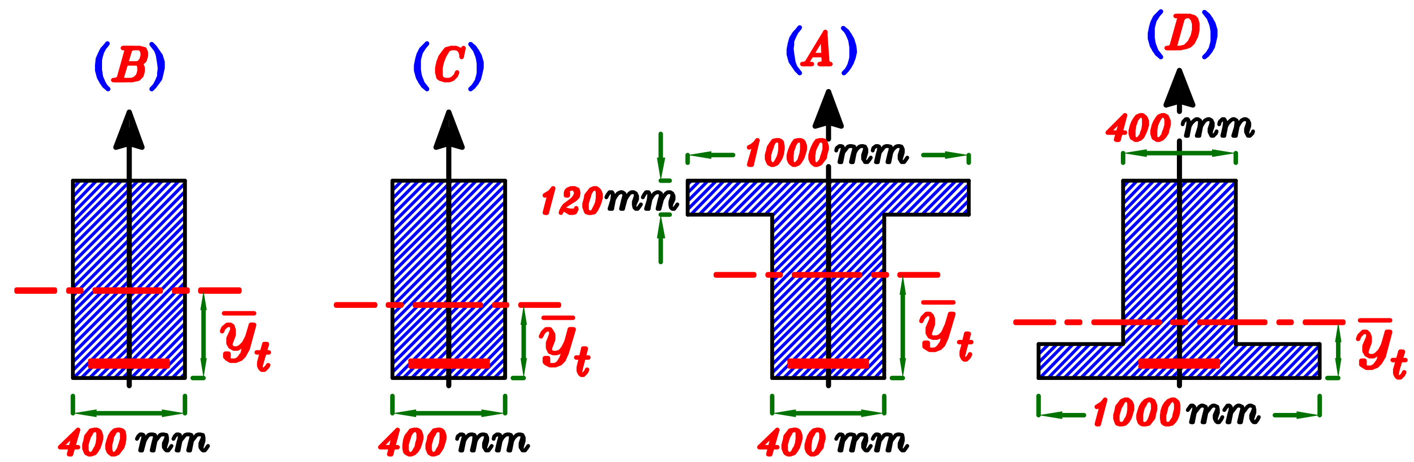
$$M_{cr.} = \frac{F_{ctr} * I_g}{\bar{y}_t}$$

F_{ctr} قيمتها ثابته

I_g كلما زادت تزيد $M_{cr.}$

$\bar{y}_t$ كلما زادت تقل $M_{cr.}$

الفرق فى $M_{cr.}$ يكون صغير عند اختلاف $\bar{y}_t$ لانها تحسب مساحه فى مسافه
الفرق فى $M_{cr.}$ يكون كبير عند اختلاف I_g لانها تحسب مساحه فى مربع المسافه



$A_s = 4 \phi 16$ | $A_s = 7 \phi 16$ | $A_s = 4 \phi 16$ | $A_s = 4 \phi 16$

$$M_{cr.}(B) < M_{cr.}(C) < M_{cr.}(A) < M_{cr.}(D)$$

الاسباب

فرق A_s

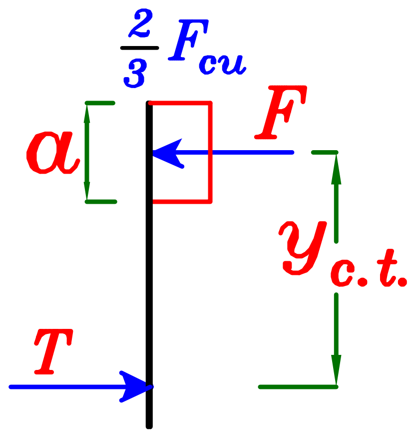
$M_{cr.}(B) < M_{cr.}(C) \xrightarrow{\text{Cause}}$ $I_g(B) < I_g(C)$, $\bar{y}_t(B) > \bar{y}_t(C)$

$M_{cr.}(C) < M_{cr.}(A) \xrightarrow{\text{Cause}}$ $I_g(C) < I_g(A)$, $\bar{y}_t(C) < \bar{y}_t(A)$

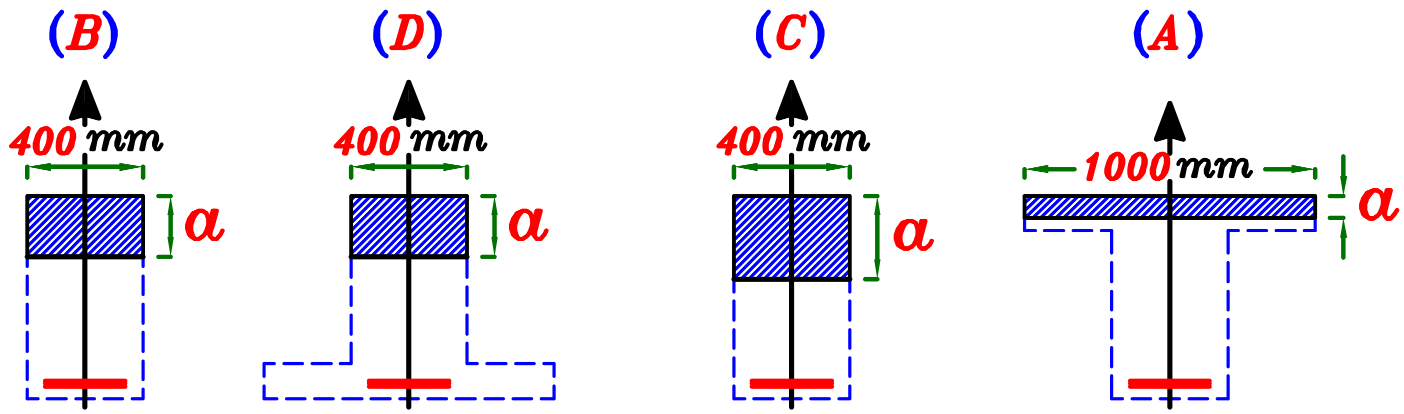
$M_{cr.}(A) < M_{cr.}(D) \xrightarrow{\text{Cause}}$ $\bar{y}_t(A) > \bar{y}_t(D)$, $I_g(A) \simeq I_g(D)$

For Ultimate Moment.

$$\boxed{F = \frac{2}{3} F_{cu} * a * b}$$



$$\boxed{\begin{aligned} M_{ult} &= Force * Distance \\ M_{ult} &= F * y_{c.t.} \end{aligned}}$$



$A_s = 4\phi 16$ $\quad$ $A_s = 4\phi 16$ $\quad$ $A_s = 7\phi 16$ $\quad$ $A_s = 4\phi 16$

$$\boxed{M_{ult}(B) = M_{ult}(D) < M_{ult}(C) < M_{ult}(A)}$$

الاسباب

$$M_{ult}(B) = M_{ult}(D) \xrightarrow{Cause} F(B) = F(D)$$

$$y_{c.t.}(B) = y_{c.t.}(D)$$

$$M_{ult}(B,D) < M_{ult}(C) \xrightarrow{Cause} F(B) = F(D) < F(C)$$

لان $A_s(C)$ أكبر ← $a\,(C)$ أكبر ← $F\,(C)$ أكبر

$$M_{ult}(C) < M_{ult}(A) \xrightarrow{Cause} F(C) < F(A)$$ غالباً

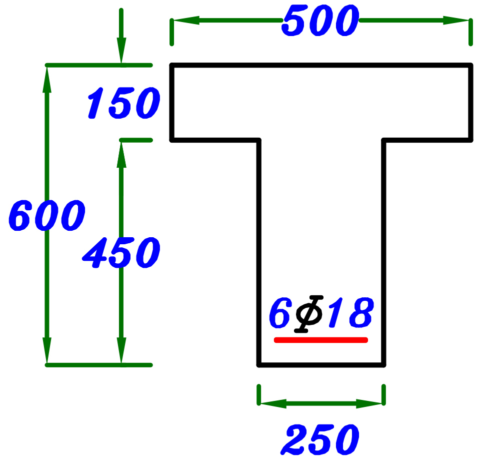
لان غالبا $b\,(A)$ أكبر ← $F\,(A)$ أكبر

Example.

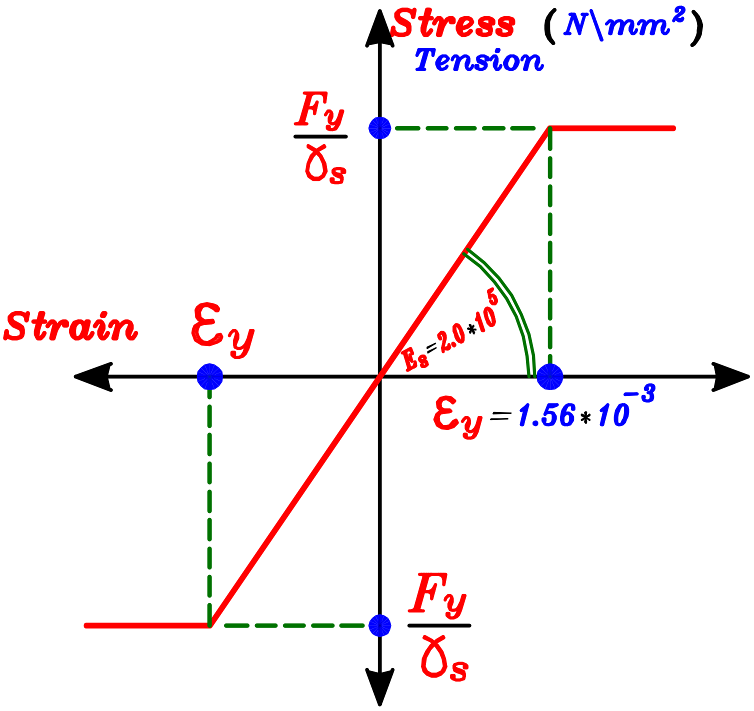
Use the data in the given Idealized Stress–Strain Curves For concrete and steel.

to calculate C_b , C_{max} , a_{max} , $M_{U.L.}$

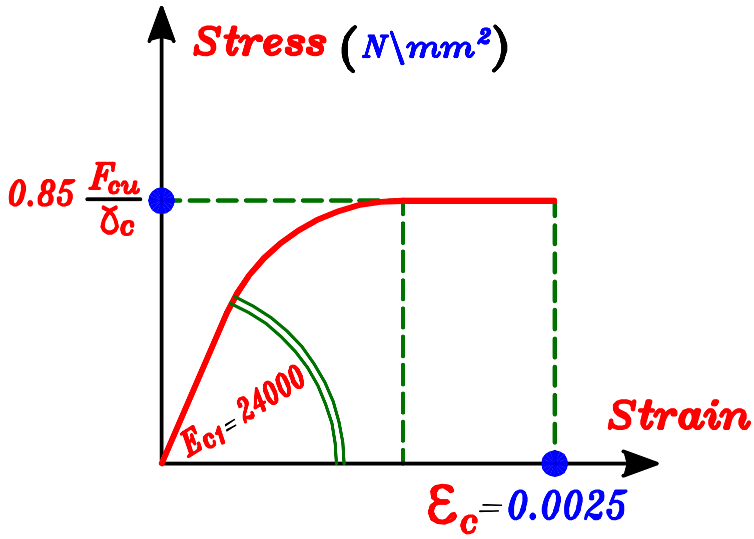
For the given section.



$F_{cu} = 25 \; N \backslash mm^2$



Idealized Stress–Strain Curve For Steel.



Idealized Stress–Strain Curve For Concrete.

Solution.

From Curves $\varepsilon_c = 0.0025 \xrightarrow{\text{بدلاً من}} \varepsilon_c = 0.003$

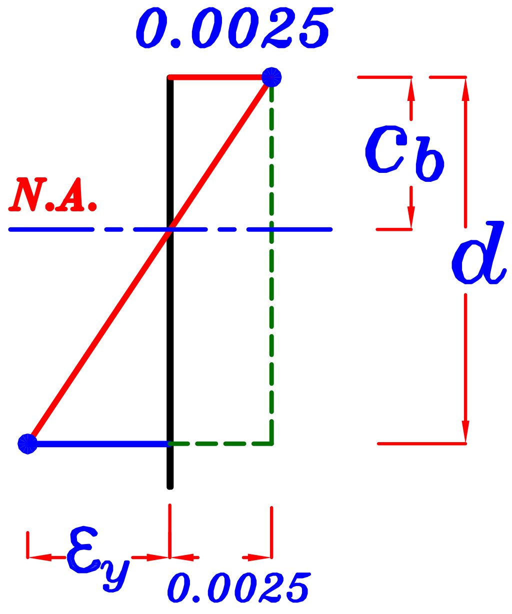
max concrete stress $= 0.85 \dfrac{F_{cu}}{\gamma_c} \xrightarrow{\text{بدلاً من}} \dfrac{2}{3} \dfrac{F_{cu}}{\gamma_c}$

max steel stress $= \dfrac{F_y}{\gamma_s} = \varepsilon_y * E_s = 1.56 * 10^{-3} * 2.0 * 10^{5} = 312 \; N \backslash mm^2$

من تشابه المثلثات

$$\frac{c_b}{d} = \frac{0.0025}{0.0025 + \varepsilon_y}$$

$$\frac{c_b}{d} = \frac{0.0025}{0.0025 + 1.56 * 10^{-3}} = 0.615$$



$$\therefore \boxed{c_b = 0.615\ d}$$

$$\therefore \boxed{c_{max} = \frac{2}{3} c_b = 0.41\ d}$$

$$\boxed{a_{max} = 0.8\ c_{max} = 0.328\ d}$$

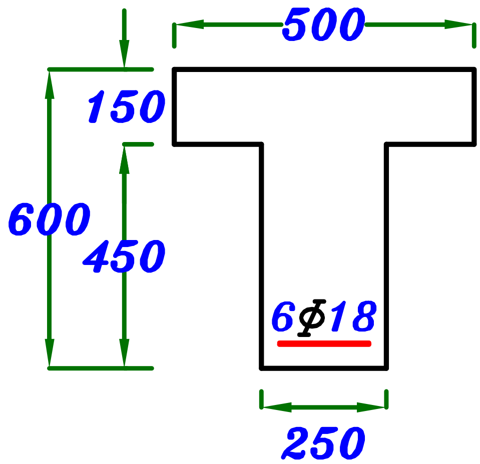
$A_s = 6\phi18 = 1526\ mm^2$

$d = 550\ mm$

$a_{min} = 0.1\ d = 55.0\ mm$

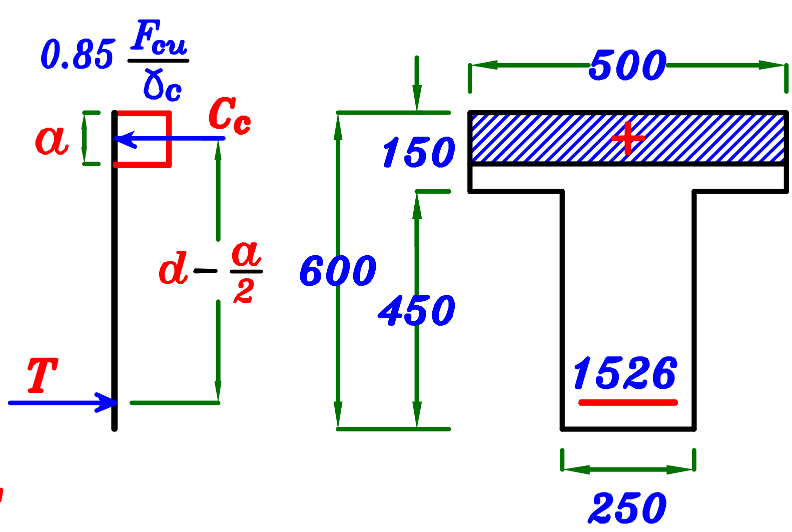
$a_{max} = 0.328\ d = 0.328 * 550$

$= 180.4\ mm$



assume $a \leqslant t_s$

$a < 150$ mm



From equilibrium eqn. $C_c = T$

$$0.85\,\frac{F_{cu}}{\gamma_c} * a * B = F_s * A_s \text{ ----- } a\,,\,F_s$$

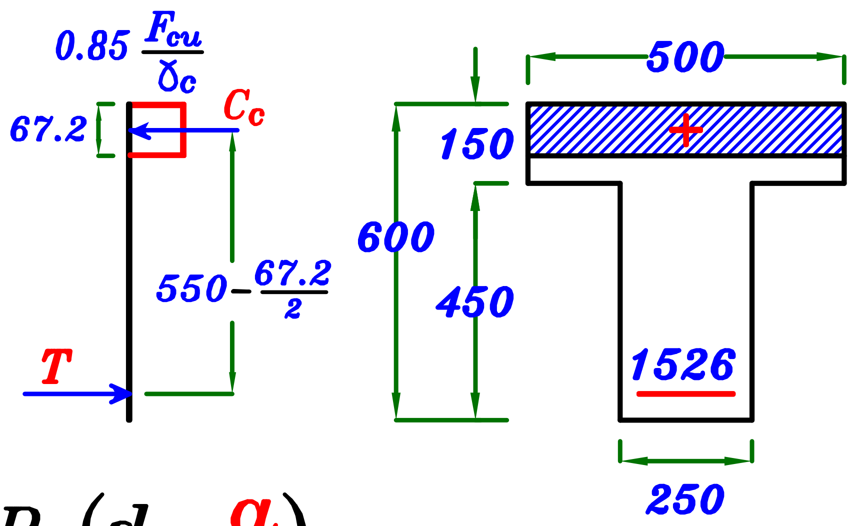
assume $F_s = \frac{F_y}{\gamma_s} = 312$ $N\backslash mm^2$ (Under reinforced Sec.)

$$\therefore\ 0.85\left(\frac{25}{1.5}\right)(a)(500) = (312)(1526) \longrightarrow a = 67.2 \text{ mm}$$

$$\therefore\ a = 67.2 \text{ mm} < t_s \quad \therefore \text{ o.k.}$$

$$a_{min} < a < a_{max} \quad \therefore \text{ o.k.}$$

$$M_{U.L.} = C_c * \left(d - \frac{a}{2}\right)$$



$$M_{U.L.} = 0.85\,\frac{F_{cu}}{\gamma_c} * a * B\,\left(d - \frac{a}{2}\right)$$

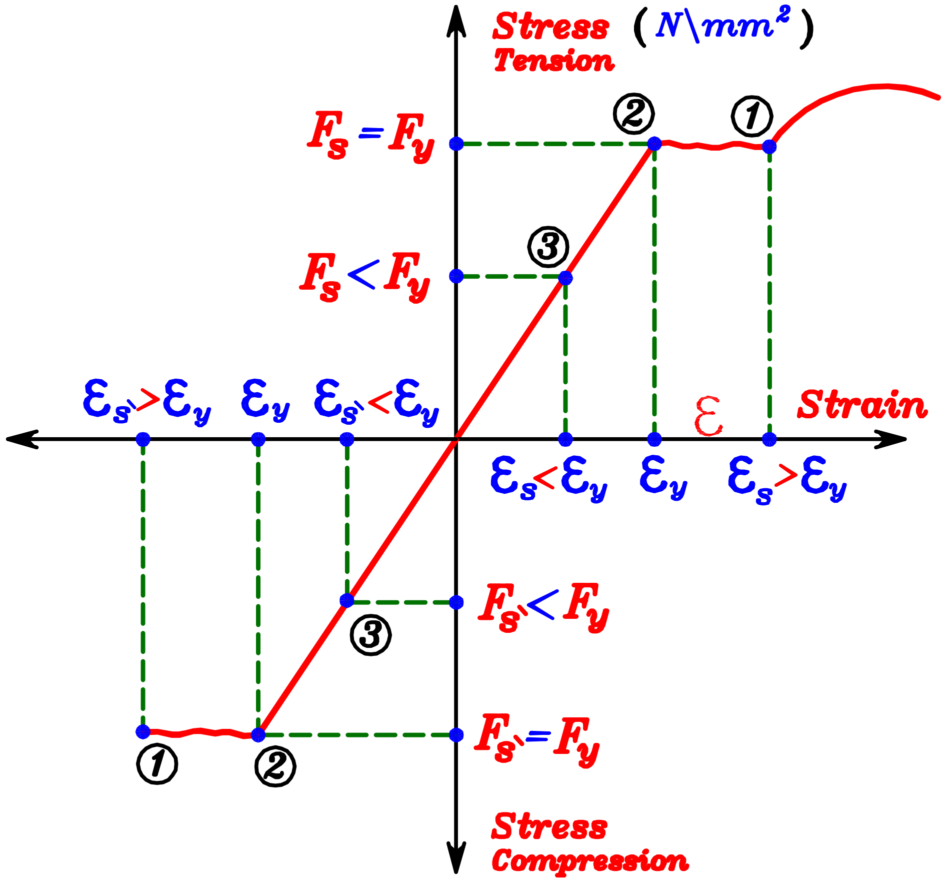
$$M_{U.L.} = 0.85\left(\frac{25}{1.5}\right)(67.2)(500)\left(550 - \frac{67.2}{2}\right) = 245806400 \text{ N.mm}$$

$$= 245.8 \text{ kN.m}$$

$$\boxed{M_{U.L.} = 245.8 \text{ kN.m}}$$

شكل الـ Sterss–strain curve للحديد فى الـ Compression

هو نفس شكل الـ Sterss–strain curve للحديد فى الـ Tension



max. stress (Concrete) = F_{cu}

max. stress (Tension Steel) = F_y

max. stress (Compression Steel) = F_y

max. strain (Concrete) = $\varepsilon_c = 0.003$

strain at yield (Tension Steel) $\varepsilon_s = \varepsilon_y = \dfrac{F_y}{E_s} = \dfrac{F_y}{2*10^5}$

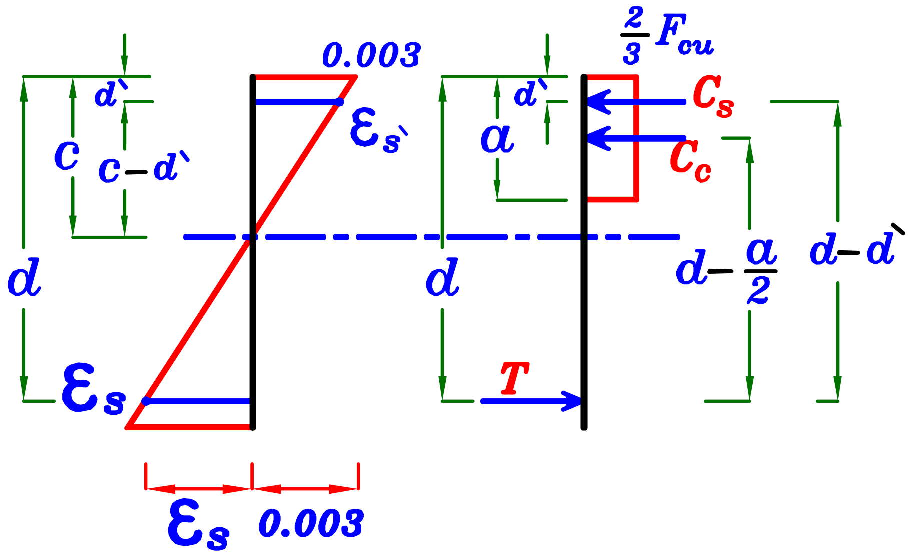
strain at yield (Compression Steel) $\varepsilon_{s`} = \varepsilon_y = \dfrac{F_y}{E_s} = \dfrac{F_y}{2*10^5}$

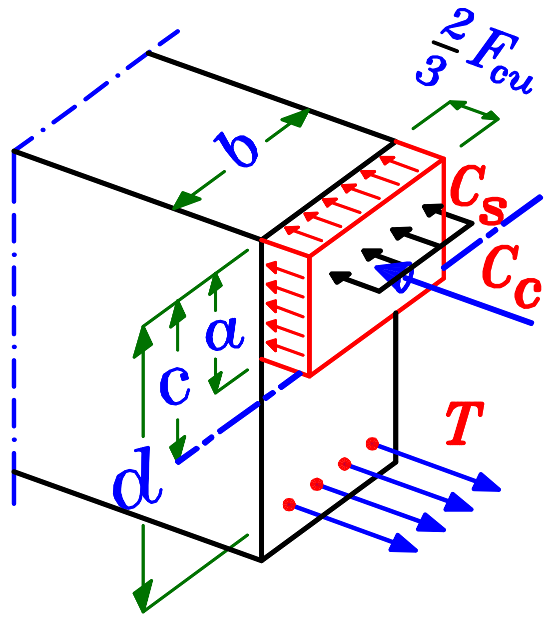
Note.

When $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$

When $\varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = F_y$

IF there is compression steel.





Compression Force.

$$C_c = \frac{2}{3} F_{cu} * (a * b)$$

$$C_s = A_{s`} * F_{s`}$$

Tension Force.

$$T = A_s * F_s$$

When

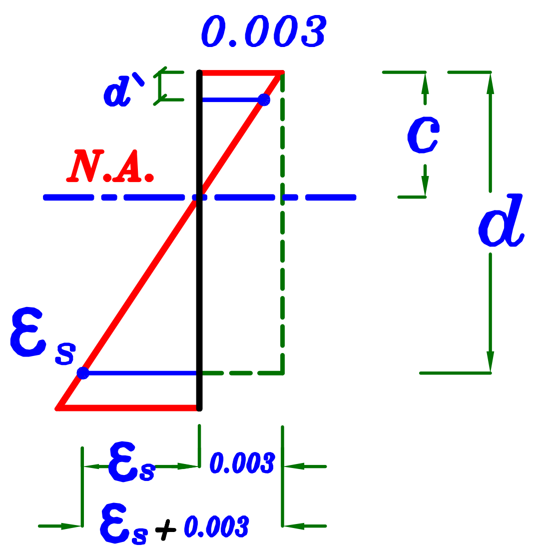
$$\varepsilon_s \geq \varepsilon_y \longrightarrow F_s = F_y$$

$$\varepsilon_{s`} \geq \varepsilon_y \longrightarrow F_{s`} = F_y$$

Equilibrium Equation.

$$C_c + C_s = T$$

$$\boxed{\frac{2}{3} F_{cu} * (a * b) + A_{s`} * F_{s`} = A_s * F_s}$$

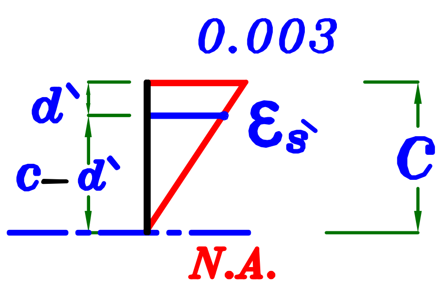


Compatibility Equations.

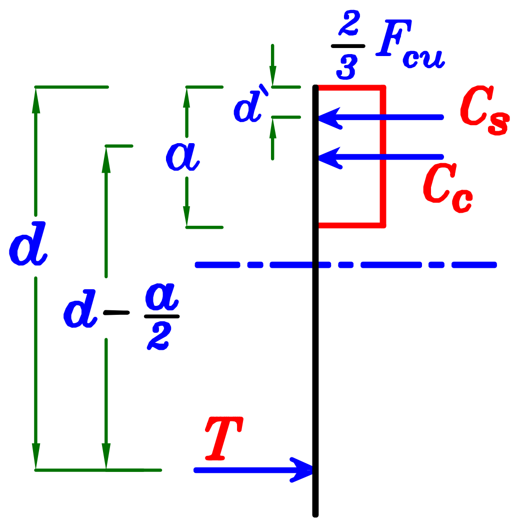
$$\boxed{C = 1.25\,a = \frac{600}{600 + F_s} * d}$$

$$\frac{\varepsilon_{s`}}{0.003} = \frac{c-d`}{c} \qquad \because \varepsilon_{s`} = \frac{F_{s`}}{2*10^5}$$

$$\boxed{\frac{F_{s`}}{600} = \frac{c-d`}{c} = \frac{1.25\,a - d`}{1.25\,a}}$$



Steps to determine M_{ult}



① Get $C_b = \dfrac{600}{600 + F_y} * d$

② Use equilibrium eqn. $C_c + C_s = T$

$$\frac{2}{3}F_{cu} * a * b + A_{s`} * F_{s`} = A_s * F_s \quad \text{---} \quad a, F_s, F_{s`} = ??$$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$ (Under reinforced or Balanced Sec.)

assume $\varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = F_y$ Where $\varepsilon_y = \dfrac{F_y}{E_s} = \dfrac{F_y}{2*10^5}$

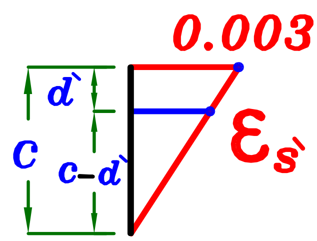
$$\therefore \frac{2}{3}F_{cu} * a * b + A_{s`} * F_y = A_s * F_y \longrightarrow \text{Get } a \longrightarrow \text{Get } C = 1.25\,a$$

* IF $C \leqslant C_b$

$\therefore$ The Section is Under reinforced or Balanced Sec. $\therefore F_s = F_y$

To check the second assumption $F_{s`} = F_y$

$$\frac{\varepsilon_{s`}}{0.003} = \frac{C - d`}{C} \quad \text{get } \varepsilon_{s`}$$

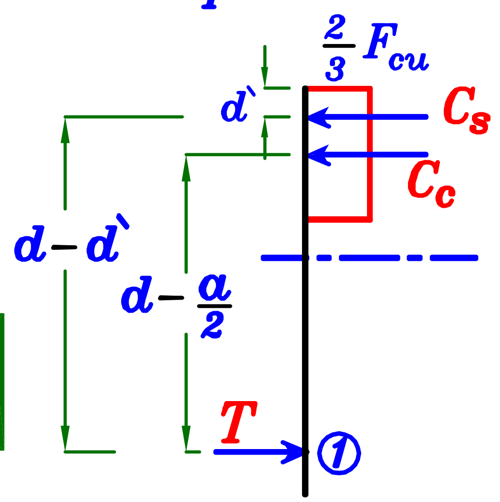


$$\text{Get } \varepsilon_y = \frac{F_y}{E_s} = \frac{F_y}{2*10^5}$$

– IF $\varepsilon_{s`} \geqslant \varepsilon_y$ $\therefore F_{s`} = F_y$ right assumption

$\therefore C_s = A_{s`} F_y$

$, C_c = \frac{2}{3} F_{cu}\, a\, b$



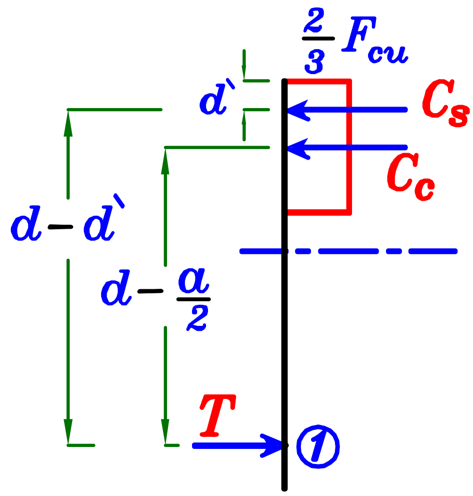
$$\therefore \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s`} F_y \left(d - d`\right)}$$

$- \ \varepsilon_{s`} < \varepsilon_y \quad \therefore F_{s`} < F_y$ *wrong assumption*

$\therefore$ *To get The right value of* $a \ , F_{s`}$

$$\frac{2}{3} F_{cu} * a * b + A_{s`} * F_{s`} = A_s * F_y \qquad \underline{a \ , F_{s`}} \ (1)$$

$$\frac{F_{s`}}{600} = \frac{1.25\,a - d`}{1.25\,a} \qquad \underline{a \ , F_{s`}} \ (2)$$



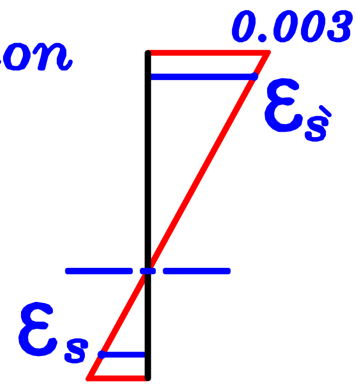
From eqns. (1) , (2) *Get* $a \ , F_{s`}$

$$\therefore \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s`} F_{s`} (d - d`)}$$

$*$ *IF* $C > C_b$

$\therefore$ *The Section is Over reinforced Sec.*

$\therefore \varepsilon_s < \varepsilon_y \quad \therefore F_s < F_y$ *wrong assumption*



IF $\boxed{\varepsilon_s < \varepsilon_y \longrightarrow \varepsilon_{s`} > \varepsilon_y}$

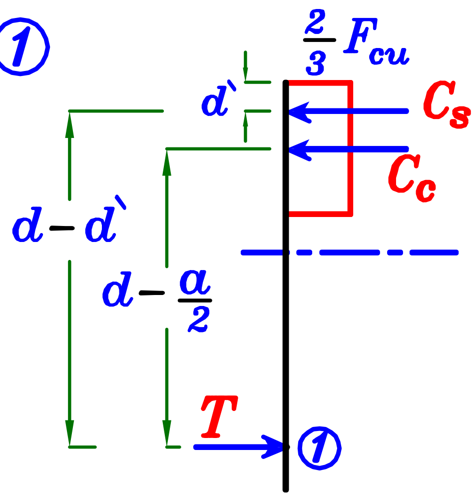
$\because \varepsilon_{s`} > \varepsilon_y \quad \therefore F_{s`} = F_y$

$\therefore$ *To get The right value of* $a \ , F_s$

ε

$$\frac{2}{3} F_{cu} * a * b + A_{s`} * F_y = A_s * F_s \qquad \underline{a \ , F_s} \ (1)$$

$$C = 1.25\,a = \frac{600}{600 + F_s} * d \qquad \underline{a \ , F_s} \ (2)$$



From eqns. (1) , (2) *Get* $a \ , F_s$

$$\therefore \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s`} F_y (d - d`)}$$

To Calculate M_{ult} (With Ten. & comp. Steel)

Get $C_b = \dfrac{600}{600 + F_y} * d$

From equilibrium eqn. $\frac{2}{3} F_{cu} * (a * b) + A_{s`} * F_{s`} = A_s * F_s$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$ (The section is under reinforced or Balanced Sec.)

assume $\varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = F_y$ Where $\varepsilon_y = \dfrac{F_y}{E_s} = \dfrac{F_y}{2*10^5}$

$\therefore \frac{2}{3} F_{cu} * (a * b) + A_{s`} * F_y = A_s * F_y \longrightarrow$ Get $a \longrightarrow$ Get $C = 1.25\, a$

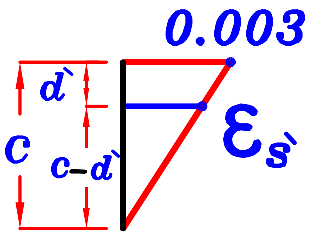
IF $\boxed{C}$

IF $C \leqslant C_b$

Under or Balanced Sec. $\therefore F_s = F_y$

To check $F_{s`} = F_y$

From $\dfrac{\varepsilon_{s`}}{0.003} = \dfrac{C - d`}{C}$ get $\varepsilon_{s`}$



$\varepsilon_{s`} \geqslant \varepsilon_y$

$\therefore F_{s`} = F_y$

$$\therefore \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s`} F_y (d - d`)}$$

$\varepsilon_{s`} < \varepsilon_y$ $\therefore F_{s`} < F_y$

to get a , $F_{s`}$

$$\frac{2}{3} F_{cu} * a * b + A_{s`} * F_{s`} = A_s * F_y \qquad a \text{ , } F_{s`} \quad (1)$$

$$\frac{F_{s`}}{600} = \frac{1.25\, a - d`}{1.25\, a} \qquad a \text{ , } F_{s`} \quad (2)$$

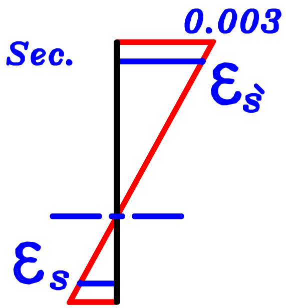
$$\therefore \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s`} F_{s`} (d - d`)}$$

IF $C > C_b$

Over Reinforced Sec.

$\varepsilon_s < \varepsilon_y \longrightarrow F_s < F_y$

$\therefore \varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = F_y$



$$\frac{2}{3} F_{cu} * a * b + A_{s`} * F_y = A_s * F_s \qquad a \text{ , } F_s \quad (1)$$

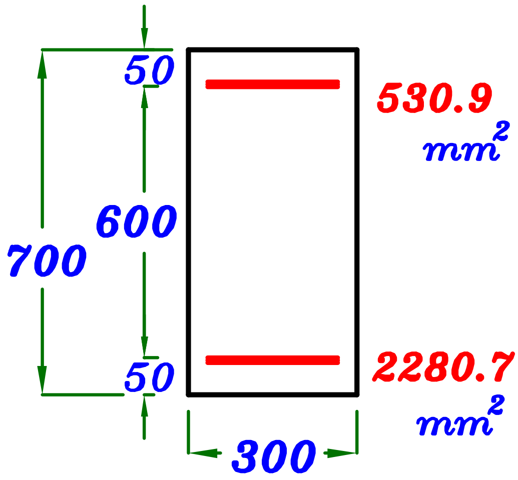
$$C = 1.25\, a = \frac{600}{600 + F_s} * d \qquad a \text{ , } F_s \quad (2)$$

From (1) , (2) get a , F_s

$$\therefore \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s`} F_y (d - d`)}$$

Example.

Data.



$F_{cu} = 25 \ N\backslash mm^2$, st. 360/520

Req.

Calculate M_{ult}.

Solution.

$d = 650 \ mm$, $d` = 50 \ mm$

① $$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 650 = 406.25 \ mm$$

② From equilibrium eqn. $C_c + C_s = T$

$$\frac{2}{3} F_{cu} * (a * b) + A_{s`} * F_{s`} = A_s * F_s$$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$ (Under or Balanced Sec.)

assume $\varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = F_y$

$$\frac{2}{3} (25)(a)(300) + (530.9)(360) = (2280.7)(360)$$

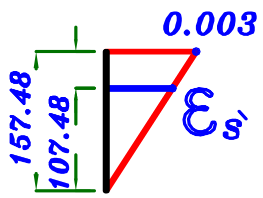
$\longrightarrow a = 125.98 \ mm \longrightarrow C = 1.25 \ a = 157.48 \ mm < C_b$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

and the First assumption is right $F_s = F_y$

To check if the second assumption is right or wrong. $F_{s`} = F_y$

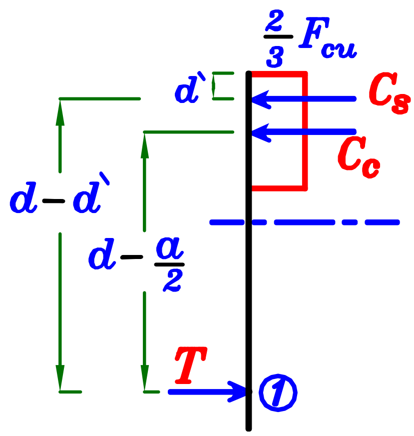
Get $\varepsilon_y = \frac{F_y}{2*10^5} = \frac{360}{2*10^5} = 1.8 * 10^{-3}$



From $\frac{\varepsilon_{s`}}{0.003} = \frac{C - d`}{C} = \frac{107.48}{157.48} \longrightarrow \varepsilon_{s`} = 2.047 * 10^{-3}$

$\therefore \varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = F_y$ $\therefore$ The second assumption is right.

$$\therefore M_{ult} = \frac{2}{3} F_{cu} \, a \, b \left(d - \frac{a}{2}\right) + A_{s`} F_y (d - d`)$$



$$= \frac{2}{3}(25)(125.98)(300)\left(650 - \frac{125.98}{2}\right) + (530.9)(360)(650 - 50)$$

$= 484431999 \ N.mm = 484.43 \ kN.m$

$\therefore$ $\boxed{M_{ult} = 484.43 \ kN.m}$

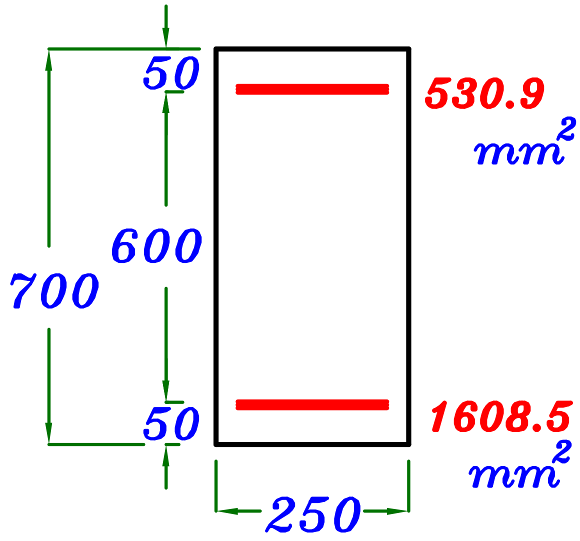
Example.

Data.

$F_{cu} = 25 \ N\backslash mm^2$

st. 360/520

Req. Calculate $M_{ult.}$



Solution.

$d = 650 \ mm \ , \ d` = 50 \ mm$

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 650 = 406.25 \ mm$

② From equilibrium eqn. $C_c + C_s = T$

$$\frac{2}{3} F_{cu} * (a * b) + A_{s`} * F_{s`} = A_s * F_s$$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$ (Under or Balanced Sec.)

assume $\varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = F_y$

$$\frac{2}{3} (25) (a) (250) + (530.9) (360) = (1608.5) (360)$$

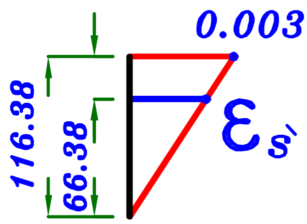
$\longrightarrow a = 93.1 \ mm \longrightarrow C = 1.25 \ a = 116.38 \ mm < C_b$

$\therefore$ **The Section is Under Reinforced Sec.**

and the First assumption is right $F_s = F_y$

To check if the second assumption is right or wrong. $F_{s`} = F_y$

Get $\varepsilon_y = \dfrac{F_y}{2 * 10^5} = \dfrac{360}{2 * 10^5} = 1.8 * 10^{-3}$



From $\dfrac{\varepsilon_{s`}}{0.003} = \dfrac{C - d`}{C} = \dfrac{66.38}{116.38} \longrightarrow \varepsilon_{s`} = 1.711 * 10^{-3}$

$\therefore \varepsilon_{s`} < \varepsilon_y \longrightarrow F_{s`} < F_y \therefore$ The second assumption is wrong.

To Get the right value of $a, F_{s`}$

* **From equilibrium eqn.**

$$\frac{2}{3} F_{cu} * a * b + A_{s`} * F_{s`} = A_s * F_y$$

$$\frac{2}{3}(25)(a)(250) + (530.9)(F_{s`}) = (1608.5)(360)$$

$$F_{s`} = 1090.71 - 7.848\ a \qquad a, F_{s`} \text{ --------- } (1)$$

* **From compatibility eqn.**

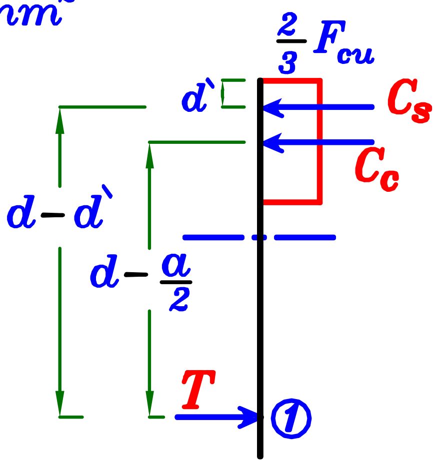
$$\frac{F_{s`}}{600} = \frac{1.25\,a - d`}{1.25\,a} \qquad a, F_{s`} \text{ --------- } (2)$$

From eqns. (1), (2)

$$\frac{(1090.71 - 7.848\,a)}{600} = \frac{1.25\,a - 50}{1.25\,a} \longrightarrow a = 94.78\ mm$$

$$, F_{s`} = 1090.71 - 7.848\,(94.78) = 346.87\ N\backslash mm^2$$

$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s`} F_{s`} (d - d`)$$



$$\therefore M_{ult} = \frac{2}{3}(25)(94.78)(250)\left(650 - \frac{94.78}{2}\right) + (530.9)(346.87)(650 - 50)$$

$$= 348472702\ N.mm = 348.47\ kN.m$$

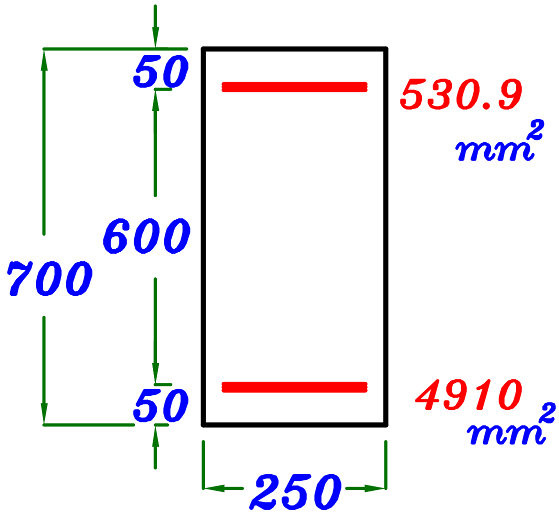
$$\therefore \boxed{M_{ult} = 348.47\ kN.m}$$

Example.

Data.

$F_{cu} = 25\ N\backslash mm^2$ st. 360/520

Req. Calculate $M_{ult.}$



Solution. $d = 650\ mm$, $d' = 50\ mm$

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 650 = 406.25\ mm$

② From equilibrium eqn. $C_c + C_s = T$

$\frac{2}{3} F_{cu} * (a * b) + A_{s'} * F_{s'} = A_s * F_s$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$ (Under or Balanced Sec.)

assume $\varepsilon_{s'} \geqslant \varepsilon_y \longrightarrow F_{s'} = F_y$

$\frac{2}{3}(25)(a)(250) + (530.9)(360) = (4910)(360)$

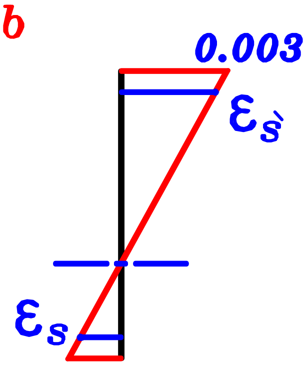
$\longrightarrow a = 378.35\ mm \longrightarrow C = 1.25\ a = 472.94\ mm > C_b$

$\therefore$ $\boxed{\text{The Section is Over Reinforced Sec.}}$

and the First assumption is wrong $F_s < F_y$

But the second assumption will be right $F_{s'} = F_y$

To Get the right value of a, F_s



$\frac{2}{3}(25)(a)(250) + (530.9)(360) = (4910)\ F_s$ $\quad a, F_s$ ------①

$1.25a = \dfrac{600}{600 + F_s} * 650$ ------------------ a, F_s ②

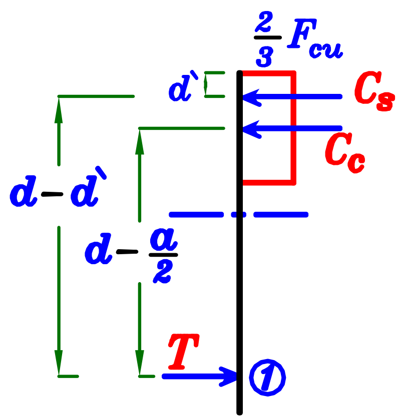
$\Big\}$ $a = 337.31\ mm$, $F_s = 324.96\ N\backslash mm^2$

$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s'} F_y (d - d')$

$= \frac{2}{3}(25)(337.31)(250)\left(650 - \frac{337.31}{2}\right) + (530.9)(360)(650 - 50)$

$= 791184741\ N.mm = 791.18\ kN.m$

$\therefore$ $\boxed{M_{ult} = 791.18\ kN.m}$



Example.

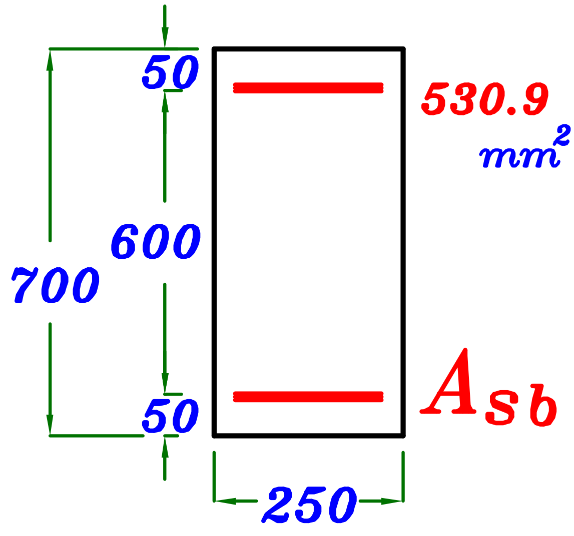
Data. $F_{cu} = 25 \ N\backslash mm^2$

st. $360/520$

Req. Calculate A_{sb}

To make the sec. is balanced Sec.

and then get M_b



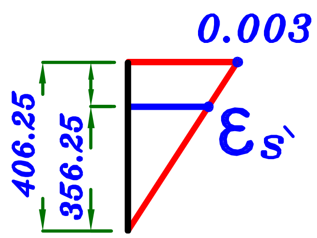
Solution.

For Balanced Sec. $C = C_b$, $a = a_b = 0.8\,C_b$, $F_s = F_y$

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 650 = 406.25 \ mm$

② $a = a_b = 0.8\,C_b = 0.8 * 406.25 = 325 \ mm$

③ Get $\varepsilon_y = \dfrac{F_y}{2*10^5} = \dfrac{360}{2*10^5} = 1.8 * 10^{-3}$



From $\dfrac{\varepsilon_{s`}}{0.003} = \dfrac{C - d`}{C} = \dfrac{356.25}{406.25} \longrightarrow \varepsilon_{s`} = 2.63 * 10^{-3}$

$\therefore \ \varepsilon_{s`} > \varepsilon_y \longrightarrow F_{s`} = F_y$

④ From equilibrium eqn. $C_c + C_s = T$

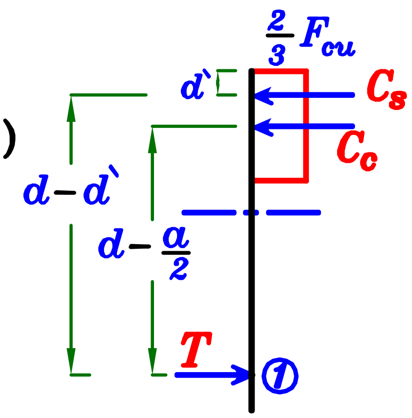
$$\frac{2}{3} F_{cu} * (a_b * b) + A_{s`} * F_y = A_{sb} * F_y$$

$$\frac{2}{3}(25)(325)(250) + (530.9)(360) = A_{sb}(360) \quad \therefore \ \boxed{A_{sb} = 4292.4 \ mm^2}$$

$$\therefore \ M_{ult} = \frac{2}{3} F_{cu}\, a_b\, b \left(d - \frac{a_b}{2}\right) + A_{s`} F_y (d - d`)$$

$$M_{ult} = \frac{2}{3}(25)(325)(250)\left(650 - \frac{325}{2}\right) + (530.9)(360)(650 - 50)$$

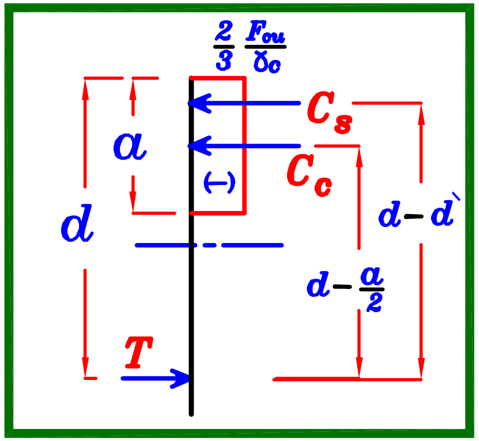
$$M_{ult} = 774830650 \ N.mm = 774.83 \ kN.m$$



$$\therefore \ \boxed{M_{ult} = 774.83 \ kN.m}$$

To Calculate $M_{U.L.}$ (With Ten. & Comp.Steel)

Calculate $a_{max} = \frac{2}{3} C_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600+(F_y \backslash \delta_s)}\right] * d$



From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\delta_c} * a * b + A_{s`} * F_{s`} = A_s * F_s$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = \frac{F_y}{\delta_s}$ (Under reinforced Sec.)

assume $\varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = \frac{F_y}{\delta_s}$ where $\varepsilon_y = \frac{(F_y \backslash \delta_s)}{2*10^6}$

$\therefore \frac{2}{3}\frac{F_{cu}}{\delta_c} * a * b + A_s * \frac{F_y}{\delta_s} = A_{s`} * \frac{F_y}{\delta_s} \longrightarrow$ Get a

IF $\boxed{a}$

IF $a \leqslant 0.1\,d$

take $a = 0.1\,d$, neglect $A_{s`}$

because $F_{s`}$ is very small.

$\therefore M_{U.L.} = A_s \frac{F_y}{\delta_s}\left(d - \frac{a}{2}\right)$

$\therefore M_{U.L.} = A_s \frac{F_y}{\delta_s}\left(d - \frac{0.1d}{2}\right)$

$\therefore M_{U.L.} = A_s F_y\, d\, \frac{1}{1.15}\left(1 - \frac{0.1}{2}\right)$

$\therefore \boxed{M_{U.L.} = 0.826\, A_s\, F_y\, d}$

$0.1\,d < a < a_{max.}$

The First assumption is right $F_s = \frac{F_y}{\delta_s}$

To check $F_{s`} = \frac{F_y}{\delta_s}$ From $\frac{\varepsilon_{s`}}{0.003} = \frac{c - d`}{c}$

$\boxed{\varepsilon_{s`}}$

IF $\varepsilon_{s`} \geqslant \varepsilon_y \quad \therefore F_{s`} = \left(\frac{F_y}{\delta_s}\right)$

$\boxed{M_{ult} = \frac{2}{3}\left(\frac{F_{cu}}{\delta_c}\right) a\, b \left(d - \frac{a}{2}\right) + A_{s`}\left(\frac{F_y}{\delta_s}\right)(d - d`)}$

IF $\varepsilon_{s`} < \varepsilon_y \quad \therefore F_{s`} < \left(\frac{F_y}{\delta_s}\right)$

To Get $a, F_{s`}$

$\frac{2}{3}\frac{F_{cu}}{\delta_c} * a * b + A_{s`} * F_{s`} = A_s * \frac{F_y}{\delta_s}$ $\quad a, F_{s`}$ ①

$\frac{F_{s`}}{600} = \frac{1.25\,a - d`}{1.25\,a}$ $\quad a, F_{s`}$ ②

$\therefore \boxed{M_{U.L.} = \frac{2}{3}\left(\frac{F_{cu}}{\delta_c}\right) a\, b \left(d - \frac{a}{2}\right) + A_{s`} F_{s`}(d - d`)}$

IF $a > a_{max.}$

Take $a = a_{max.}$, $F_{s`} = \frac{F_y}{\delta_s}$

$\therefore \boxed{M_{U.L.} = \frac{2}{3}\frac{F_{cu}}{\delta_c}\, a_{max.}\, b \left(d - \frac{a_{max.}}{2}\right) + A_{s`}\left(\frac{F_y}{\delta_s}\right)(d - d`)}$

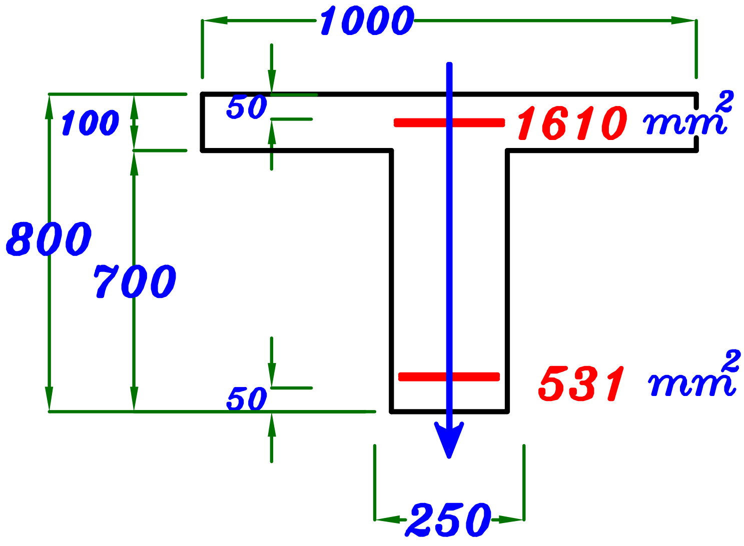
Example.

Data.

$F_{cu} = 25 \; N\backslash mm^2$

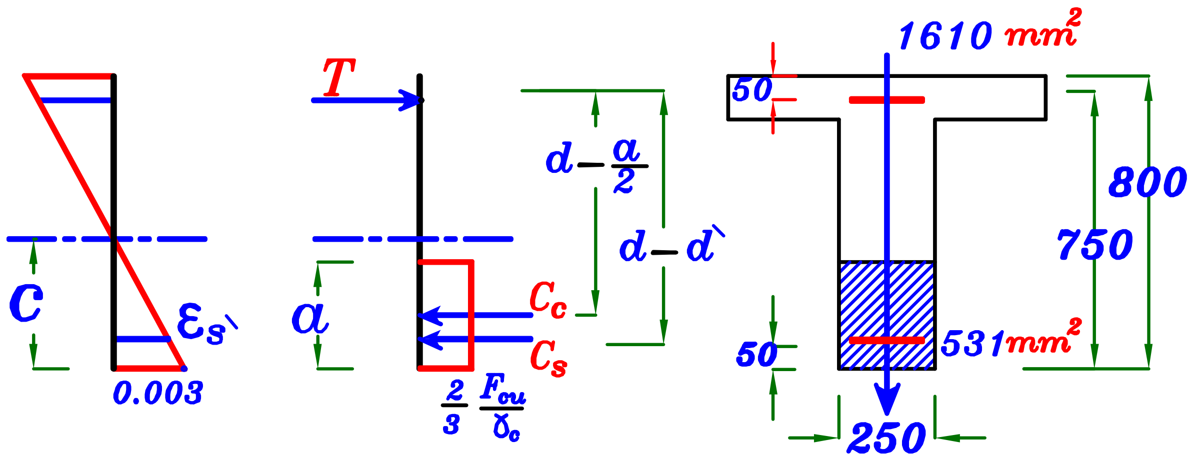
st. 360/520

Req. Calculate $M_{U.L.}$



Solution. $0.1\,d = 75$ mm

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y\backslash \delta_s)}\right] * d = 0.35\,d = 0.35 * 750 = 262.5 \text{ mm}$$



From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\delta_c} * a * b + A_{s`} * F_{s`} = A_s * F_s$

assume $\varepsilon_s > \varepsilon_y \longrightarrow F_s = \frac{F_y}{\delta_s}$ (Under reinforced Sec.)

assume $\varepsilon_{s`} > \varepsilon_y \longrightarrow F_{s`} = \frac{F_y}{\delta_s}$

$$\therefore \; \frac{2}{3}\frac{F_{cu}}{\delta_c} * a * b + A_{s`} * \frac{F_y}{\delta_s} = A_s * \frac{F_y}{\delta_s}$$

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)(250) + (531)\left(\frac{360}{1.15}\right) = (1610)\left(\frac{360}{1.15}\right)$$

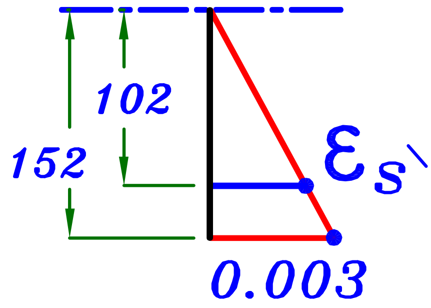
$$\longrightarrow a = 121.6 \text{ mm}$$

$\therefore\ 0.1\,d < a < a_{max.}$ **Right assumption** $F_s = \frac{F_y}{\gamma_s}$

To check IF $F_{s`} = \frac{F_y}{\gamma_s}$ **or not**

Get $\varepsilon_y = \frac{F_y/\gamma_s}{E_s} = \frac{360/1.15}{2*10^5} = 1.565*10^{-3}$

$C = 1.25\,a = 1.25 * 121.6 = 152$ mm



From $\frac{\varepsilon_{s`}}{0.003} = \frac{C-d`}{C}$

$\therefore\ \frac{\varepsilon_{s`}}{0.003} = \frac{102}{152} \longrightarrow \varepsilon_{s`} = 2.013*10^{-3}$

$\therefore\ \varepsilon_{s`} > \varepsilon_y \longrightarrow F_{s`} = \frac{F_y}{\gamma_s}$

$$\therefore\ M_{U.L.} = \frac{2}{3}\frac{F_{cu}}{\gamma_c}\,a\,b\left(d-\frac{a}{2}\right) + A_{s`}\frac{F_y}{\gamma_s}(d-d`)$$

$$\therefore\ M_{U.L.} = \frac{2}{3}\left(\frac{25}{1.5}\right)(121.6)(250)\left(750-\frac{121.6}{2}\right) + (531)\left(\frac{360}{1.15}\right)(750-50)$$

$$= 349154705 \text{ N.mm} = 349.15 \text{ kN.m}$$

$$\boxed{M_{U.L.} = 349.15 \text{ kN.m}}$$